Name: _____

CSE/MATH 455 Exam II November 21, 1997 Professor Arnold

Do 5 of the 6 problems and omit 1. State which problem you omit.
Work neatly and carefully. You must *show your work* to get credit.

1. Find the Cholesky decomposition of $\begin{pmatrix} 4 & -2 & 6 \\ -2 & 2 & -2 \\ 6 & -2 & 15 \end{pmatrix}$.

2. Find the Hermite cubic interpolant to $f(x) = \sin x$ at $x = -\pi/2$ and $x = \pi/2$.

3. Let A be a nonsingular $n \times n$ matrix, b an n-vector, x the exact solution to $Ax = b$, and x^* an approximate solution. Suppose that $\|A\| = 6$, $\|A^{-1}\| = 3$, $\|b\| = 10$, $\|x^*\| = 3$, and $\|Ax^* - b\| = .001$ (where the 1-norm is used for all norms). What is the largest possible value of the relative error $\|x - x^*\|/\|x\|$?

4. Write an algorithm which takes an upper triangular $n \times n$ matrix U and an n-vector x and overwrites x with the product $y = Ux$. How many multiplications does your algorithm require?

5. Let $x_0 < x_1 < \cdots < x_n$. Prove that if $p(x)$ is the Lagrange interpolating polynomial to a function $f(x)$ at the points $x_0, x_1, \ldots, x_{n-1}$ and $q(x)$ is the Lagrange interpolating polynomial to $f(x)$ at the points $x_1, x_2, \ldots, x_n$, then

$$r(x) = \frac{x_n - x}{x_n - x_0} p(x) + \frac{x - x_0}{x_n - x_0} q(x)$$

is the Lagrange interpolating polynomial to $f(x)$ at all the points $x_0, x_1, \ldots, x_n$.

6. Find the formula with which to replace ??? below so that the resulting function $s(x)$ is a natural cubic spline on the interval $[-3, 1]$ with knots at -3, -2, -1, 0, and 1.

$$s(x) = \begin{cases} 1 - 2x + 9x^2 + x^3, & x \in [-3, -2], \\ 9 + 10x + 15x^2 + 2x^3, & x \in [-2, -1], \\ 6 + x + 6x^2 - x^3, & x \in [-1, 0], \\ ???, & x \in [0, 1]. \end{cases}$$