Name: _____

CSE/MATH 455 Final Exam December 17, 1997 Professor Arnold

Do 8 of the 10 problems and omit 2. State which problems you omit.
Work neatly and carefully. You must *show your work* to get credit.

1. a) Complete this algorithm for evaluating a polynomial, using Horner's method.

input: integer n, real numbers a_0, a_1, ... , a_n, x
output: $p = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$

b) How many additions and multiplications does this algorithm require?

2. Find the LU decomposition of $\begin{pmatrix} 4 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 2 & 3 \end{pmatrix}$.

3. Suppose that $g(x)$ is a C^2 function defined on the real line which has a fixed point x^* for which $g'(x^*) = 0$. Assume also that $|g''(x)| \leq N$ for all real numbers x. Starting from some number x_0, define a sequence x_n by $x_{n+1} = g(x_n)$. Prove that $|e_{n+1}| \leq M|e_n|^2$, where $e_n = x_n - x^*$. Give an expression for M.

4. a) Write down the formulas for the secant method applied to the function $f(x) = \exp(\cos x) - x^3$, which has a unique real root x^*. (You don't need to simplify.)

b) If the starting values for the secant method are chosen sufficiently close to the root, what can said about $\lim_{n\to\infty} \left| \dfrac{x^* - x_{n+1}}{x^* - x_n} \right|$?

5. Find *both* Newton's form and Lagrange's form of the polynomial of lowest degree passing through the points $(-2, 9)$, $(-1, 0)$, $(1, 0)$, $(2, 15)$. (You don't need to simplify.)

6. If $e^{0.2}$ is estimated by interpolation from the values $e^0 = 1$, $e^{0.1} = 1.1052$, and $e^{0.3} = 1.3499$, is the approximate value greater than or less than $e^{0.2}$? Find (reasonably realistic) upper and lower bounds on the absolute value of the error. Justify your answer.

   greater than or less than? _____

   upper bound _____

   lower bound _____

7. a) Define what is meant by the condition number of a nonsingular matrix A.

b) Use the condition number to make a precise statement describing the sensitivity of solutions of the linear system $Ax = b$ to changes in b.

8. a) Write down the simple quadrature rule on $[0,1]$ that comes from linear interpolation of the integrand at two points: $x = 0$ and $x = c$ (with c any given point in $[0,1]$).

b) What value of c makes the degree of precision of this rule as high as possible? What is the degree of precision achieved?

c) For the best value of c determined in part b), write down the corresponding rule scaled to an arbitrary interval $[a,b]$.

9. a) Suppose that $f(x)$ is a smooth function on the real line. The difference between $f[0, h, 2h]$ and $\frac{1}{2}f''(0)$ will decrease like what power of h as h decreases to zero? Justify your answer (but you don't need to give a formal proof).

Answer: h to the power _____

b) Answer the same question except for the difference between $f[-h, 0, h]$ and $\frac{1}{2}f''(0)$.

Answer: h to the power _____

10. a) Write the simple trapezoidal rule approximation for $\int_0^h f(x)\,dx$.

b) Now divide the interval $[0, h]$ into 4 subintervals of length $h/4$ and write the composite trapezoidal rule approximation for this integral.

c) Show how to use Richardson extrapolation to combine the approximations of parts a) and b) to get a more accurate approximation.