CSE/MATH 455, Arnold Lab 3 September 26, 1997

Root finding routines

Preparation: For this lab you need to copy the following files from my directory into yours: `bisect.m`, `bisect0.m`, `newton.m`, `newton0.m`, `root.m`, `root0.m`. Use the Unix command "`cp ~dna2/*.m .`".

The Matlab routines bisect.m and newton.m are root-finding routines based on the bisection method and Newton's method. The routine `root.m` is a hybrid secant/bisection algorithm. These contain print statements that report on the algorithm as it progresses. The routines `bisect0.m`, `newton0.m`, and `root0.m` are the same except without the print statements.

You should also know about the routine `fzero` which comes with Matlab. It is like `root.m` but more sophisticated. For practical work, this is probably the one to use.

1. a) Plot the function $f(x) = x^3 - 2x + 2$ on the interval $[-2, 2]$ using `fplot('x^3-2*x+2',[-2,2])`. Notice the root near -1.75. Then run `newton('x^3-2*x+2','3*x^2-2',-1)` to find the root to machine precision using Newton's method starting from $x = -1$. Note that you have to supply the derivative. How many function evaluations did Newton's method need? _____

b) If you increase the starting guess from -1 to 0, does Newton's method still converge? _____ Roughly, how large can the starting guess be to have convergence? _____

c) What about decreasing the starting guess, e.g., down to -2. _____ How low can you go? _____

2. Solve the same equation using the bisection method starting with $[-2, 2]$ as the bracketing interval: `bisect('x^3-2*x+2',[-2,2])`. How many iterations? _____ Now doing it using root: `root('x^3-2*x+2',[-2,2])`. How many? _____

3. Now let's try some harder functions. For each one, first use `fplot` to plot the function and then use `root` to find the root. Indicate how many iterations. You may also want to see how Newton's method and/or the bisection method does.

a) This one has a long flat region: $f(x) = x\exp(-x^2) + .1$ on $[-1, 15]$
Iterations: _____

b) This one gets very large near the right end point: $f(x) = \tan(x) - 30x$ on $[1, 1.57]$.
Iterations: _____

c) This one has a discontinuity at 0: $f(x) = \text{sign}(x)(2 + \sin x^3)$ on $[-3, 3]$. What does `root` do?

d) This one is pretty hairy: $f(x) = x^3 - 2x + 2 + .1\sin 1000x$ on $[-2, 2]$.

e) This one is even nastier near the root: $f(x) = x^3 - 2x + .5\sin(1/x)$ on $[-2, 3]$.

4. Can you find any problems for which `root` needs more iterations than bisection? (Hint: try a multiple root.)

5. Can you find any problems for which `root` fails?

6. Can you find problems for which Matlab's `fzero` is noticeably better than `root`?