MATH 502 Second exam May 9, 1997 Professor Arnold

Do 3 problems, omit 1. State which problem you omit. Partial credit will be awarded, but errors and unacknowledged omissions count negatively.

1. Suppose that $x, x_1, x_2, \ldots$ are elements in a Hilbert space such that x_n converges to x weakly and $\|x_n\|$ converges to $\|x\|$. Prove that x_n converges to x in norm. Give a counterexample in Banach space.

2. Let X and Y be Banach spaces and set

$$U = \{\, T \in B(X,Y) \,|\, T^*(Y^*) = X^* \,\}.$$

Prove that U is open in $B(X,Y)$.

3. Prove that the sum of a closed subspace and a finite dimensional subspace of a Banach space is closed.

4. Prove that any compact subset of the complex plane is the spectrum of a bounded linear operator on a Banach algebra.