MATH 502, Spring '97 Homework 1 Arnold

§I.2: 2, 4 (Notation: this means problems 2 and 4 from the end of Chapter I, §2 of Conway.)

§I.4: 2, 3, 5, 7

§II.3: 4, 7

§II.6: 1

Let $SO(2,\mathbb{R})$ denote the set of 2×2 orthogonal matrices with determinant 1. (A matrix is orthogonal if it has orthonormal rows and orthonormal columns, or, equivalently, if its transpose equals its inverse. Such matrices all have determinant ± 1. Let C denote the set of all real 2×2 matrices of the form aQ with $a \in \mathbb{R}$, $Q \in SO(2,\mathbb{R})$.

a) Prove that C is a field.

b) Prove that C is isomorphic to $\mathbb{C}$.