§IV.5: 1, 5, 6, 8

§IV.6: 4, 7, 10, 11

(The next two problems are from the summer '90 qualifying exam.)

1. Let D be the open unit disc, γ the usual curve traversing its boundary, and suppose that f is an analytic function on $\mathbb{C} \setminus D$ satisfying $\lim_{z\to\infty} f(z) = 5$. Prove that

$$\frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w-z}\, dw = 5 - f(z), \qquad z \in \mathbb{C} \setminus D.$$

2. Let D be the open unit disc and let a and b be points in D. Give an explicit expression for a one-to-one analytic function f taking D onto itself with $f(a) = b$. Verify that f satisfies these conditions.