MATH 502, Spring '97 Homework 6 Arnold

§V.3: 9

§VI.1: 2, 7, 8

§VI.2: 3

§VII.1: 6

(The next three problems are from the spring '96 qualifying exam.)

1. Let ϕ be an entire function and suppose that

$$|\phi(z)| \leq 1 + \sqrt{|z|}$$

for every $z \in \mathbb{C}$. Prove that ϕ is constant.

2. Consider a function ϕ analytic in $D(0,1)$ and mapping into $D(0,1)$. Prove that if $\alpha \in D(0,1)$ and $\phi(\alpha) = 0$, then $|\alpha| \geq |\phi(0)|$.

3. Calculate $\displaystyle\int_{-\pi}^{\pi} \frac{\exp(e^{2it})(e^{it} - 1/2)}{1 - 1/(2e^{it})}\, dt$.