MATH 502, Spring '97 Homework 7 Arnold

§VII.4: 3

§X.2: 3

1. (From May 1996 qualifier) Prove that there is a function U, harmonic and bounded in $B(0,1)$ for which there is no function V harmonic and bounded in $B(0,1)$ so that $U+iV$ is holomorphic.

2. (From May 1993 qualifier) Show that a holomorphic map of a bounded, simply connected domain into itself which has two fixed points is the identity.

3. Let G be a region in the complex plane and u a real-valued continuous function on G. As we defined it in class, u is *subharmonic* if whenever $\bar{B}(a,r) \subset G$, then

$$u(a) \leq \frac{1}{2\pi} \int_0^{2\pi} u(a + re^{i\theta})\, d\theta.$$

An alternate definition is that u is subharmonic if whenever G_0 is a subregion of G and v is a harmonic function on G_0, then $u - v$ satisfies the maximum principle on G_0 (i.e., it has no maximum on G_0 unless it is constant). Prove the equivalence of the two definitions.

4. Suppose that u is C^2 on G. Prove that it is subharmonic if and only if $\Delta u \geq 0$ on G.