1. Prove that any infinite dimensional normed linear space admits a discontinuous linear functional.

2. Prove that a continuous linear operator between normed linear spaces is uniformly continuous.

3. Let l^∞ denote the set of sequences $\boldsymbol{x} = (x_n)_{n\in\mathbb{N}}$ with the supremum norm $\|\boldsymbol{x}\| = \sup_n |x_n|$. Prove that this is a Banach space. Prove that it is not separable, i.e., that it does not have a countable dense subset.

4. Let $c_0 \subset l^\infty$ denote the set of sequences which tend to zero. Prove that c_0 is a separable Banach space.

5. Prove that a normed linear space is complete if and only if $\sum_{n=1}^\infty x_n$ exists in the space whenever $\sum_{n=1}^\infty \|x_n\| < \infty$.

6. Let X be a normed linear space and S a proper closed subspace. Prove that there exist $x_n \in X$ with $\|x_n\| = 1$ and $\lim_{n\to\infty} \operatorname{dist}(x_n, S) = 1$.

7. Prove that a Banach space is finite dimensional if and only if the closed unit ball is compact.