MATH 502 Midterm March 7, 1997 Professor Arnold

Do 3 problems, omit 1. State which problem you omit. Partial credit will be awarded, but errors and unacknowledged omissions count negatively.

1. Prove or provide a counterexample (with justification): if f is analytic on $B(0,1)$ and continuous on its closure, then f extends to an analytic function on $B(0,R)$ for some $R > 1$.

2. (a) Suppose that f is analytic and nowhere zero on some bounded region of the complex plane, continuous on its closure, and of constant modulus on its boundary. Prove that f is constant on the region.

(b) Can the hypothesis that f is nowhere zero be dropped?

(c) Can we replace the hypothesis that the region is bounded by that its complement is unbounded?

3. How many roots (counting multiplicity) does the function $p(z) = z^6 + z^3 + 5z^2 - 2$ have in the annulus $1 < |z| < 2$?

4. Find $\int_0^{2\pi} \sin(\exp e^{i\theta})\, d\theta$.