In preparation for this assignment study the program poisson_convergence1.py and make sure you understand how it works.

Make your own program which, for any given values of n and r, solves the Poisson equation on the unit square using a mesh of size n (i.e., the mesh produced by "UnitSquareMesh(n, n)") and Lagrange elements of degree r. Your program should output the following quantities:

n	r	# elts	# DOFs	H^1 err	% err

Here "H^1 err" refers to the H^1 seminorm of the error, and "% err" to the same quantity expressed as a percentage of the H^1 seminorm of the exact solution.

As a test case take as the exact solution $u(x,y) = \sin \pi x \sin 2\pi y$, and compare the errors you get to those produced by poisson_convergence1.py.

Once you are sure your program is working, switch the exact solution to be $u(x,y) = \sin \pi x \sin 10\pi y$. Using $r = 1$, find a value of n such that the H^1 seminorm error is about 1%, say between 0.9% and 1.1%. Start a table with the columns labelled as shown above and fill out the first row based on this value of n and r. Now switch to $r = 2$ and again find what value of n is needed to achieve a 1% error. Then do $r = 3$ and $r = 4$. In this way add three more rows to the table

Consider the results. Submit the table, a few sentences summarizing the results, and your program (by email).

A tip: Knowing n and r you can figure out the number of cells and the number of DOFs. But you can also get them from FEniCS. If the mesh is named "mesh", then you can use "mesh.num_cells()" to get the number of elements. If the finite element space is named "Vh", then "len(Vh.dofmap().dofs())" is the number of DOFs.