1. Consider the *box scheme* for the advection equation $\partial u/\partial t + \partial u/\partial x = 0$ on the interval $[0, 2\pi]$ with periodic conditions (so solutions are 2π-periodic functions on the whole real line), using mesh size $h = 2\pi/N$ and a timestep $k > 0$:

$$\frac{(U_n^{j+1} + U_{n+1}^{j+1}) - (U_n^j + U_{n+1}^j)}{2k} + \frac{(U_{n+1}^{j+1} + U_{n+1}^j) - (U_n^{j+1} + U_n^j)}{2h} = 0.$$

a) Draw the stencil diagram for the method (analogous the diagram on the left of Figure 5.2 of the notes for the downwind difference scheme).

b) Is the method implicit or explicit?

c) Show that the truncation error is $O(k^2 + h^2)$. (Hint: expand around the center of a box, rather than around a mesh point.)

d) Compute the amplification matrix G (for which $U^{j+1} = GU^j$, with U^j the vector of mesh values at time jk). State it in terms of the shift matrices K_+ and K_-.

e) Find the eigenvalues of G and determine whether the method is conditionally stable (in which case give the condition), unconditionally stable, or unconditionally unstable.

f) Can the same method be used to compute a wave travelling to the left?

g) Run the box scheme by adding a function definition `unew = bs(u)` to the program advection1d.py. Turn in the function definition and the plot of the advected profile at the final time $T = 2$.

h) Optional, extra credit. Precisely state and prove a second-order convergence result for the box scheme in the discrete L^2 norm.