Year	*Population*
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298
1980	226,540,825
1990	248,709,873

Theorem. Given $n+1$ distinct numbers x_0, x_1, ... , x_n, and $n+1$ numbers y_0, y_1, ... , y_n, there exists a unique polynomial $p_n(x)$ of degree $\leq n$ such that

$$p_n(x_i) = y_i, \quad i = 0, 1, \ldots, n.$$

Theorem (Runge). Let $f(x) = \dfrac{1}{1+x^2}$ and let $p_n(x)$ be the polynomial of degree $\leq n$ interpolating $f(x)$ at $n+1$ equally space points from -5 to 5. Then

$$\lim_{n\to\infty} p_n(x) = f(x)$$

if and only if $|x| < 3.63\ldots$.

Theorem (Weierstrass 1885). Let $f(x)$ be any continuous function on a closed bounded interval $[a, b]$. Then there exist polynomials $p_n(x)$ of degree $\leq n$ such that

$$\lim_{n\to\infty} p_n(x) = f(x)$$

uniformly for $x \in [a, b]$.

Theorem. If $f(x)$ is C^{n+1} and $p_n(x)$ is the polynomial of degree $\leq n$ interpolating $f(x)$ at $x_0,\ x_1,\ \ldots,\ x_n$, then

$$f(x)-p_n(x) = \frac{1}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)f^{(n)}(\xi)$$

for some ξ.

(Follows from repeated application the mean value theorem.)

Pafnuty Lvovitch Chebyshev (1821–1894)

Pafnutii L'vovitsch Tschebysheff
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Theorem (Bernstein 1918). Equidistant interpolation to $f(x) = |x|$ on $[-1, 1]$ does not converge at any point except $x = -1$, 0, and 1.

Theorem (Bernstein 1916). For any Hölder continuous function, Chebyshev interpolation converges uniformly.

For functions with continuous derivatives of order up to p, Chebyshev interpolation converges with order almost $O(1/n^p)$.

Theorem (Faber 1914). Given a triangular array of points

$$\begin{array}{ccccc} & & x_0^0 & & \\ & x_0^1 & & x_1^1 & \\ x_0^2 & & x_1^2 & & x_3^2 \\ & & \vdots & & \end{array}$$

in $[a, b]$ and a continuous function $f(x)$ on $[a, b]$, let $p_n(x)$ be the polynomial of degree $\leq n$ with interpolates f at the $n+1$ points x_0^n, x_1^n, ... , x_n^n. Then no matter how the points are chosen, there exits a continuous function f for which the p_n do *not* converge uniformly to f.

Theorem (Bernstein 1931). For any such triangular array of points there is a continuous function f and a point $c \in [a, b]$ such that $p_n(c)$ does not converge to $f(c)$.

Theorem (Erdös and Vértesi, 1980). For any such triangular array there is a continuous function for which $p_n(c)$ does not converge to $f(c)$ for almost any $c \in [a, b]$.

Theorem (Marcinkiewicz, Grünewald 1937). For Chebyshev interpolation there is a continuous function for which the interpolating polynomials do not converge at *any* point.