Newsjournal of the Society for Industrial and Applied Mathematics

siam news

Volume 40/ Number 10
December 2007

Applied Math Enters the Digital Age

By Barry A. Cipra

Cry, baby, cry
Put your finger in your eye
—*Mother Goose*

You've got to hand it to Francisco Valero-Cuevas. Not many grad students have the nerve to give their advisers the finger. In 1997, when Valero-Cuevas did so, it got him a PhD from Stanford. Now a professor of biomedical engineering at the University of Southern California, he described his extended applied mathematical studies of the human index finger in an invited address at the 2007 SIAM Conference on Applications of Dynamical Systems.

The human hand is a fascinating neuromuscular contrivance. Its ability to coordinate multiple degrees of freedom and act with simultaneous delicacy and force is the envy of roboticists. Structurally, each finger is a set of three hinged rods, connected by a complicated skein of taut, gristly strings to seven meaty actuators in the palm and forearm. (The only muscles in the fingers themselves are the ones responsible for raising goosebumps on the knuckles. Otherwise, finger flesh consists of a sack of skin housing fat and a mesh of capillaries, with a network of touch- and temperature-sensitive neurons on one side and a continuously extruding keratin plate on the other.) But the precise layout of tendons and their connections to the muscles is unlike anything a mechanical engineer is likely to concoct. Indeed, aspects of the tendon network have puzzled students of anatomy since the days of Leonardo da Vinci and the 16th-century anatomist Vesalius.

Valero-Cuevas and colleagues are beginning to get a grip on some of these anatomical mysteries. Combining laboratory experiments on human hands (some living, some cadaveric), mathematical analysis, and computer simulations, the researchers have shown that the intricate arrangement of muscle and tendon is key to human dexterity, the ability to grasp and manipulate objects by dynamically regulating force in any direction. Their research should help roboticists design more versatile mechanisms. It could also help surgeons restore more functionality as they repair injured hands; therapists could make use of it in developing and implementing rehabilitation regimens, and neurologists in understanding how disease, aging, stroke, and trauma impair hand function. Valero-Cuevas is looking for applied mathematics students to join his research efforts.

Manipulating a Hypercube

A cartoon model of a joint has a single muscle flexing the joint via a one-dimen-

See **Finger Dynamics** *on page 4*

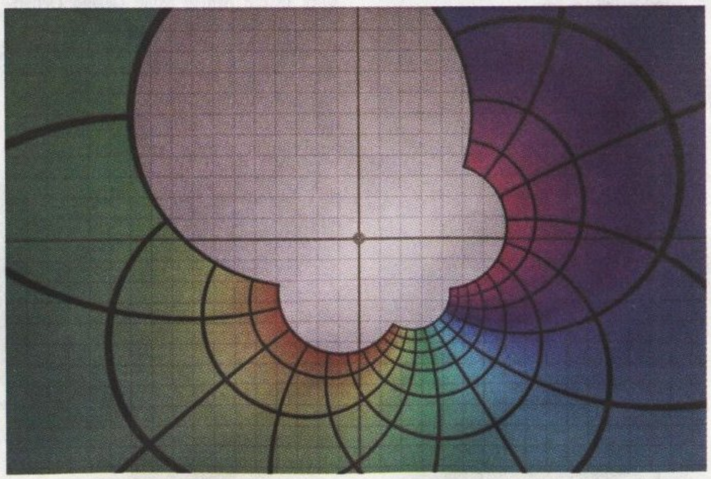
Prize-winning Video Brings Möbius Transformations to Life

By Michelle Sipics

Each year since 2003, *Science* magazine and the National Science Foundation have sponsored a contest, challenging researchers to find new ways of visualizing scientific data. As their motivation, the sponsors cite the role that "bringing data to life visually" can play in promoting both better understanding of scientific concepts in the public and communication between scientists working in different disciplines.

As in previous years, math was in the spotlight in this year's competition, thanks in part to the contribution of mathematicians Douglas Arnold and Jonathan Rogness of the University of Minnesota. Their entry, "Möbius Transformations Revealed," tied for an honorable mention in the competi-

See **Visualization** *on page 16*



The image of the unit square under a Möbius transformation. The transformation maps the square conformally onto an exterior region whose boundary consists of four circular arcs, the images of the edges of the square.

Visualization

continued from page 1

tion's noninteractive media category. The short video, set to Schumann's *Kinderscenen*, shows animated Möbius transformations seen from both a two- and a three-dimensional viewpoint.

(In 2006, awards to mathematicians and their entries ranged from honorable mentions to first in their respective categories. See "The Human Heart Laid Bare" and "Visualizing Mathematical Surfaces," *SIAM News*, November 2006; http://www.siam.org/news/.)

Arnold and Rogness's video should prove useful to mathematics students everywhere. As Arnold points out, "Möbius transformations are the simplest non-polynomial maps of the complex plane, and an excellent source of conformal maps. . . . They are studied in virtually any class on complex analysis." But what encouraged Arnold and Rogness to create the video in the first place?

Appropriately, it was a complex journey.

"I had produced a much simpler animation of a Möbius transformation for a class I gave back in 1997," explains Arnold, who has been director of the Institute for Mathematics and its Applications since 2001. A Canadian film director named Jean Bergeron, who was making a documentary on the mathematics in the work of the Dutch artist M.C. Escher, found Arnold's ten-year-old animation on the Web.

This January, Bergeron asked if he could use those old animations, which would have to be reproduced at higher resolution, Arnold recalls. "I did that, but it got the idea of depicting Möbius transformations through animation back in my mind."

Enter Jonathan Rogness, who has expertise in developing mathematical applets for educational use. An assistant professor of mathematics, he also works in the university's Institute of Technology Center for Educational Programs.

"I like to use applets when I think it can help students understand or visualize a tricky concept," Rogness says. "When using vector fields to talk about fluid flow, for example, we'll use an applet which lets us watch the particles move around, analyze their trajectories, and so on."

Always on the lookout for nice visualization projects, Rogness says that Arnold's idea appealed to him for two reasons.

First, "Möbius transformations result in some very striking images, and I thought it could be a way to convey the beauty of mathematics to non-mathematicians," he explains. "Second, I felt this could help students get a better understanding of Möbius transformations. Textbooks might show a picture of a square turned inside-out by a Möbius transformation, but it can be hard for students to understand how the lines in the original square morphed into the resulting picture.

"Sometimes as mathematicians we forget that things which seem clear or obvious to us are not at all clear to non-professionals," Rogness adds.

With the animations of Möbius transformations, the video depicts a theorem that Arnold says he learned only recently: "All Möbius transformations can be obtained through inverse stereographic projection, followed by a rigid motion of the sphere of projection, followed by forward stereographic projection back to the plane." If you had to read that through a few times, you'll appreciate the video. A single viewing suffices to explain why the judges of the 2007 Science and Engineering Visualization Challenge chose to honor it : Just a few minutes of video beautifully illustrate the 29-word theorem.

The video's reach extends beyond *Science* subscribers: It has also been viewed more than 60,000 times on YouTube as this issue of *SIAM News* goes to press. What's more, a portion of "Möbius Transformations Revealed" appears in Bergeron's recently released film.

At the end of May, Arnold sent the just completed animation for "Möbius Transformations Revealed" to Bergeron. Bergeron replied, "Wow! Very interesting indeed . . . [but] I am in my last days of final edit." An hour later, he wrote again: "Your rendering is truly beautiful, as compared to any previous versions. Ah! I wonder if I could take the time to re-visit the final edit."

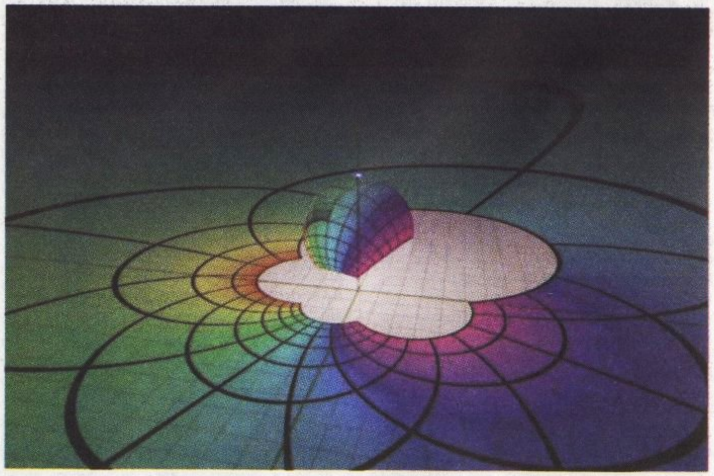
At that point, Bergeron decided that he would use a portion of "Möbius Transformations Revealed" in his film, rather than the re-rendered animations from 1997—but only if Arnold and Rogness could re-render their video in HD within 24 hours.

"That meant producing about 500 frames, for a total of about 3GB of data," Arnold says. He and Rogness offered their terms.

"Yes, we would do it, but our payment was that he would hold the U.S. premiere at the IMA. He agreed, and his film, titled *Achieving the Unachievable*, was shown for the first time in the U.S. on November 1, at the IMA, with Bergeron present for a Q&A at the end."

That's how some ten-year-old animations for a mathematics course turned into an award-winning video, which then made its way into a documentary on the solution of a math problem involving the work of M.C. Escher.

Michelle Sipics is a contributing editor at SIAM News.



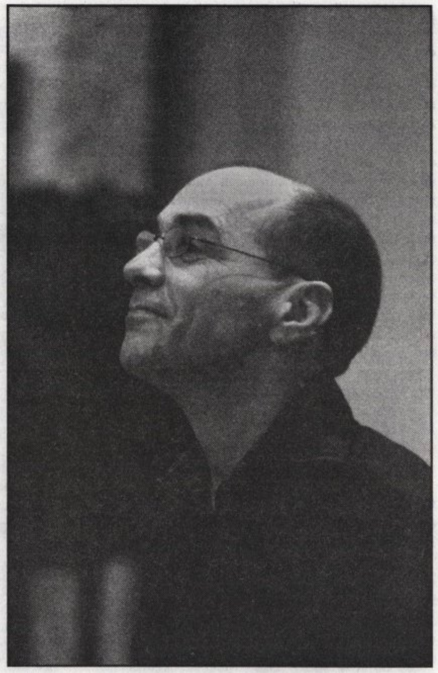
When viewed in three dimensions, the image of the square under the Möbius transformation can be obtained by stereographic projection from a sphere that has been rotated and translated after the square has been projected onto it. Video frames courtesy of Doug Arnold and Jonathan Rogness.

Film Bridges Artistic and Mathematical Intuitions

On Thursday evening, November 1, 2007, the Institute for Mathematics and its Applications at the University of Minnesota hosted the U.S. premiere of the film *Achieving the Unachievable* by the Canadian filmmaker Jean Bergeron. The film—which runs a little under an hour—is devoted to the work of the well-known Dutch artist Maurits Cornelis Escher (1898–1972), in particular his unfinished work "Print Gallery." The art world has long been puzzled by the blurry white circular patch in the center of this lithograph. There has been considerable speculation about why the artist left his masterpiece unfinished in this way and whether it could be completed.

About five years ago, a team assembled by Hendrik Lenstra, a mathematician at the University of Leiden in the Netherlands, discovered that an infinite sequence of ever-decreasing copies of the outer portions of the painting, subject to a subtle transformation, which they deduced, fits perfectly in the blank disc. (See "M.C. Escher: More Mathematics Than Meets the Eye," by Sara Robinson; http://www.siam.org/news/news.php?id=474.) Interviews with Lenstra shown in the film suggest a fantastic bridge that grew between the intuitions of the artist and the mathematician. The film also contains interesting interviews with other artists and mathematicians, including Douglas Hofstadter of the University of Indiana (author of *Gödel, Escher, Bach*), who had conjectured that the completion of "Print Gallery" ultimately obtained by Lenstra would be impossible.

The film illustrates many of the mathematical structures that enter into Escher's work, including Möbius transformations—which, as described in the accompanying article, have also been a recent interest of IMA director Doug Arnold. He and Jonathan Rogness, an assistant professor of mathematics at the University of Minnesota, developed animations of Möbius transformations that not only won a prize in this year's *Science* visualization contest but were also incorporated into Bergeron's film. The film was received enthusiastically by the audience of around seven hundred people, including many high school students as well as members of the university and surrounding community. Jean Bergeron, who wrote and directed the film, attended the premiere, after which he took questions from the audience.—*Donald Kahn, University of Minnesota.*



Canadian filmmaker Jean Bergeron in Minneapolis, at the U.S. premiere of his film Achieving the Unachievable. *Photo by Donald Kahn.*