Homework #4. FM 5012 (Due Thursday April 24)

Consider the following model for European call options:

$$(E)\quad \begin{cases} \dfrac{\partial f}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2}{\partial S^2} f + rS\dfrac{\partial f}{\partial S} = rf & \text{in } (0,S_M)\times(0,T), \\ f(S,T) = \max\{S-K,0\} & \forall S \in (0,S_M), \\ f(0,t) = 0 & \forall t \in (0,T), \\ f(S_M,t) = S_M - K e^{-r(T-t)} & \forall t \in (0,T). \end{cases}$$

Take $K=1$, $S_M=2$, $r=0.1$ and $\sigma=0.2$. Find the value of the option at time $t=0$.

(8pts) ① Do this by transforming our problem to a problem involving a heat equation for the function "F" (as in page 3 of the notes on finite differences), solving it numerically with the Crank-Nicolson scheme, and then finding "f" from the values of "F".

(Note that, as indicated in page 2, we have to carry out the change of variable $S = e^x$, which maps the interval $(0,S_M)$ into $(-\infty, \ln S_M)$. Since we cannot use finite differences in such an interval, we must replace it by an interval of the form $(-x_0, \ln S_M)$. Accordingly, the boundary condition at $-x_0$ would then be $F(-x_0,t)=0$ $\forall t \in (0,T)$. You would have to choose x_0 so that your approximation to $f(\cdot, t=0)$ is sufficiently accurate. If $|x_0|$ is too small, the approximation might not be good enough and if $|x_0|$ is very big you might be wasting computational resources. Carry out several experiments to conclude with confidence that your choice of x_0 is sensible.)

(16pts) ② Devise a finite difference scheme for the model (E) for European call options <u>without transforming</u> it into a heat equation. Find an approximation to the value of the option at time $t=0$. Compare this approximation with the one obtained in the previous exercise.