Differential Equations with Applications
MATH 4512

Instructor: Bernardo Cockburn. Office: Vincent Hall 327
Fall 2014 schedule: MWF 10:10-11:00, Vincent Hall 1

• **Objective**: In this course, we study several basic aspects of differential equations modeling phenomena of interest in physics and engineering.

• **Textbook**: *Differential Equations*, by P. Blanchard, R.L. Devaney and G.R. Hall, Brooks/Cole, New York, 2012 (4th. edition). We will cover the first seven chapters.

• **Grade**: 40% homeworks (due on Sept. 22, Oct. 6, Oct. 27, Nov. 17, and Dec. 8), 30% midterms (on Sep. 29 10%, Oct. 31 10% and Nov. 24 10%, at class time) and 30% final (on Dec. 17).

Office hours. By appointment **only**. To get one, just send me an e-mail to cockburn@math.umn.edu.

Homeworks.
1. Problems # 6, # 7, # 8 of §1.1; # 19 of §1.2; # 8 of §1.3; # 12 of §1.4.
2. Problems # 15, # 17 of §2.1; # 21, # 23 of §2.2; # 10 of §2.3.
3. Problems # 24, 26, 28, 30, 31 and 32 of "Review Exercises for Chapter 3".
4. Problems #18, 19, 21 of §4.3, and # 21 of "Review Exercises for Chapter 4".

5 Problems

Material to be covered on the corresponding week.
(1) Sept. 3: Population growth and free fall models. (§1.1, 1.2)
(2) Sept. 8: Predator-prey, circuit models. (§1.3, 1.4)
(3) Sept. 15: Existence and uniqueness. (§1.5, 1.6)
(4) Sept. 22: Bifurcation, linear equation. (§1.7, 1.8)
(5) Sept. 29: Equilibrium points of linear 2x2 systems.(§3.1, 3.2, 3.3)
(6) Oct. 6: Damped harmonic oscillator. (§3.4, 3.5)
(7) Oct. 13: Second-order equations, eigenvalues and eigenvectors (§3.6)
(8) Oct. 20: Bifurcation and third-order equations (§3.7, 3.8)
(9) Oct. 27: Forced oscillations (§4.1, 4.2, 4.3)
(10) Nov. 3: Forced oscillations (§4.4, 4.5)
(11) Nov. 10: Nonlinear systems (§5.1, 5.2)
(12) Nov. 17: Nonlinear systems (§5.3, 5.4)
(13) Nov. 24: Nonlinear systems (§5.5, 5.6)
(14) Dec. 1: Laplace transforms (§6.1, 6.2, 6.3)
(15) Dec. 8: Laplace transforms (§6.4, 6.5, 6.6)

First MATH 4512 mid-term exam: Sep. 29, 2014
(Each problem is 10 points)

(1) Find the solution of $\frac{dy}{dt} = y + 1/2, y(t_0) = 2$.

(2) Obtain the phase diagram of the equation $\frac{dy}{dt} = 1 + \cos(y)$. Which equilibrium points are sources, sinks or nodes?

(3) Sketch various solutions of the equation $\frac{dy}{dt} = (y + 1/2)(y + t)$. What is the behavior as t increases of the solution passing through $1/2$ at $t = 0$?

(4) Draw the bifurcation diagram for the equation $\frac{dy}{dt} = r + 4y - y^3$. Find the bifurcation points, if any.

(5) State the uniqueness theorem for the problem $\frac{dy}{dt} = f(t, y), y(t_0) = y_0$. Give an example for which this problem has two solutions and explain why this does not contradict the uniqueness theorem just stated.