Math 4512 HW 5 Solution

3.3 5

(a)

$$c_1\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}+c_2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+c_3\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}+c_4\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{cases} c_1+c_2=0\\ c_3+c_4=0\\ c_1+c_3=0\\ c_2+c_4=0\end{cases}$$

It is easy to observe that $c_1=c_4=1$ $c_2=c_3=-1$ are nontrival solutions. Thus, it is linearly dependent.

(b)

$$c_1\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}+c_2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+c_3\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}+c_4\begin{bmatrix}2 & -2\\ -1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{cases} c_1+c_2+2c_4=0\\ c_3-2c_4=0\\ c_1+c_3-c_4=0\\ c_2+c_4=0\end{cases}$$

$> c_1+c_2+c_3=0$

$> c_1+c_2+c_3=0$

$\Rightarrow$ There are arbitrary many solutions. $\Rightarrow$ linearly dependent.

7.

(a)

Notice that $d^2y/dt^2 - y = 0$ is linear. And the axioms to justify a vector space $V$ implicate that if $y_1$ and $y_2$ are in $V$, then linear combination $ay_1+by_2$ is again in $V$ for any choice of constants a and b. So, $V$ is a vector space in this problem clearly.

(b)

$$d^2y/dt^2 - y = 0$$

The characteristic equation is $r^2 - 1 = 0 \Rightarrow r = \pm 1$

Thus, the two solutions are $y_1 = e^t$, $y_2 = e^{-t}$

Clearly, $y_1$ and $y_2$ are linearly independent. So $y_1, y_2$ form the basis.

10.

Let's check $x^1(t)$, $x^2(t)$, $x^3(t)$ are solutions of the ODE, which is in V. This is clear.

Then we need to show they are linearly independent.

Notice that $e^t$, $e^{2t}$, $e^{3t}$ are linearly independent, then $x^1(t)$, $x^2(t)$ and $x^3(t)$ are also linearly independent clearly. So they form a basis for V.

3.6 10.

$$A = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \quad \det A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 1 \neq 0$$

The inverse of A exists.

$$A^{-1} = \frac{\operatorname{adj} A}{\det A} = \operatorname{adj} A = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$