Numerical Analysis and Scientific Computing
MATH 8441-2

Instructor: Bernardo Cockburn. Office: Vincent Hall 327
Fall 2017 schedule: MWF 11:15-12:05, Vincent Hall 207

• **Objective**: In this course, we study the basic mathematical principles for the devising and analysis of numerical methods for approximating functions, and for solving ordinary and partial differential equations. The emphasis of the course will be on the link between the mathematical analysis of the methods and their scientific computing aspects.

• **Textbook**: Reading material not included in the references below will be provided by the instructor.

• **References**:

(1) W. Gautschi, *Numerical Analisis. An Introduction*;

(2) B. Gustafsson, H.-O. Kreiss, and J. Oliger, *Time Dependent Problems and Difference Methods*;

(3) A. Iserles, *A first course in the numerical analysis of differential equations*;

(4) C. Johnson, *Numerical Solutions of PDEs by the Finite Element Method*;

(5) R. Plato, *Concise Numerical Mathematics*;

(6) M.J.D. Powell, *Approximation theory and methods*;

(7) W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, *Numerical Recipes, The Art of Scientific Computing*;

(8) A. Quarteroni, R. Sacco and F. Salieri, *Numerical Mathematics*;

(9) R.D. Richtmyer and K.W. Morton, *Difference Methods for Initial-Value Problems*;

(10) T.J. Rivlin, *An introduction to the approximation of functions*;

(11) Y. Saad, *Iterative Methods for Sparse Linear Systems*;

(12) J.C. Strikwerda, *Finite Difference Schemes and Partial Differential Equations*;

(13) J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis.*

(14) H.F. Weinberger, *A first course in partial differential equations with complex variables and transform methods.*

• **Grade**: Average of homeworks (one homework per week). The codes for the homeworks can be written in any language and do *not* need to displayed as part of the homework.

Office hours. By appointment **only**. To get one, just send me an e-mail to cockburn@math.umn.edu.

<u>Fall Program</u>

Approximation of functions

1. Approximation in finite dimensional spaces
   1. The best approximation
   2. Other approximation operators
   3. Approximability by density

2. The L^2-projection
   1. Definition and general properties
   2. Sturm-Liouville problems
   3. Trigonometric polynomials: Fourier series
   4. Orthogonal polynomials: Chebyshev series
   5. Orthogonal polynomials: Legendre series
   6. Finite element spaces
   7. Spectral decomposition of compact, self-adjoint operators

3. Interpolation
   1. Definition
   2. By trigonometric polynomials
   3. By Chebyshev polynomials
   4. Splines

4. Filters
   1. For trigonometric approximations
   2. For finite element approximations

Approximation of ordinary differential equations

1. Introduction
   1. Basic properties of the exact solutions
   2. A priori and a posteriori error estimates
   3. Finite difference versus finite element methods
   4. Residuals and truncation errors

2. One-step finite difference methods
   1. Definition and general properties
   2. Main Examples
   3. Stability and convergence analysis
   4. Adaptive step control
   5. Stiff problems

3. One-step finite element methods
   1. The continuous Galerkin method
   2. The discontinuous Galerkin method
   3. Stability and convergence analysis
   4. Adaptive step control
   5. Stiff problems

(4) Multi-step finite difference methods
    (1) Definition and general properties
    (2) Main Examples
    (3) Stability and convergence analysis
    (4) Stiff problems

Spring Program

Approximation of second-order elliptic equations

(1) Introduction
    (1) Basic properties of the exact solutions
    (2) A priori and a posteriori error estimates
    (3) Finite difference versus finite element methods
    (4) Residuals and truncation errors

(2) Finite difference methods
    (1) Definition and general properties
    (2) The main Examples
    (3) Stability and convergence analysis

(3) Iterative methods for solving matrix equations
    (1) The classical methods
    (2) The steepest descent method
    (3) The Conjugate Gradients method
    (4) The generalized minimal residual method
    (5) Preconditioning

(4) Finite element methods
    (1) The continuous Galerkin method
    (2) Mixed, discontinuous Galerkin and hybrid type methods
    (3) Residuals and stabilization properties
    (4) Implementation by static condensation
    (5) Stability and convergence analysis
    (6) Superconvergence and postprocessing
    (7) A posteriori error estimation

(5) Extensions
    (1) The heat equation
    (2) The Stokes equations of fluid flow
    (3) The equations of linear elasticity

Approximation of the advection equation

(1) Finite difference methods
    (1) Definition of the methods
    (2) The discrete Fourier transform

   (3) Dissipation and dispersion
   (4) Stability and error analysis
   (5) The model equation

(2) Discontinuous Galerkin methods
   (1) Definition of the methods
   (2) A discrete Fourier transform
   (3) Dissipation and dispersion
   (4) Stability and error analysis
   (5) Properties in general meshes
   (6) A priori and a posteriori error analysis

(3) Extensions
   (1) Wave propagation
   (2) Friedrich systems
   (3) Convection-diffusion equations
   (4) The Oseen equations of fluid flow
   (5) The incompressible Navier-Stokes equations

Nonlinear conservation laws

(1) Scalar hyperbolic conservation laws
   (1) The weak solutions
   (2) The loss of uniqueness
   (3) Traveling waves and the Riemman problem
   (4) The vanishing viscosity (entropy) limit
   (5) The viscosity-dispersion limit
   (6) The Sobolev regularization limit

(2) Monotone finite volume methods
   (1) Definition
   (2) Monotonicity properties
   (3) Accuracy and stability analysis
   (4) Convergence to the entropy solution

(3) The discontinuous Galerkin method
   (1) Definition
   (2) Stabilization and properties
   (3) Accuracy and stability analysis
   (4) Convergence to the entropy solution

(4) Extensions
   (1) Nonlinear hyperbolic systems
   (2) Euler equations of gas dynamics
   (3) The compressible Navier-Stokes equations