

Homework #4: Rates of convergence II. Due on Friday, October 12, 2018.

1. (5 points) Find conditions on $\varphi \in L^2(0, \pi)$ for which we can write

$$\|T^{-m/2}\varphi\|_{L^2(0,\pi)} = \left\| \frac{d^m}{dx^m}\varphi \right\| \quad m = 1, 2, \dots$$

(Here T is the compact operator of the example considered in class).

2. (5 points) Set $v(x) := 1 - \cos(2x)$. Apply our error estimates with $s = 3/2$ to obtain a rate of convergence of $\|v - \pi_n v\|_{L^2(0,\pi)}$ where π_n is the L^2 -orthogonal projection into the space

$$W_n := \text{span}\{\sin(ix), i = 1, \dots, n\}.$$

3. (5 points) Repeat the above exercise but this time with $s = 3$. Do you obtain a different estimate of the rate of convergence?

4. (5 points) Fill the history of convergence table below. Do the results match the theoretical predictions of the two previous exercises?

TABLE 0.2
History of convergence of the approximation error $e_n := \|v - \pi_n v\|_{L^2(0,\pi)}$

n	e_n	α_n	C_n
2		-	-
4			
8			
...			