Homework #5. Exponential convergence. Due on Friday, October 19, 2018.

In what follows, $\pi_n v$ represents the $L^2(0,\pi)$-projection of the function $v \in L^2(0,\pi)$ into the space of trigonometric functions

$$W_n := \text{span}\{\sin jx : j = 1, \ldots, n\}.$$

1. (5 pts.) For $v(x) := \pi/2 - |x - \pi/2|$, compute the $L^2(0,\pi)$-projection $\pi_n v$. Plot the functions v and $\pi_n v$ for a few values of n in order to get an idea of how $\pi_n v$ approximates v as n increases. To find how fast $e_n := \|v - \pi_n v\|_{L^2(0,\pi)}$ converges to zero, fill the history of convergence table below. Justify your numerical results by using the theory.

TABLE 0.3
History of convergence of the orthogonal projection

n	e_n	α_n	C_n
2		-	-
4			
8			
...			

3. (5 pts.) Repeat the previous exercise for $e_n := \|v - \pi_n v\|_{L^\infty(0,\pi)}$.

4. (5 pts.) Compute by using a symbolic manipulator (or by hand) the coefficients defining the $L^2(0,\pi)$-projection $\pi_n v$ of $v(x) = 4\sin(x)/(\pi(5 - 4\cos(x)))$. Plot the functions v and $\pi_n v$ for a few values of n in order to get an idea of how $\pi_n v$ approximates v as n increases. To see if the convergence of the approximation error $e_n := \|v - \pi_n v\|_{L^2(0,\pi)}$ is exponential, fill the history of convergence table below. We assume that $e_n = D\,\exp(-r\,n)$ and estimate the exponential rate of convergence r and the constant D by

$$r_n := \ln(e_{n/2}/e_n)/(n/2), \qquad D_n := e_n/e^{-r_n\,n}.$$

Justify your numerical results by using the theory.

TABLE 0.4
History of convergence of the orthogonal projection

N	e_n	r_n	D_n
2		-	-
4			
8			
...			

4. (5 pts.) Repeat the previous exercise for $e_n := \|v - \pi_n v\|_{L^\infty(0,\pi)}$.