Homework #7: Quadrature rules. Due on Friday, November 2.

1. (5 points) We approximate the integral $\int_a^b u(s)\,ds$ by the mid-point rule and by the trapezoidal rule. Compute the Peano kernel for each of these rules and compare them by plotting them in a single figure. What can you say about the relative accuracy of these two rules?

2. (5 points) Write a code which gives you the weights and quadrature points of a quadrature of n points and $2n-1$ degree of exactness. Use the Golub-Welsch method for the Legendre polynomials. Display numerical evidence that your code is correct.

3. (5 points) Write a code which computes an approximation of the integral of an arbitrary function u on the arbitrary interval (a, b) by using the quadrature rule of n points and degree of exactness $2n-1$. (Use the results of the previous problem.) Display numerical evidence that your code is correct.

4. (5 points) Apply your code to find the integral $I := \int_0^\pi \sin(s)\,ds$. Proceed as follows. For each value of $n = 1, 2, 3, \ldots$, apply your quadrature rule to obtain Q_n. Then set $e_n := |I - Q_n|$ and display a history of convergence for e_n in terms of n. Can you justify theoretically your numerical results?