

ANALYSIS of NUMERICAL METHODS for PDEs
MATH 8445-6 - FALL 2016/SPRING 2017

Instructor: B. Cockburn

- **Objective:** This course is intended to be an introduction to the *mathematical analysis* of finite element methods for partial differential equations. The distinctive feature of this course is that it emphasizes the interplay between theory and practice, as well as the physics behind the devising of the different numerical methods.

- **References:**
 - D. Braess, Finite Elements. Theory, Fast Solvers and Applications in Solid Mechanics;
 - F. Brezzi and M. Fortin, Mixed and hybrid finite element methods;
 - P.G. Ciarlet, The Finite Element Method for Elliptic Problems;
 - C. Johnson, Numerical solutions of PDEs by the Finite Element Method;
 - R. LeVeque, Numerical methods for conservation laws;
 - R. LeVeque, Finite Volume Methods for Hyperbolic Problems;
 - A. Quarteroni and A. Valli, Numerical methods for PDEs.

- **Grade:** Average of homeworks. (One each two weeks)

- **Office hours:** By appointment (cockburn@math.umn.edu).

- **Program for the Fall:**
 - Linear hyperbolic conservation laws (R. LeVeque + Notes)
 - The original Discontinuous Galerkin method.
 - * The elementwise weak formulation
 - * Upwinding and local conservativity
 - * The approximate divergence
 - * the local energy identity
 - * Existence and uniqueness of the approximation
 - * Computation of the approximation
 - * Stabilization mechanisms
 - * Convergence properties
 - The space discretization by the DG method.
 - * The method of lines
 - * Runge-Kutta time-marching methods
 - * The stability condition
 - * Dissipativity and dispersion
 - * Convergence properties
 - Extension to Friedrich's systems
 - * The general form
 - * The wave equation
 - * Maxwell's equations
 - * Upwinding and the Riemann problem
 - * Setting the boundary conditions
 - * Stabilization effects

- * Convergence properties
- Non-linear conservation laws (R. LeVeque + Notes)
 - The entropy solution of the scalar conservation law
 - * Strong solutions
 - * Weak solutions
 - * Entropy-satisfying discontinuities
 - * Solution of the Riemann problem.
 - * Characterization of the entropy solution
 - Monotone schemes.
 - * Definition
 - * Monotonicity and artificial viscosity
 - * Maximum principles
 - * Total variation boundedness
 - * Discrete entropy inequality
 - * Convergence to the entropy solution
 - The discontinuous Galerkin method.
 - * Special Runge-Kutta time-marching methods
 - * Maximum principles
 - * Total variation boundedness
 - * The slope limiter as artificial viscosity
 - * Convergence to the entropy solution
 - Application to the Euler equations of gas dynamics
- The continuous Galerkin method for the Poisson equation. (Johnson,Ciarlet)
 - Variational formulation.
 - A geometric interpretation.
 - A minimization problem.
 - Static condensation.
 - The Neumann problem.
 - Examples of finite elements.
 - Error analysis.
 - Regularity of the exact solution.
 - Error estimates in the L^2 -norm.
 - Abstract formulation and error analysis

•Program for the Spring:

- Mixed and HDG methods for steady-state diffusion. (Notes)
 - Devising the methods.
 - * A characterization of the exact solution
 - * Definition of the methods
 - * The approximate gradient and divergence
 - Existence and uniqueness.
 - * The local energy identity
 - * Conditions for well posedness
 - * Two extreme cases
 - The role of the interelement jumps
 - * The relation between the residuals

- * Stabilization by the jumps
- Four formulations of the methods
 - * Formulation with all the variables
 - * A stabilized hybrid-mixed method
 - * A primal-hybrid formulation
 - * A formulation for the approximate trace
- Main examples
 - * The LDG-H methods
 - * Mixed and superconvergent HDG methods
 - * New stabilization functions
 - * The EDG methods
- Error analysis.
 - * The energy argument
 - * The duality argument
 - * Superconvergence
 - * Approximation of linear functionals
- Convection-diffusion problems (Johnson + Notes)
 - The model convection-diffusion problem
 - Continuous Galerkin methods.
 - Streamline-diffusion methods.
 - HDG methods
 - Time-dependent problems
- Mixed and HDG methods for Fluid flow (Johnson, Brezzi and Fortin + Notes)
 - The Stokes system of fluid flow.
 - Different approaches to incompressibility
 - The Oseen’s equations.
 - The incompressible Navier-Stokes equations.
 - The compressible Navier-Stokes equations.
- Mixed and HDG methods for Elasticity (Notes)
 - The equations of linear elasticity.
 - Methods with weakly symmetric stresses.
 - Methods with strongly symmetric stresses.
 - Nonlinear elasticity.
 - Wave propagation in elastic media.
 - * Two different approaches
 - * Superconvergent methods
 - * BDF, DIRK, ERK and SSPRK time-marching
 - * Symplectic time-marching methods.