

HOMEWORK ASSIGNMENT N5, MATH 4567, SPRING 2014
Due on May 5 (Monday)

Problem 1. Find all eigenvalues and associated eigenfunctions for the Sturm-Liouville problem

$$\begin{aligned}y''(x) + \lambda y(x) &= 0, & 0 \leq x \leq \pi, \\y(0) + y'(0) &= 0, \\y(\pi) + y'(\pi) &= 0.\end{aligned}$$

Problem 2. a) Solve the temperature problem in the slab

$$\begin{aligned}u_t &= k u_{xx}, & u &= u(x, t), \quad 0 \leq x \leq \pi, \quad t \geq 0 \quad (k > 0) \\u_x(0, t) &= 0, \\u_x(\pi, t) &= 0, \\u(x, 0) &= (1 + \cos^2 x)^2.\end{aligned}$$

b) Find the temperatures in the long run: Show that there is a finite limit $C = \lim_{t \rightarrow \infty} u(x, t)$ which is independent of x . Find the constant C .

Note. The initial temperatures may be represented as a cosine polynomial of degree 4.

Problem 3. Use the Fourier transform to solve the temperature problem in the upper half-plane

$$\begin{aligned}u_t &= k u_{xx}, & u &= u(x, t), \quad -\infty < x < \infty, \quad t \geq 0, \quad u \text{ is bounded,} \\u(x, 0) &= e^{-2x^2}.\end{aligned}$$

Hint. First formulate a general theorem about the boundary value problems

$$\begin{aligned}u_t &= k u_{xx}, & u &= u(x, t), \quad -\infty < x < \infty, \quad t \geq 0, \quad u \text{ is bounded,} \\u(x, 0) &= f(x).\end{aligned}$$

Problem 4. Use the Fourier transform to solve the boundary value problem, involving the Laplace equation:

$$\begin{aligned}\Delta u &= 0, & u &= u(x, y), \quad x, y \geq 0, \quad u \text{ is bounded,} \\u_x(0, y) &= 0, \\u(x, 0) &= \frac{1}{1+x^2}.\end{aligned}$$

Hint. First formulate a general theorem about boundary value problems of the form

$$\begin{aligned}\Delta u &= 0, & u &= u(x, y), \quad x, y \geq 0, \quad u \text{ is bounded,} \\u_x(0, y) &= 0, \\u(x, 0) &= f(x).\end{aligned}$$

Note that the Fourier transform for the functions $f(x) = e^{-x^2/(2\sigma^2)}$ and $f(x) = \frac{1}{1+x^2}$ are known and have been evaluated in class.