

MATH 4567, SPRING 2019
HOMEWORK PROBLEMS No. 3
Due on March 27

Problem 1 (6 points). Using elementary arguments, solve the boundary value problem

$$\begin{aligned}u_{xx} &= -y^2 \cos x, & u &= u(x, y), \quad 0 \leq x \leq \pi, \\u(0, y) &= y^2, \\u(\pi, y) &= \pi \sin y - y^2.\end{aligned}$$

Problem 2 (6 points). a) Solve the temperature problem:

$$\begin{aligned}u_t &= k u_{xx}, & u &= u(x, t), \quad 0 < x < 1, \quad t > 0 \quad (k > 0 \text{ parameter}) \\u_x(0, t) &= u_x(1, t) = 0, \\u(x, 0) &= \frac{1}{2} x^2.\end{aligned}$$

Your answer will have the form of an infinite functional series.

b) Write down separately the first 4 terms of that functional series.

Problem 3 (7 points). a) Solve the boundary value problem:

$$\begin{aligned}u_{tt} &= \frac{1}{2} u_{xx}, & u &= u(x, t), \quad 0 < x < 1, \quad t > 0 \\u(0, t) &= u(1, t) = 0, \\u_t(x, 0) &= 0, \\u(x, 0) &= x(1 - x).\end{aligned}$$

Your answer will have the form of an infinite functional series.

b) Write down separately the first 4 terms of that functional series and evaluate this partial sum at time $t = 1$ for the point $x = \frac{1}{2}$.

Problem 4 (6 points). Solve directly for the eigenvalues and normalized eigenfunctions:

- a) No. 1 on page 225;
- b) No. 2 on page 225;
- c) No. 3 on page 225.