

# Supplementary material for “WRAPD: Weighted Rotation-aware ADMM for Parameterization and Deformation”

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## 1 THRESHOLD FOR REWEIGHTING

As discussed in Section 5.1 of the main paper, we update ADMM’s global matrix only when the candidate weights differ significantly from the current weights. In particular, we flag the entire mesh for reweighting if the ratio of any element’s squared candidate weight to its current squared weight exceeds  $\gamma$  (or falls below  $\gamma^{-1}$ ). Small values of  $\gamma$  will trigger refactorization more often, improving convergence per iteration but increasing the computational cost, while larger values will do the opposite. Here, we evaluate the effect of the choice of  $\gamma$  on the overall performance of WRAPD.

In Figure 1, we plot the computation time needed for WRAPD to reach the early, middle, and late phases of convergence. As can be seen, the computational cost is fairly insensitive to  $\gamma$  over a broad range of values, only rising steeply when  $\gamma$  is very small (excessive refactorization) or very large (poor convergence).

## 2 MAXIMUM WEIGHT CLAMPING STRATEGY

When updating weights, we also clamp the squared candidate weight to lie in a bounded range  $[\beta_{\min} w_0^2, \beta_{\max} w_0^2]$ . For parameterization, we fix the lower bound  $\beta_{\min} = 1$  and gradually increase the upper bound  $\beta_{\max} = \min(10 \times 1.5^k, 10^9)$  as the iterations progress. Here we evaluate the benefits of this “easing” process.

Parameterization problems are initialized by mapping to a disk, giving many elements extremely high distortion in the initial state. Without weight clamping, the weights for such elements grow excessively large in the very earliest iterations. This leads to various numerical difficulties: the global system matrix becomes ill-conditioned in the presence of weights spanning several orders of magnitude, and convergence is slowed since elements with overestimated weights are unable to make progress.

An example is shown in Figure 2, where WRAPD without easing takes much longer to find a flip-free state. Among the nine meshes

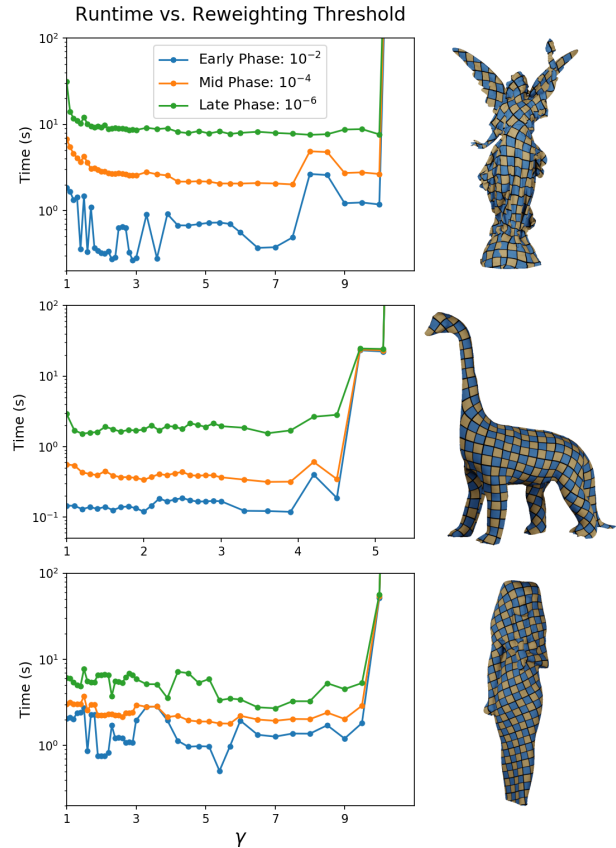


Fig. 1. Comparing reweighting thresholds using a symmetric Dirichlet parameterization example. *Left:* Runtime cost vs. reweighting threshold parameter  $\gamma$ . Plotted are the times required to reach the early, middle, and late phases of convergence. We define these thresholds at energy errors of  $10^{-2}$ ,  $10^{-4}$ , and  $10^{-6}$ , respectively. *Right:* Original mesh textured with the minimum energy solution.

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shown in Figure 11 of the main paper, this was the only one that exhibited this behavior. However, such difficulties occur more frequently in the larger parameterization dataset from Liu et al. [2018]. In Table 1, we report the time to early and late convergence, along with the percentage of meshes that failed to converge. Without easing, the failure rate for WRAPD+CM reaches 17.9% on the most challenging D3 dataset, indicating that such meshes failed to reach an injective state. With easing, the failure rate of WRAPD+CM on D3 drops to only 2.3%.

