Math 8701 – Fall 2013 – Problem Set 8

1. Find the residues at all singularities for the following functions:

a) $\dfrac{z}{z^4+1}$

b) $\dfrac{\sin z}{z^2(\pi - z)}$

c) $\dfrac{ze^{iz}}{(z-\pi)^2}$

d) $\dfrac{z^3+5}{(z^4-1)(z+1)}$

2. Let C denote the unit circle, traversed counter-clockwise. Compute each of the integrals:

$$\int_C \frac{e^{\pi z}}{4z^2+1}dz, \qquad \int_C \frac{e^z}{(z^2+z-3/4)^2}dz$$

3. Riemann integrals of a real variable θ of the form

$$\int_0^{2\pi} f(\cos\theta, \sin\theta)d\theta$$

can sometimes be solved by changing variables to a complex contour integration.

Indeed, setting $z = e^{i\theta}$ then as θ runs from 0 to 2π, z traverses the unit circle counterclockwise. Moreover,

$$\cos\theta = \frac{z^2+1}{2z}, \quad \sin\theta = \frac{z^2-1}{2iz}, \quad d\theta = \frac{dz}{iz}.$$

Show that such a change of variables allows one to evaluate:

$$\int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$$

via a complex contour integral, upon using the residue theorem.

4. Problems from Ahlfors, Section 4.5.2: **p. 154, problems 1 and 2.**