Math 8702 – Spring 2014 – Take Home Midterm
DUE: Monday, March 24

1. Prove that a continuous function on a region is harmonic if and only if it satisfies the mean value property.

2. Construct an analytic, one-to-one, onto map from $\{z : |z| < 1, \mathrm{Re}(z) > 0\}$ to $\{z : |z| < 1\}$.

3. For each of the following parts, determine whether the given family of functions is normal.

a) $\{f_n := n^{-1} \cos nz : n \in \mathbb{Z}\}$ defined on the interior of the first quadrant $\{z = x + iy : x, y > 0\}$.

b) The set of holomorphic maps $g : D \to U$ such that $g(0) = 0$, with

$$D = \{z : |z| < 1\} \quad U = \{z : -2 < Rez < 2\}.$$

4. Recall that the *order* of a doubly periodic meromorphic function is the number of poles (counted with multiplicity) in a fundamental domain. Let f be a doubly periodic function of order m so that f' is doubly periodic, say of order n. Show that $m + 1 \leq n \leq 2m$.

5. Give a complex structure on the set

$$X := \{[x : y : z] \in \mathbb{P}^2(\mathbb{C}) : zx^2 = y(y - z)(y - 2z)\}$$

so that X is a Riemann surface.