Name: ______________________

Problem Set 11
Math 4281, Fall 2013
Due: Friday, November 22

Read Sections 11.1 (skip Theorem 11.2) and 9.1 (up to Theorem 9.5) in your textbook.

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1. Show that the Klein four-group $\mathcal{V}$ is not isomorphic to $\mathbb{Z}_4$.
2. Prove that $\mathbb{Z}_7^\times \cong \mathbb{Z}_6$. (It is crucial to remember that we multiply in $\mathbb{Z}_7^\times = U(7)$ and add in $\mathbb{Z}_6$.)
3. a. Prove that $\mathbb{Z}_{12}^\times \cong \mathcal{V}$.
   b. Prove that $\mathbb{Z}_{15}^\times \cong \mathbb{Z}_{16}^\times \cong \mathbb{Z}_{20}^\times$. What about $\mathbb{Z}_{24}^\times$?
4. Show that $\phi \colon \mathbb{R} \to \mathbb{C}^\times$ given by $\phi(t) = \operatorname{cis}(2\pi t)$ is a homomorphism. Then describe its kernel and image.
5. Let $a \in G$ be fixed, and define $\phi \colon G \to G$ by $\phi(x) = axa^{-1}$. Prove that ϕ is a homomorphism. Under what circumstances is ϕ an isomorphism?
6. Let $\zeta = \operatorname{cis}\left(\frac{2\pi}{n}\right)$. Prove that the dihedral group D_n is isomorphic to the subgroup of $\mathrm{G}L_2(\mathbb{C})$ obtained by taking all products of the two matrices $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$ and $\begin{bmatrix} \zeta & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$ and their inverses. (In other words, by taking the subgroup generated by these two matrix elements.)

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________