Name: ______________________

Problem Set 3
Math 4281, Fall 2013
Due: Friday, September 27

Read Sections 16.1, 16.2, and 4.2 in your textbook. (In Section 16.2, we will only use a special case of reading from Theorem 16.4 to Theorem 16.6, as abbreviated in class notes. You will not be responsible for the more general results stated in this portion of the text.)

Complete the following items, staple this page to the front of your work, and turn your assignment in at the beginning of class on Friday, September 27.

1. Let $a, m \in \mathbb{Z}$ with $m > 0$.
   a. Prove that $\gcd(a, m) = 1$ if and only if $[a] = \overline{a} \in \mathbb{Z}_m$ is a unit.
   b. Prove that if $[a] = \overline{a} \in \mathbb{Z}_m$ is a zero divisor, then $\gcd(a, m) > 1$, and conversely, provided $m \nmid a$.
2. Complete the following exercises in your textbook.
   pp. 262-3 #1(f), 3(d,e), 12
3. Suppose R is an integral domain, $c, x, y \in R$ and $c \neq 0$. Prove that if $cx = cy$, then $x = y$.
4. Let p be a prime number. Use the fact that $\mathbb{Z}_p$ is a field to prove that $(p-1)! \equiv 1 \pmod{p}$. (Hint: Pair elements in $\mathbb{Z}_p$ with their multiplicative inverses; see #4 of Problem Set 2).
5. Complete the following exercises in your textbook.
   p. 72 #15(b), 16(b), 17(c), 18(d,e)
6. Prove the following properties of the modulus of a complex number. Let $z, w \in \mathbb{C}$.
   a. $|zw| = |z||w|$
   b. $|\overline{z}| = |z|$
   c. $|z|^2 = z\overline{z}$
   d. $|z + w| \leq |z| + |w|$ (This is called the *triangle inequality*. Why?)
7. Find the sixth roots of $-3i$. Express your answers in the (exact) form $z = a + bi$ without trigonometric functions whenever possible, and then plot them in the complex plane.

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________