Name: ____________________

# Problem Set 10
## Math 4281, Spring 2014
Due: Wednesday, April 9

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**Groups**

1. Which of the following are groups?

   a. $\{0, 2, 4, 6\} \subseteq \mathbb{Z}_{10}$, with operation addition
   b. $\{z \in \mathbb{C} \mid |z| = 1\}$, with operation multiplication
   c. $\{x \in \mathbb{Q} \mid 0 < x \leq 1\}$, with operation multiplication
   d. $\mathbb{Z}$ with operation $a \bullet b = a + b + 1$

2. Give a counterexample to "if $G$ is a group and $a, b, c \in G$ with $ab = bc$, then $a = c$."

3. Let $G$ be a group with identity element $e$.

   a. Prove that $(ab)^2 = a^2b^2$ for all $a, b \in G$ if and only if $G$ is abelian.
   b. Prove that if every element $a \in G$ is such that $a^2 = e$, then $G$ is abelian.

4. Let $G$ be a group and fix $a \in G$. Prove that $C_a = \{x \in G \mid ax = xa\}$ is itself a group, called the centralizer of $a$.

**Cyclic groups**

5. a. List all of the generators of $\mathbb{Z}_{20}$.
   b. List the elements of the subgroups $\langle 3 \rangle$ and $\langle 7 \rangle$ in $U(20) = \mathbb{Z}_{20}^{\times}$.
   c. Find all subgroups of $\mathbb{Z}_{18}$ and $U(11) = \mathbb{Z}_{11}^{\times}$.

6. a. Let $a$ be an element in a group. If $|a| = n$, show that $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$.
   b. Let $a$ be an element in a group. Suppose that $|a| = 24$. Find a generator of $\langle a^{21} \rangle \cap \langle a^{10} \rangle$.

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| Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code. |

Signed: ____________________