Name: ____________________

Problem Set 13
Math 4281, Spring 2014
Due: Wednesday, May 7

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1. In this exercise, you will prove *Cayley's Theorem*, which says that every group is isomorphic to a subgroup of a permutation group.
   Let G be a finite group of order n. Let $\mathrm{Perm}(G)$ denote the group of permutations of G, i.e.,

$$\mathrm{Perm}(G) = \{\pi\colon G \to G \mid \pi \text{ is a one-to-one and onto map of sets}\}.$$

   a. Show that $\mathrm{Perm}(G)$ is a group.
   b. For each $a \in G$, let $L_a\colon G \to G$ be defined by $L_a(g) = ag$. Prove that L_a is a permutation of G, i.e., $L_a \in \mathrm{Perm}(G)$.
   c. Define $\phi\colon G \to \mathrm{Perm}(G)$ by $\phi(a) = L_a$. Prove that ϕ is a one-to-one group homomorphism.
   d. Use (b) to prove that G is isomorphic to a subgroup of S_n.

Galois theory

2. Determine the Galois group and the corresponding subgroups and intermediate fields for $f(x) = x^5 - 1 \in \mathbb{Q}[x]$. Check for normal subgroups and Galois extensions of $\mathbb{Q}$. (Hint: We can factor $f(x) = (x-1)(x^4 + x^3 + x^2 + x + 1)$.)

3. Determine the Galois group and the corresponding subgroups and intermediate fields for $f(x) = x^3 + 2 \in \mathbb{Q}[x]$. Check for normal subgroups and Galois extensions of $\mathbb{Q}$.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________