# $2^{\text{nd}}$ midterm for MATH 1272: Calculus II, section 030

**Name:** **Section Number:**
**ID #:** **Teaching Assistant:**

**Instructions:**

- *Please don't turn over this page until you are directed to begin.*
- There are 6 problems (and problems 1, 3, 5, and 6 have multiple parts) on this exam.
- There are 7 pages to the exam, including this page. All of them are one-sided. If you run out of room on the page that you're working on, use the back of the page.
- Please show all your work. Answers unsupported by an argument will get little credit.
- Scientific calculators are allowed. No books or notes are allowed. Please turn off your cell phones.

## Grading summary

| Problem: | 1 | 2 | 3 | 4 | 5 | 6 | total |
|---|---|---|---|---|---|---|---|
| Possible: | 10 points | 10 points | 15 points | 10 points | 15 points | 20 points | 80 points |
| Grade: | | | | | | | |

## Some helpful formulas

| | | |
|---|---|---|
| $\sin^2(x) + \cos^2(x) = 1$ | $\tan^2(x) + 1 = \sec^2(x)$ | $1 + \cot^2(x) = \csc^2(x)$ |
| $2\sin^2(x) = 1 - \cos(2x)$ | $2\cos^2(x) = 1 + \cos(2x)$ | $2\sin(x)\cos(x) = \sin(2x)$ |
| $\int \tan(x)\ dx = \ln\lvert\sec(x)\rvert + C$ | $\int \sec(x)\ dx = \ln\lvert\sec(x) + \tan(x)\rvert + C$ | $\int \csc(x)\ dx = \ln\lvert\csc(x) - \cot(x)\rvert + C$ |

1. **(10 points total, 5 points each)** Consider the differential equation

$$y' = x + y^2.$$

(a) Sketch a direction field for this differential equation in the region $-2 \le x \le 2$ and $-2 \le y \le 2$.

(b) Use Euler's method with step size 0.2 to estimate $y(0.4)$ where $y(x)$ is the solution of the differential equation with initial value $y(0) = 0$.

2. **(10 points)** Solve the differential equation

$$e^{-y}y' + \cos x = 0$$

with the initial condition $y(0) = 1$.

3. **(15 points total, 5 points each)** Consider a population of rabbits whose initial population is $P(0) = 50$, and whose population $P(t)$ at time $t$ (in years) satisfies the differential equation

$$\frac{dP}{dt} = 0.4P - 0.001P^2$$

(a) What is the carrying capacity of the population?

(b) What is $P'(0)$?

(c) When will the population reach half the carrying capacity?

4. **(10 points)** Solve the differential equation

$$(1+s)\frac{dv}{ds} + v = 1 + s$$

where $s > 0$, and $v(0) = 5$.

5. **(15 points total, 5 points each)** Populations of birds and insects are modelled by the differential equations

$$\frac{dx}{dt} = 4x - 0.2xy \quad \text{and} \quad \frac{dy}{dt} = -2y + 0.08xy$$

(a) Which of the variables, $x$ or $y$, represents the bird population and which represents the insect population? Explain why.

(b) Find the equilibrium solutions, and explain their meaning.

(c) Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

6. **(20 points total, 5 points each)** Consider the curve in polar coordinates $(r, \theta)$ defined by the equation $r = \sin(2\theta)$.

   (a) Sketch a graph of the curve in Cartesian (i.e., $(x, y)$) coordinates. **Note:** you do *not* need to find a *formula* for the curve in Cartesian coordinates.

   (b) At what values of $\theta$ with $0 \leq \theta \leq 2\pi$ does the curve intersect itself?

   (c) Find the area of one loop of the curve (that is, between two points of intersection).

   (d) Find the slope of the curve corresponding to the point $\theta = \pi/4$.