Homework #10 for MATH 5345H: Introduction to Topology

November 27, 2017

Due Date: Monday 4 December in class.

1. Let X be a completely regular space, and let Y be any compactification[1] of X. Let $\beta(X)$ be the Stone-Čech compactification of X. Show that there is a continuous surjective map $g : \beta(X) \to Y$ which is the identity on X (a common subspace of both Y and $\beta(X)$). **Hint:** You may need to employ some of the theorems of section 38 of Munkres which we did not discuss in class to prove this result.

2. Suppose that X and Y are spaces and that Y is contractible. Show that any two continuous maps from X to Y are homotopic.

3. Suppose that X_0, X_1, Y_0, Y_1 are spaces such that X_0 is homotopy equivalent to Y_0 and X_1 is homotopy equivalent to Y_1. Show that $X_0 \times X_1$ is homotopy equivalent to $Y_0 \times Y_1$.

4. Let S^n be the n-sphere, defined as usual as

$$S^n = \left\{ (x_1, \cdots, x_{n+1}) \in \mathbb{R}^{n+1} \mid \sum_{i=1}^{n+1} x_i^2 = 1 \right\}.$$

Define the *antipodal map* $\mathbf{a} : S^n \to S^n$ by

$$\mathbf{a}(x_1, x_2, \cdots, x_{n+1}) = (-x_1, -x_2, \cdots, -x_{n+1}).$$

Show that if n is odd[2], then $\mathbf{a}$ is homotopic to the identity on S^n. **Hint:** If $n = 2k-1$ is odd, S^n is a subspace of $\mathbb{C}^k$. Multiplication by elements of $\mathbb{C}$ of norm 1 could be helpful here.

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[1] Recall that this means that Y is a compact, Hausdorff space which contains X as a subspace, and X is dense in Y

[2] If n is even, then it can be shown that $\mathbf{a}$ is *not* homotopic to the identity. Unfortunately, we will not have the time to develop the tools in this course to show this fact.