Homework #6 for MATH 5345H: Introduction to Topology

October 23, 2017

Due Date: Monday 30 October in class.

1. Let $f : X \to Y$ be a homeomorphism. Show that X is compact if and only if Y is.
2. Show that any set equipped with the finite complement topology is compact.
3. Let X and Y be spaces, and let $x_0 \in X$, $y_0 \in Y$ be closed points. Let $X \amalg Y$ be the disjoint union of X and Y (so that an open set of $X \amalg Y$ is the union of an open set of X and an open set of Y) and define the *wedge sum* $X \vee Y$ by

$$X \vee Y = (X \amalg Y)/(x_0 = y_0).$$

   The notation means the quotient of $X \amalg Y$ by the equivalence relation $\sim$ with $x_0 \sim y_0$ and no other nontrivial equivalence classes - so $X \vee Y$ is obtained by joining X and Y at a point. The space $X \vee Y$ depends on the choices of x_0 and y_0, but we suppress this from the notation.

   If X and Y are connected, show that $X \vee Y$ is connected. If X and Y are compact, show that $X \vee Y$ is compact.
4. Let $p : X \to Y$ be a quotient map, and assume Y to be connected. Assume further that for each point $y \in Y$, the subspace $p^{-1}(\{y\}) \subseteq X$ (called the *fibre* over y) is connected (when given the subspace topology). Show that X is connected. Is the converse true? That is, if X is connected, must its fibres be, too?