Homework #8 for MATH 5345H: Introduction to Topology

November 8, 2017

Due Date: Wednesday 15 November in class.

1. Let X be a metric space, equipped with the metric topology.
   (a) Show that if X is separable, then it is second countable.
   (b) Show that if X is Lindelöf, then it is second countable.
2. Show that the one point compactification of $\mathbb{R}$ is the circle.
3. Give $\mathbb{Q}$ the subspace topology of $\mathbb{R}$. Is it locally compact? Prove your answer.
4. Recall the wedge sum from Homework 6: if A and B are two topological spaces, $a_0 \in A$ and $b_0 \in B$, then $A \vee B$ is the quotient space of $A \coprod B$ under the relation $a_0 \sim b_0$ (the points a_0 and b_0 are usually called *basepoints* of A and B). Let X and Y be locally compact, Hausdorff spaces, and let X^*, Y^* be their one point compactifications. Show that there is a homeomorphism

$$(X \coprod Y)^* \cong X^* \vee Y^*$$

where the basepoints of X^* and Y^* are taken to be the points at infinity.