# Homework #2 for MATH 5345H: Introduction to Topology

September 13, 2019

**Due Date:** Friday 20 September in class.

1. Suppose $A_1, A_2, A_3, \cdots$ are sets such that for each $n$, the intersection

$$A_1 \cap A_2 \cap \cdots \cap A_n$$

is nonempty. Is it always the case that the infinite intersection

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots$$

is nonempty? If so, prove it. If not, give a counterexample.

2. Let $S, T$ and $U$ be sets. Let $f : S \to T$ and $g : T \to U$ be functions. For each part, give a proof or a counterexample.

   (a) If $f$ and $g$ are injective, must $g \circ f$ be injective?

   (b) If $g \circ f$ is injective, must $f$ be injective?

   (c) If $g \circ f$ is injective, must $g$ be injective?

3. Let $A$ be a set, and write $P(A)$ for the *power set* of $A$;

$$P(A) = \{S \mid S \subseteq A\}$$

Assuming that $A$ has $n$ elements, show that $P(A)$ has $2^n$ elements.

**Also do these problems** from Munkres' *Topology*: ch.1, §3, # 1, 4, 11.