

# Homework #2 for MATH 8301: Manifolds and Topology

September 11, 2017

**Due Date:** Monday 18 September in class.

1. Prove that the image of a connected space under a continuous map is connected. Is the same true with the word “connected” replaced by “path connected?”
2. Prove that an open subset of a  $d$ -dimensional manifold is also a  $d$ -dimensional manifold.
3. Prove that a disjoint union of two  $d$ -dimensional manifolds is also a  $d$ -dimensional manifold.
4. This problem establishes the classification of 0-dimensional manifolds.
  - (a) Prove that 0-dimensional manifolds all carry the discrete topology.
  - (b) Show that a nonempty, connected, 0-dimensional manifold is a single point.
  - (c) Show that if  $M$  is a compact 0-dimensional manifold, then the cardinality of  $M$  is finite.
  - (d) Define a function

$$\text{card} : \{\text{homeomorphism classes of compact 0-dimensional manifolds}\} \rightarrow \mathbb{Z}_{\geq 0}$$

by sending a manifold to its cardinality. Show that  $\text{card}$  is well-defined, and a bijection.