Target Tracking Error

A utility-based model for active management

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Background

- Active Asset Management
- Net Asset Value
- Relative Performance
- Tracking Error
- Information Ratio

Objective

Inform the control of the deployment of tracking error to maximize investor utility

Performance Process

$$d\Pi_t = \lambda(v)\cdot v\,dt + v\,dW_t$$

- v = tracking error is the controlling variable
- λ = information ratio is a function of the tracking error
- *caveat:* normal increments

Dilution from Scale

$$\frac{d\lambda}{dv} = -\kappa \cdot \frac{\lambda}{v}$$

- κ is measure of elasticity
  - κ = 0 scales perfectly
  - κ = 1 does not scale at all

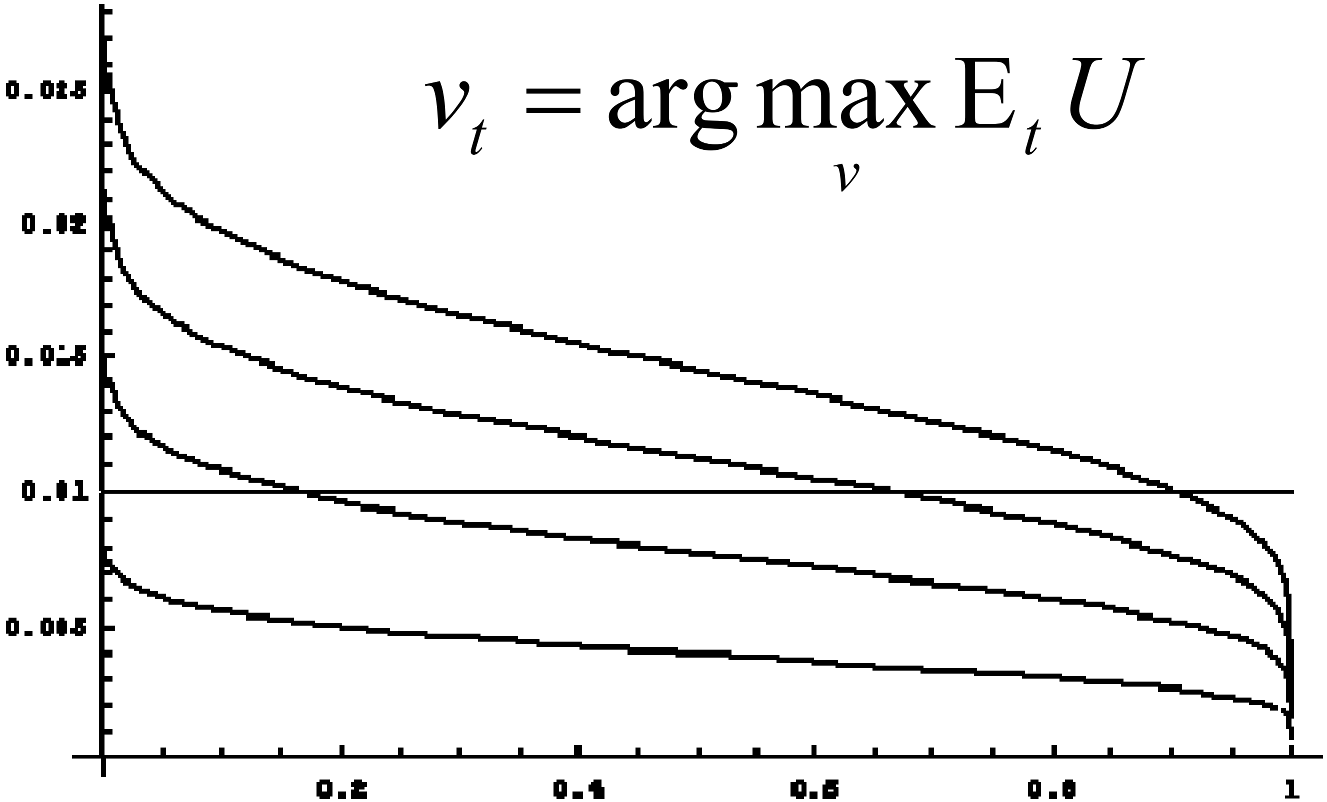
Investor Utility

$$U = \log \Phi\left(\frac{\Pi_T - \mu \cdot T}{\sigma \cdot \sqrt{T}}\right)$$

- Logarithm of the rank quantile
- Depends on
  - horizon
  - competition

Optimality Condition

$$v_t = \arg\max_v \mathrm{E}_t\, U$$

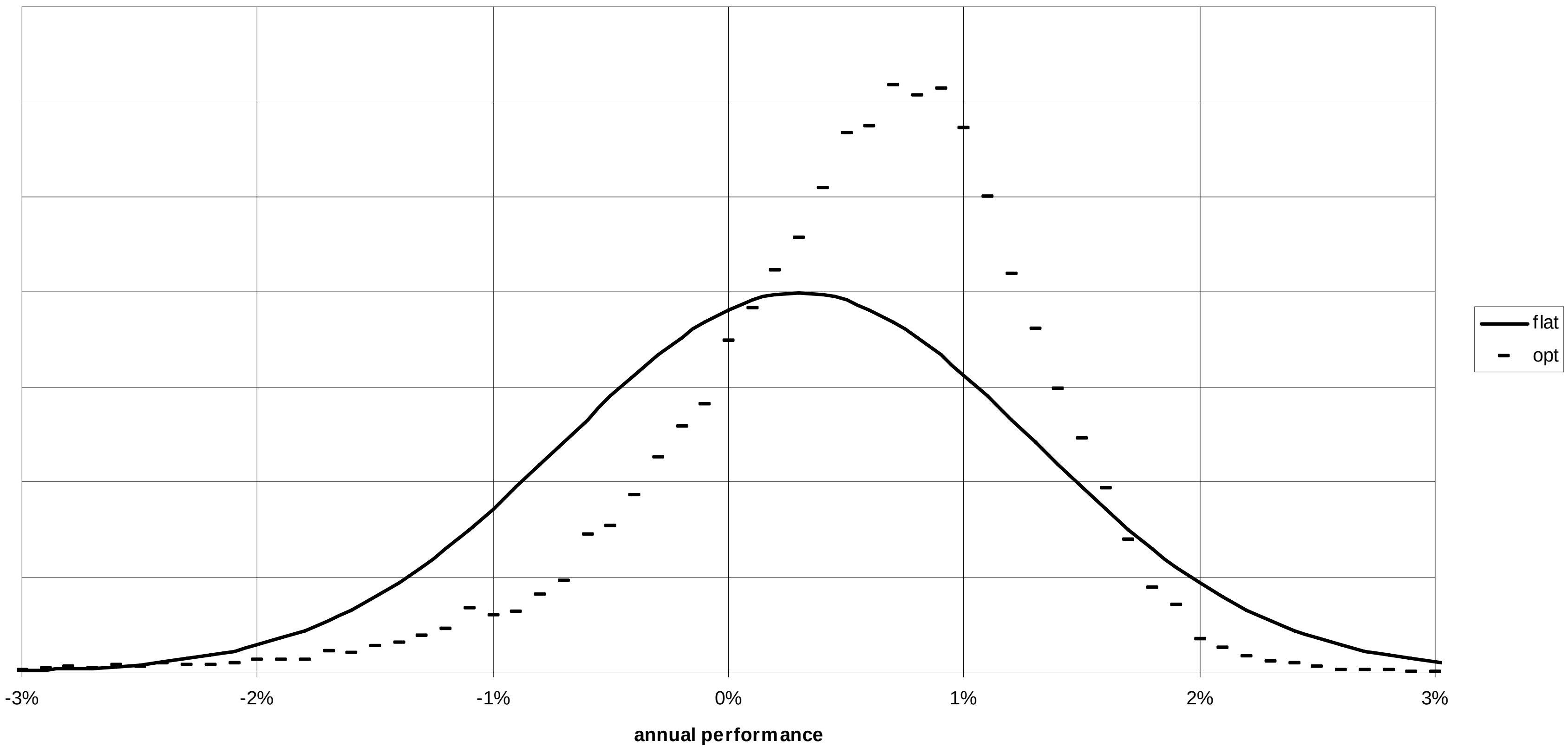


Myopic Optimality

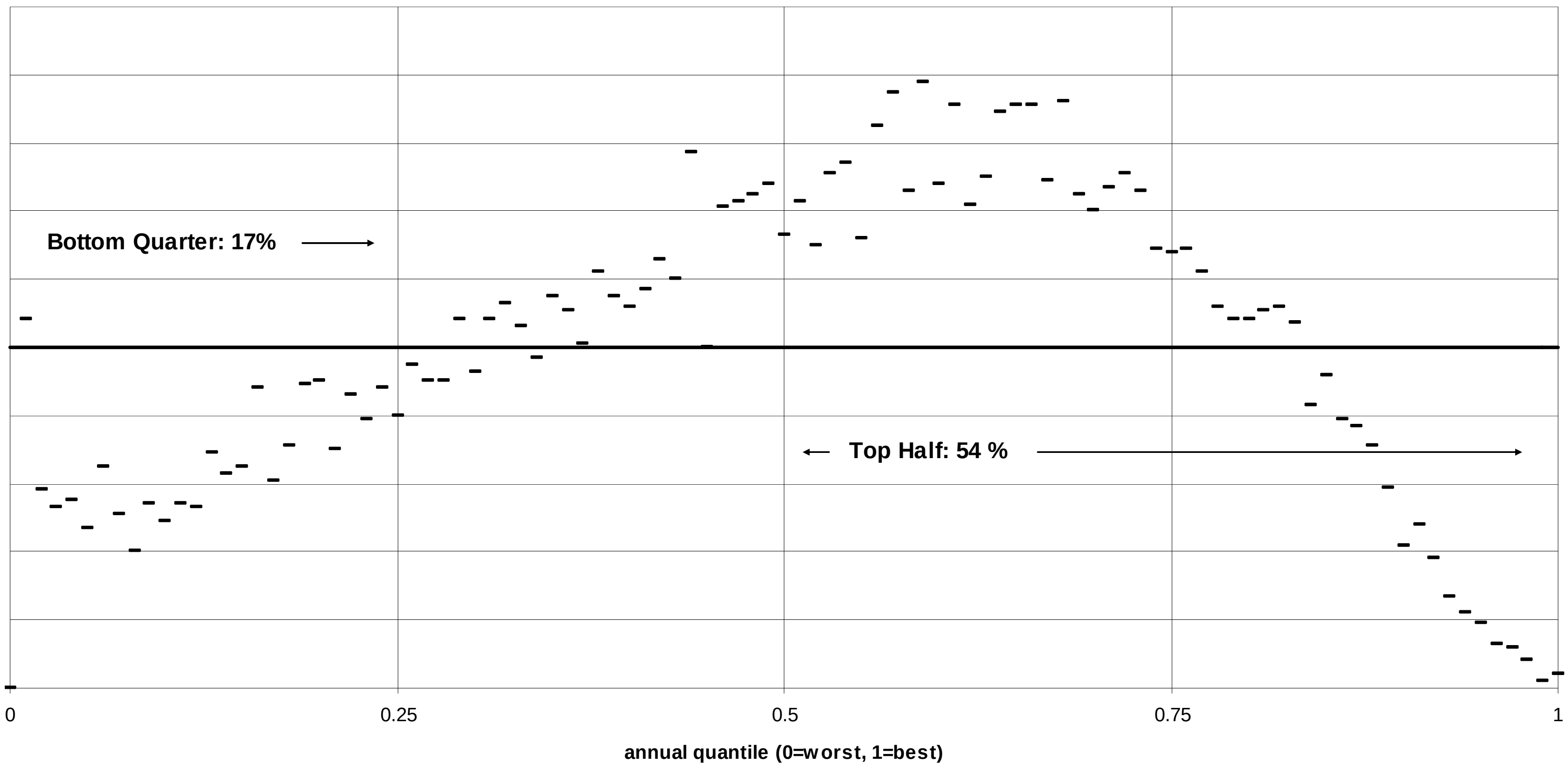
$$\lim_{t \to T} v_t = \frac{\sigma}{\kappa + \dfrac{\dfrac{\Phi' \circ \Phi^{-1}(q_t)}{q_t} + \Phi^{-1}(q_t)}{\lambda \cdot \sqrt{T}}}$$

- analytic solution – big benefit!
- does not suffer from fiduciary criticism of arbitrary fixed horizon

Simulation



Simulation



Bottom Quarter: 17%
Top Half: 54 %
0
0.25
0.5
0.75
1
annual quantile (0=worst, 1=best)

Transaction Costs

$$\lambda \cdot \sqrt{T} = \gamma \cdot \sqrt{N}$$

$$\gamma \approx 2 \cdot p - 1 - \rho$$

- γ = information coefficient
- N = breadth × T
- ρ = average round-trip transaction cost
- p = batting average

Questions?

Implied Track Record

A bayesian estimator for the information ratio

Background

- The “Information ratio” or “Sharpe ratio” is the market price of risk for asset allocation or active fund management
- It has the form of a t-statistic (average ÷ standard deviation)
- It is essentially impossible to observe using unbiased estimators

Objective

Present an information ratio estimator with a coherent trade-off between bias and standard error

Hidden Mixture Model

$$X_i | \mu, \sigma \overset{\text{iid}}{\sim} \mathrm{N}\left(\mu, \sigma^2\right)$$

$$\mu | \sigma \sim \mathrm{N}\left(\mu_0, \frac{\sigma^2}{\lambda_0}\right)$$

$$\frac{1}{\sigma^2} \sim \Gamma\left(\alpha_0, \beta_0\right)$$

- return observations are independent normals
- four hidden parameters

Parameter Updates

$$\mu_n = \frac{\lambda_0 \cdot \mu_0}{\lambda_0 + n} + \frac{1}{\lambda_0 + n} \cdot \sum_{i=1}^{n} x_i$$

$$\lambda_n = \lambda_0 + n$$

$$\alpha_n = \alpha_0 + \tfrac{1}{2} \cdot n$$

$$\beta_n = \beta_0 + \tfrac{1}{2} \cdot$$

$$\left\{ \sum_{i=1}^{n} x_i^2 - \frac{1}{\lambda_0 + n} \cdot \left(\sum_{i=1}^{n} x_i \right)^2 + \frac{\lambda_0 \cdot \mu_0}{\lambda_0 + n} \cdot \left(n \cdot \mu_0 - 2 \cdot \sum_{i=1}^{n} x_i \right) \right\}$$

- Model is invariant under sampling

Process Version

$$\mu_T = \frac{\mu_0 \cdot \lambda_0 + x_T - x_0}{\lambda_0 + T}$$

$$\lambda_T = \lambda_0 + T$$

$$\alpha_T = \alpha_0 + \tfrac{1}{2} \cdot \frac{T}{\Delta t}$$

$$\beta_T = \beta_0 + \tfrac{1}{2} \cdot \left\{ \sum_{i=1}^{T/\Delta t} \frac{\Delta x_{t_i}^2}{\Delta t} + \mu_0^2 \cdot \lambda_0 - \frac{\left(\mu_0 \cdot \lambda_0 + x_T - x_0\right)^2}{\lambda_0 + T} \right\}$$

- Process is sampled discretely
  - *This can also be done for aperiodic sampling*

Posterior Moments

$$\mathrm{E}\left[\frac{\mu}{\sigma}\middle| x_{t_i}\right] = \mu_T \cdot \sqrt{\frac{\alpha_T}{\beta_T}} \cdot \frac{\Gamma\left(\alpha_T + \frac{1}{2}\right)}{\Gamma(\alpha_T) \cdot \sqrt{\alpha_T}}$$

$$\mathrm{var}\left[\frac{\mu}{\sigma}\middle| x_{t_i}\right] = \frac{1}{\lambda_T} + \mathrm{E}\left[\frac{\mu}{\sigma}\middle| x_{t_i}\right]^2 \cdot \left[\left(\frac{\Gamma(\alpha_T) \cdot \sqrt{\alpha_T}}{\Gamma\left(\alpha_T + \frac{1}{2}\right)}\right)^2 - 1\right]$$

- Use expected value as the estimator
- Use sqrt variance as the standard error

The Estimation Problem

$$\operatorname{var}\left[\frac{\mu}{\sigma}\middle|\left\{x_{t_i} : i = 1, \ldots, \frac{T}{\Delta t}\right\}\right] \geq \frac{1}{\lambda_0 + T}$$

- This does not depend on the sampling frequency
- Unreasonable to assume that population value is that stable!

Solution Characteristics

- Unbiased estimators are too noisy
- Need to inject information to lower the standard error
- Assumptions should be readily understandable
- Solution should be perturbative in the assumptions and unbiased in limit of sample size

"Implied Track Record"

- Assume an implied history
  - given duration
  - given average tracking error
  - given average information ratio

- The analogy is a new manager with an established history elsewhere

Prior Parameters

$$\lambda_0 = T_0$$

$$\alpha_0 = \tfrac{1}{2} \cdot \frac{T_0}{\Delta t}$$

$$\beta_0 = \mathrm{E}\left[\sigma \middle| x_0\right]^2 \cdot \tfrac{1}{2} \cdot \frac{T_0}{\Delta t} \cdot \left(1 - \frac{\Delta t}{T_0}\right)^2 \cdot \left[\frac{\Gamma\left(\frac{1}{2} \cdot \frac{T_0}{\Delta t}\right) \cdot \sqrt{\frac{1}{2} \cdot \frac{T_0}{\Delta t}}}{\Gamma\left(\frac{1}{2} \cdot \frac{T_0}{\Delta t} + 1\right)}\right]^2$$

$$\mu_0 = \mathrm{E}\left[\frac{\mu}{\sigma} \middle| x_0\right] \cdot \mathrm{E}\left[\sigma \middle| x_0\right] \cdot \left(1 - \frac{\Delta t}{T_0}\right) \cdot \left[\frac{\Gamma\left(\frac{1}{2} \cdot \frac{T_0}{\Delta t}\right) \cdot \sqrt{\frac{1}{2} \cdot \frac{T_0}{\Delta t}}}{\Gamma\left(\frac{1}{2} \cdot \frac{T_0}{\Delta t} + 1\right)}\right]^2$$

Questions?