How much math do you really need to make markets in stock options?

IMA Industrial Problems Seminar

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Affiliations

I am professionally affiliated with:

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Industry Trends

Fifty years of mathematics in finance

- **80s** stock options and Black-Scholes-Merton
- **90s** LIBOR derivatives, asset-liability management
- **00s:1** credit-enhancing securitizations
- **00s:2** counterparties, liquidity; S&P 500 volatility derivatives
- **10s:1** stress testing, model validation; high-frequency trading
- **10s:2** crypto; big data
- **20s** machine learning; AI?

Market Participants

In the US, regulations define generally non-overlapping roles for legal entities that participate in the functioning of financial markets.

- investment manager: manage portfolios of stocks, bonds, and financial derivatives on behalf of individuals and institutions; referred to as the "buy side" by members of the "sell side"
- investment bank: create publicly-traded stocks and bonds on behalf of institutions
- broker: access markets for stocks, bonds, and financial derivatives on behalf of individuals and investment managers
- dealer: make markets in stocks, bonds, and financial derivatives on behalf of brokers
- exchange: provide trading services to brokers and dealers (order book, matching, settlement, and clearing)

Feedback in Value Chains

Since finance is so fragmented and complicated, it is worth taking a moment to think about value chains.

Investment managers create value for owners[1] by:

- increasing assets under management
- increasing fund performance

Dealers create value for owners by:

- increasing executed trade volume
- increasing trade spreads
- controlling risks

There are interesting opportunities for feedback in these objectives.

[1] The customer's perspective is similar but not identical.

Market Making

Let's focus on dealers, and their function as market makers. Dealers strive to have competitive bid and ask quotations in the order book for as much of the trading session as possible under as many market conditions as possible. This puts a premium on robust processes and resiliency.

Dealers in stocks and bonds need to manage inventory. There are several thousand stocks in the US markets.

Dealers in derivatives generally do not need to manage inventory. That's good, because there are several million(!) stock options in the US markets. But they do need to manage hedges.

The effectiveness of hedging is the basis for our case study today.

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Stock Options

Profiles

Most stocks in US markets have listed puts and calls over a range of strike prices and expiration dates. Since the customer can generally buy or sell, there are four fundamental exposure profiles: long call, short call, etc. You may have heard of the option payoff. Cash settlement is a feature of index options and certain futures options; but standard stock options do not settle to cash. While the underlying interest is generally 100 shares of a particular stock, in effect the underlying is a stock trade with a particular price and quantity.

Exercise and Assignment

For every long position there is a short position. The long position holder has the right to exercise. Exercises are assigned (randomly) to holders of short positions.

Insurance Analogy

Stock options are somewhat similar to property insurance policies. Both have policy expiration dates, deductibles, premiums, and defined underlying interests. The marketplace for insurance has fewer choices: the customer can only buy, not sell (not even to close a position); and is limited to only certain affiliated underlying interests.

Ergodicity

The Law of Large Numbers is fundamental to market makers in both markets.

- In the case of insurance, the large sample is across a panel of (hopefully low concordance) underlying claims.
- In the case of options, the large sample is across (certainly low concordance) non-overlapping periodic innovations in a timeseries of the price of the underlying interest.

Dynamic Control Problem

The (hopefully) degenerate r.v. in insurance is in a panel dimension, so valuations are only loosely tied to *seriatim* policies. In options markets, each position can be individually valued and hedged according to the solution of a dynamic control problem.

Black-Scholes-Merton

This can be solved exactly under reasonable, if somewhat simplistic, assumptions:

- the underlying interest can be continuously bought and sold at a (single) price (less anticipated dividends) whose logarithm is a Brownian motion with a particular (fixed) diffusion rate
- cash can be borrowed and invested, without possibility of default, at a fixed interest rate for all tenors up to the option term to expiration
- the market is free of the possibility of arbitrage

Black-Scholes-Merton

For a long call option without the right of early exercise (i.e. "European-style"), the solution to the control problem under the BSM assumptions is to maintain a position (per call contract sold) at time t (measured in years from inception), of

$$100\,\Phi\left(\frac{\log\left((S_t - D_t)/X\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

shares in the underlying interest, where X is the option strike price, $(S_t)_{t\in[0,T)}$ is the spot price process, $(D_t)_{t\in[0,T)}$ is the anticipated discounted dividend process, r is the (non-defaultable, annual) interest rate, σ is the (annual) volatility rate, T is the initial option term to expiration (in years), and $\Phi(\cdot)$ is the standard normal df.

Black-Scholes-Merton

If you collect the correct premium up front and follow this recipe, after T years you will end up (almost surely) with a position of either: exactly 100 shares long of the stock and $-X$ in cash if $S_T \geq X$; or nothing at all if $S_T < X$. This is exactly what you need to meet the settlement obligation of any possible arbitrage-free exercise/assignment.
The solution for the other three profiles is similar.

Delta Hedging

This procedure is called Delta hedging. The name inspired by the symbol used in quantitative finance for the first partial of the derivative value wrt the underlying spot price

$$\Delta_t = \frac{\partial f(t, S_t)}{\partial S_t}$$

Since σ is essentially a free parameter, it is a surprisingly general framework for the manufacture of options positions.

Case Study

Let's examine the effectiveness of Delta hedging by looking in detail at a historical case. Let's examine Sep 2023 puts and calls on 3M Company common stock struck at $100 per share. I picked these contracts because the underlying interest is the equity of a large, stable, company. And it is has Minnesota in its name! I picked the strike price to be close to the stock price over the period. In fact, the stock closed at $101.06 at expiration on Sep 15, 2023. So the call expired in-the-money and the put expired worthless.

3M Company

- scheduled quarterly dividends of $1.5 per share were paid on May 18 and Aug 18, 2023.
- quarterly financial results became public on Apr 25 and Jul 25, 2023.

Case Study

Let's pretend we are a dealer, and that we were successful in matching with brokers near the end of the trading session on Mar 17, 2023, and that each established position was held six months until exercise or expiration.

The premiums we received were:

\$/contract	long call	short call	long put	short put
initial premium	1000	-955	795	-765

From an accounting perspective, this can be classified as gross revenue.

The ambient stock share price was at \$102.40/102.42.
This translates to an implied volatility rate of about 31.9%p.a. for the call and 31.5%p.a for the put.

Case Study

fixed volatility

For the first simulation, let's trade in MMM (the 3M stock symbol) daily based on the Delta hedge recipe above with the volatility rate fixed at the initial value.
After the settlement of any obligations from exercise/assignment any residual value in our accounts is net revenue.

$/contract	long call	short call	long put	short put
gross revenue	1000	-955	795	-765
hedging costs	718	-709	643	-634
intrinsic value	106	-106	0	0
net revenue	176	-140	152	-131

The recipe worked well for the longs, but not so well for the shorts. We really want positive net revenue for all trades.
We are market makers, not prop traders!

Case Study

implied volatility

If we lost money hedging both the short call and the short put, maybe the volatility rate declined over the period. In fact, the short-term ATM ivol did decline by about 6 points after each of the earnings announcements (which is a typical phenomenon). So perhaps the constant volatility assumption is the problem. Let's try again, this time using the implied volatility each day.

$/contract	long call	short call	long put	short put
gross revenue	1000	-955	795	-765
hedging costs	709	-701	688	-678
intrinsic value	106	-106	0	0
net revenue	185	-148	107	-87

Unfortunately, this is not much better.

Case Study

implied volatility with early exercise

But, the previous example exposes an interesting phenomenon. The call price one week before expiration actually fell to the intrinsic value. This means that it should be exercised early. Maybe factoring in early exercise will turn the short call net revenue positive.

$/contract	long call	short call	long put	short put
gross revenue	1000	-955	795	-765
hedging costs	164	-111	688	-678
intrinsic value	624	-624	0	0
net revenue	212	-220	107	-87

Nope.

Leverage Effect

These challenges suggest a deeper problem with Delta hedging; perhaps there is an important feature in the market that does not fit into the BSM worldview. For example, what if equity returns are skewed? We see this in econometric studies and it has earned a name: the leverage effect.

If implied volatility varies with the underlying spot price, then there is a second term in the option spot exposure:

$$\frac{df}{dS_t} = \overbrace{\frac{\partial f}{\partial S_t}}^{\text{Delta}} + \overbrace{\frac{\partial f}{\partial \sigma}}^{\text{Vega}} \frac{d\sigma}{dS_t}$$

Vega for long (short) options is always positive (negative), and the leverage effect means that the $d\sigma/dS_t$ is negative. So generally the leverage effect lowers the effective Delta for long positions and raises it for short positions.

Case Study

leverage effect

In this version, I calculate the Delta and Vega using the current implied volatility, and $d\sigma/dS_t$ fixed at -0.01 for simplicity.

\$/contract	long call	short call	long put	short put
gross revenue	1000	-955	795	-765
hedging costs	735	-727	728	-718
intrinsic value	106	-106	0	0
net revenue	159	-122	67	-47

We are still not covering our costs on the short positions, but the shortfall is less.

Perhaps there are other material features that we have missed (such as the drop in volatility upon earnings announcements). Perhaps the market maker cannot expect every trade to be profitable.

Revising the Dynamic Control Problem

I was walking near the campus of the University of Chicago last summer, and I saw a (bearded) older gentleman wearing a maroon t-shirt that said,

That's all well and good in practice,

I do think the dynamic control approach to hedging (and pricing) options is useful, and it is probably worth adapting the stochastic model to simulate the negative skewness of the leverage effect, even at the cost of abandoning a closed-form solution.

But keep in mind there is some subtlety to this, since the presence of the logarithm limits us to processes whose finite increments are Gumbel (having finite moments).

Conclusion

Thank you!