Foundational quant topics for clearing

Promsies Kept & Promises Broken

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The risk-free rate in Black-Scholes

The Black-Scholes value of a European-style exercise call option on an asset is

$$c_t = e^{-r(T-t)}\,\mathsf{E}^{\mathbb{Q}^T}\left[\left.(S_T - K)^+\right|\mathcal{F}_t\right]$$

in the usual notation, with $\mathbb{Q}^T$ denoting the risk-neutral forward probability measure and r denoting the risk-free interest rate $\left(\frac{1}{T-t}\int_t^T f_{t'}\,dt' \text{ if you are picky}\right)$.

Vulnerable Derivatives

But what if there is a chance that the issuer of this option will fail? In that case the theoretical value is,

$$c_t' = e^{-r(T-t)}\,\mathsf{E}^{\mathbb{Q}^T}\left[(S_T - K)^+\chi_{\{\tau>T\}}\,|\,\mathcal{F}_t\right]$$

where τ is the stopping time corresponding to issuer default.

The risk-free rate in Black-Scholes

If $t < \tau$ and the default process is independent of the underlying asset value process, this can be written as

$$c'_t = e^{-(r+h)(T-t)} \operatorname{E}^{\mathbb{Q}^T} \left[(S_T - K)^+ \middle| \mathcal{F}_t \right]$$

where h is the hazard rate of the default process under $\mathbb{Q}^T$.

Put-Call Parity

Where does the r come from? Would you even know it if the market were pricing at $r + h$? "Of course," you might be tempted to say: "I can observe put and call prices, and I know that

$$c_t - p_t = e^{-\delta(T-t)} S_t - e^{-r(T-t)} K$$

So I can regress $c_T(K) - p_t(K)$ against K and pick off the implied carry and the discount."

The risk-free rate in Black-Scholes

But actually

$$c'_t - p'_t = e^{-(\delta+h)(T-t)} S_t - e^{-(r+h)(T-t)} K$$

so using put-call parity to measure r endogenously will not work. r never appears separately from h in this analysis.

Implied Volatility

But there is an important equivalence built in to the Black-Scholes argument that we should not ignore: Under $\mathbb{Q}^T$, the (total) drift rate of S_t is supposed to be r, not $r+h$. The hedging argument is fundamental. The return on the (instantaneously) hedged portfolio should not depend on the creditworthiness of the derivative issuer.

- If we substitute $r+h$ for r throughout, then our implied volatilities will be wrong.

The credit-worthiness of derivative issuers

Obviously the potential defaultability of derivative issuers is an important topic whose significance goes beyond model calibration issues.

- Derivatives dealing is a flow business. Perceptions of potential interruptions for any reason are disruptive.

So what basis do we have to say that $h = 0$, that derivative contracts in the marketplace are effectively default-free?

Central Counterparty

The modern answer to this question is the central counterparty. This presumably emerged from the same collective phenomenon that led insurance companies to recognize the value of mutualization.

Central derivative counterparty

It should not be surprising that complete clearing first evolved in fringe marketplaces away from the influence of firms that dominated the 19th century financial centers of London, New York, and Chicago.

History

Peter Norman claims that the world's first derivative central counterparty was developed in 1882 by the commodities exchange of Le Harve, the Parisian port city.

In 1891, the first American central counterparty came about when the Le Harve model was copied by the Minneapolis Grain Exchange.

An alternative to the "over-the-counter" model

Capital center banks were not interested in complete clearing at first, probably because of the reasonable assumption that they were too big to fail. But customers demanded it and eventually got it by 1916.

Central derivative counterparty

We have about a century of experience with central counterparties; and their visibility and importance has grown over that time. In recent years, market regulators have taken interest in their role as key components of the infrastructure of the financial markets.

Finanical market infrastructure

The international CPSS-IOSCO group identifies several key financial infrastructure service types

- payment systems
- securities depositories
- securities settlement systems
- trade repositiories
- central derivative counterparties

Joint principles

The same report (dated April 2012) outlines 24 principles national regulators should consider as relevant for financial market infrastructures. Let's focus on two.

Principle 1: legal basis

A central counterparty must be able to assume market risks brought about by a derivatives trade, and it must be able to fund any necessary mitigation in the event of a failure.

- novation of a bilateral trade: the legal mechanism by which the central counterparty inserts itself between the long and short parties to a derivatives trade.
- perfection of a collateral claim: the legal mechanism by which the central counterparty assumes ownership of a defaulting party's collateral pledges with regard to bankruptcy protections.

Joint principles

Why are legal topics of interest to quants? Because derivatives are nothing but legal contracts subject to the same enforcement mechanisms available through securities law and commercial codes. Learn about them!

Principle 6: Margin

A central counterparty must plan to have on hand sufficient financial resources to eliminate the market risk of a failing member's positions.

- portfolio margin: The minimum net asset value (premiums and collateral) a clearinghouse needs to eliminate a failing member's market risk as quickly as possible under the widest possible range of circumstances.
- guaranty fund: Additional liquid resources a clearinghouse must raise from members in order to be considered well-capitalized.

Market risk

These are topics in market risk measurement, which is clearly a quant discipline. For example, consider liquid markets where risk can be eliminated over a short horizon. We can define the net asset value of the date T positions in a member's account (including collateral) at time $T+\tau$ as the random variable $P_{T+\tau}$.

Expected shortfall

The expected shortfall at confidence $1-\alpha$ is the conditional average of the lowest α quantile,

$$\mathrm{ES}(\alpha,\tau) \triangleq \frac{1}{1-\alpha}\int_0^{1-\alpha} F^{-1}_{P_{T+\tau}|\mathcal{F}_T}(q)\,dq$$

where $F(\cdot)$ represents the distribution function.

Perhaps one approach to margin is to demand collateral such that $\mathrm{ES}(\alpha,\tau)>0$ each morning.

Market risk

This may be a difficult quantity to estimate, but the theory of extreme values offers a useful approximation for sufficiently high confidence levels.

Extreme value theory

Under certain assumptions, the distribution function $F_X(x) \triangleq \mu(\{X < x\})$ of a real random variable $X : \Omega \mapsto \mathbb{R}$ can be (partially) approximated by

$$F_X(x) \approx \begin{cases} \theta \left(1 + \xi \frac{\eta - x}{\beta}\right)^{-1/\xi} & x \leq \eta \\ ? & \text{otherwise} \end{cases}$$

with tail parameter $0 < \xi < 1$, scale parameter $\beta > 0$, threshold parameter η, and tail mass $0 < \theta < 1$.

If this can be fit for some $\theta > 1 - \alpha$, then the definition of expected shortfall above can be easily evaluated in terms of these parameters.

Risk management of central counterparties

Obviously, this is only a piece of the whole. The CPSS-IOSCO report has sections on a whole panoply of risks to consider:

- systemic risk
- legal risk
- credit risk
- liquidity risk
- general business risk
- custody and investment risks
- operational risk

some of which are amenable to mathematical analysis, but much of which is not.

Questions & Discussion

Research proposal

Leland 1994

In 1994, Hayne Leland wrote an influential paper on corporate capital structure (the mixture of equity and debt financing a firm chooses). He assumed that underlying assets followed a (continuous) geometric brownian motion process and that debt obligations were perpetual

I have been playing around with the ideas in this paper for some years.

- I wrote an analysis of it in 2008 that attracted the attention of my current boss!
- I saw something weird when I tried to incorporate corporate actions.

Leland's paper has been extensively cited in subsequent work.

Research proposal

Gerber-Shiu 1998

In 1998, Hans Gerber and Elias Shiu wrote a (somewhat less influential) paper on valuing perpetual options for jump processes.

Without much effort, Leland's model can be extended to jumpy assets in this manner.

- In fact, it has probably already been done. A literature review is called for.

Øksendal-Sulem 2009

In 2009, Bernt Øksendal and Agnès Sulem wrote a book on extending the Bellman-Jacobi-Hamilton method for optimal control to jumpy processes.

Research proposal

I would like to frame the capital structure problem in these terms, again limited to perpetual debt, and look again at this curiosity I saw with respect to enterprise value of the firm.

- In Leland's model, it is possible for a firm to emerge from bankruptcy (airlines come to mind).
- I suspect there is hidden value in the equity shares that the firm issues upon emerging from bankruptcy that is not showing up in the enterprise value of the firm.

Suggestions and offers of collaboration are welcome.

Thanks!