FM 5031 Assignment

Instructor: John A. Dodson

Due: September 26, 2007

Instructions

This assignment is the basis for half of your grade for this module this semester. It is due at the beginning of lecture on September 26. You may use any materials or software on this assignment; but cite your sources, document your techniques, and justify your answer. Inadequate documentation will lower your grade. Insufficient citations or plagiarism may be considered academic dishonesty.

You are required to work alone and not to discuss this assignment with anyone other than the instructor before you turn in your solution.

Notice

I will be out of town from Sunday morning, September 23, through Wednesday afternoon, September 26; consequently I will not be available for office hours on Tuesday, September 25. Please contact me for alternate arrangements if you need to meet me before Sunday. I will be able to check e-mail and the class web log while I am away.

Problem Statement

What initial mixture of a risky and a riskless investment will maximize expected return under a drawdown constraint?

Definitions

- The risky and the riskless asset values evolve according to geometric brownian motion with given rates of drift and volatility.
- There are no dividends or other cashflows.
- "Drawdown" is defined pathwise as the maximum peak-to-trough decline in percentage terms over any holding period within the investment horizon.
- The drawdown constraint is defined probabilistically as the maximum drawdown at 95% confidence considering all possible paths.
- No portfolio re-balancing. The investment allocation is fixed at the beginning of the period.
- The initial investment is $10,000,000.
- The horizon for the problem is one year.

Parameters

You may assume the following values are known with certainty and fixed over the term of the problem.

quantity	value
annual riskless rate	5%
annual risky return	10%
annual risky volatility	30%
drawdown constraint	25%

Appendix

Geometric Brownian Motion

Geometric brownian motion is a type of stochastic process defined by $S(0,\omega) = S_0$ for all $\omega \in \Omega$ and

$$dS(t,\omega) = \mu \cdot S(t,\omega)\, dt + \sigma \cdot S(t,\omega) \cdot dZ(t,\omega) \quad \forall\, t > 0, \omega \in \Omega$$

where ω is the sample, $Z(t,\omega)$ is a realization of a standard Brownian motion, and the constant parameters μ and σ are the drift and volatility rates.

The solution to this stochastic differential equation is

$$S(t,\omega) = S_0 \cdot e^{\left(\mu - \frac{1}{2}\cdot\sigma^2\right)\cdot t + \sigma \cdot Z(t,\omega)} \quad \forall\, t \geq 0, \omega \in \Omega$$

Drawdown

Drawdown is a pathwise quantity defined by

$$\max_{0<t\leq T} 1 - \frac{\Pi(t)}{\max_{0<t'<t} \Pi(t')}$$

for portfolio total value $\Pi(t)$ and investment horizon $T > 0$.