Risk & Asset Allocation
Quiz for Week 3

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September 17, 2014

2 Say you are trying to hedge an investment. The future profit on the investment is X. The future profit on the hedge is Y per unit, which has (Pearson's) correlation $0 < \rho < 1$ with X. By what fraction can the standard deviation of the net future profit be optimally reduced?

Consider the portfolio with profit/loss $\Pi = X - \theta Y$ for fixed θ. The variance of this is

$$\operatorname{var}\Pi(\theta) = \operatorname{var}X - 2\rho\,\theta\sqrt{\operatorname{var}X\operatorname{var}Y} + \theta^2\operatorname{var}Y$$

This is minimized for $\theta = \theta^\star$ where

$$\left.\frac{d}{d\theta}\operatorname{var}\Pi\right|_{\theta^\star} = -2\rho\sqrt{\operatorname{var}X\operatorname{var}Y} + 2\,\theta^\star\operatorname{var}Y = 0$$

or

$$\theta^\star = \rho\sqrt{\frac{\operatorname{var}X}{\operatorname{var}Y}}$$

where

$$\begin{aligned}\operatorname{var}\Pi\left(\theta^\star\right) &= \operatorname{var}X - 2\rho^2\operatorname{var}X + \rho^2\operatorname{var}X\\ &= \left(1-\rho^2\right)\operatorname{var}X\end{aligned}$$

so

$$\frac{\sqrt{\operatorname{var}\Pi(0)} - \sqrt{\operatorname{var}\Pi\left(\theta^\star\right)}}{\sqrt{\operatorname{var}\Pi(0)}} = 1 - \sqrt{1-\rho^2}$$

This is surprisingly inefficient. For example, if the correlation between the investment and the hedge is 0.9, the standard deviation can be reduced by only about 56%.