2 Say you are trying to hedge an investment. The future profit on the investment is $X$. The future profit on the hedge is $Y$ per unit, which has (Pearson’s) correlation $0 < \rho < 1$ with $X$. By what fraction can the standard deviation of the net future profit be optimally reduced?

Consider the portfolio with profit/loss $\Pi = X - \theta Y$ for fixed $\theta$. The variance of this is

$$\text{var} \, \Pi(\theta) = \text{var} \, X - 2 \rho \theta \sqrt{\text{var} \, X \, \text{var} \, Y} + \theta^2 \text{var} \, Y$$

This is minimized for $\theta = \theta^*$ where

$$\frac{d}{d\theta} \text{var} \, \Pi \bigg|_{\theta^*} = -2\rho \sqrt{\text{var} \, X \, \text{var} \, Y} + 2 \theta^* \text{var} \, Y = 0$$

or

$$\theta^* = \rho \sqrt{\frac{\text{var} \, X}{\text{var} \, Y}}$$

where

$$\text{var} \, \Pi(\theta^*) = \text{var} \, X - 2 \rho^2 \text{var} \, X + \rho^2 \text{var} \, X$$

$$= (1 - \rho^2) \text{var} \, X$$

so

$$\frac{\sqrt{\text{var} \, \Pi(0)} - \sqrt{\text{var} \, \Pi(\theta^*)}}{\sqrt{\text{var} \, \Pi(0)}} = 1 - \sqrt{1 - \rho^2}$$

This is surprisingly inefficient. For example, if the correlation between the investment and the hedge is 0.9, the standard deviation can be reduced by only about 56%.