Opperational Risk

MFM Practitioner Module: Quantitiative Risk Management

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Introduction

The quantitative analysis of operational risk is still in its infancy. Chapter 13 covers the current state of the art for regulatory capital calculations, but more importantly it draws parallels with the nature of loss in a property/casualty insurance setting and introduces some potentially useful classical results and techniques from actuarial science.

It is important to note that unlike market and credit risk, one will never be compensated for bearing operational risk. Generally, the regulatory interest is to incent financial firms to identify and shed operational risk by internalizing it in minimum capital requirements.

- One challenge is that data is beginning to suggest that operational risk is hugely leptokurtokic, and that the risk measures that we have been using so far may not yield useful results once that is fully recognized.

Basic Indicator Approach

The most basic regulatory measure under the current Basle Committee guidance for the bank capital requirement for operational risk is a fixed fraction of trailing three-year gross income, according to

$$RC_{BI}^{t} = 15\% \times \frac{\sum_{i=1}^{3} \chi_{\{GI^{t-i}>0\}} GI^{t-i}}{\sum_{i=1}^{3} \chi_{\{GI^{t-i}>0\}}}$$

This clearly has nothing to do with observed or hypothetical operational losses.

- The weighted average should probably be interpreted merely as a proxy for the scale of the bank's business.
- It is intended to be simple but conservative.

Standardized Approach

Under the standardized approach, the annual gross income is allocated into eight business lines, $GI^t = \sum_{j=1}^{8} GI_j^t$

1. ($\beta_1 = 18\%$) corporate finance
2. ($\beta_2 = 18\%$) trading & sales
3. ($\beta_3 = 12\%$) retail banking
4. ($\beta_4 = 15\%$) commercial banking
5. ($\beta_5 = 18\%$) payment & settlement
6. ($\beta_6 = 15\%$) agency services
7. ($\beta_7 = 12\%$) asset management
8. ($\beta_8 = 12\%$) retail brokerage

$$RC_S^t = \frac{1}{3} \sum_{i=1}^{3} \max \left[0, \sum_{j=1}^{8} \beta_j GI_j^{t-i} \right]$$

Loss Distribution Approach

A more sophisticated "advanced measurement" approach is developing around proprietary operational loss databases.

Losses are classed into seven event types $\ell = 1, \ldots, 7$:

1. internal fraud
2. external fraud
3. employment practices & workplace safety
4. clients, products & practices
5. damage to physical assets
6. business disruption & systems failures
7. execution, delivery & process management

Data is collected on the loss magnitudes, $X_{t-i,k}^{j,\ell}$, and incident counts, $N_{t-i}^{j,\ell}$; and $X^{j,\ell}$ and $N^{j,\ell}$ are given some sort of probability distribution. Then

$$RC_{AM} = \sum_{j=1}^{8} VaR_{0.999}\left(\sum_{\ell=1}^{7}\sum_{k=1}^{N^{j,\ell}} X_k^{j,\ell}\right)$$

Compound Sums

The loss distribution approach to operational risk is evocative of the insurance analytics that actuaries employ in a non-life (property/casualty) setting. In particular, the random variable defined by

$$S_N = \sum_{k=1}^{N} X_k$$

where N is a random variable with sample space $0, 1, \ldots,$ and X_k are i.i.d. with distribution function $G(x)$ with $G(0) = 0$ is termed a compound sum.

While there are few exact results about the distribution of compound sums, there are useful results for

- low moments
- the tail index
- approximations

Compound Sums

Low Moments

If $\mathrm{E}\,N < \infty$ and $\mathrm{E}\,X_1^2 < \infty$, then

$$\begin{aligned} \mathrm{E}\,S_N &= (\mathrm{E}\,N)\,(\mathrm{E}\,X_1) \\ \mathrm{var}\,S_N &= (\mathrm{var}\,N)\left(\mathrm{E}\,X_1^2\right) + (\mathrm{E}\,N)\,(\mathrm{var}\,X_1) \end{aligned}$$

Tail Index

If there exists $\varepsilon > 0$ such that $\mathrm{E}(1+\varepsilon)^N < \infty$, and if $G(x) = 1 - x^{-1/\xi}L(x)$ where $\xi > 0$ and $L(\cdot)$ "slowly varying" in the sense of Karamata's theorem, then

$$\lim_{x\to\infty} \frac{1 - F_{S_N}(x)}{1 - G(x)} = \mathrm{E}\,N$$

which means that S_N inherits the tail index ξ of X_1.

Panjer Recursion

If X_1 is discrete, in particular if its sample space is $1, 2, \ldots,$ then S_N has the sample space $0, 1, 2, \ldots$

Panjer class

A large range of parametric random variables for N can be characterized by

$$\mathrm{P}\{N = i\} = \left(a + \frac{b}{i}\right) \mathrm{P}\{N = i - 1\}$$

for all $i \geq 1$ for some reals a and b.

Panjer Recursion

If $N \sim \mathrm{Panjer}(a, b)$, then $\mathrm{P}\{S_N = 0\} = \mathrm{P}\{N = 0\}$ and

$$\mathrm{P}\{S_N = r\} = \sum_{i=1}^{r} \left(a + \frac{bi}{r}\right) \mathrm{P}\{X_1 = i\} \mathrm{P}\{S_N = r - i\}$$

for $r \geq 1$.