Credit Risk

MFM Practitioner Module: Quantitative Risk Management

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Introduction

The quantification of credit risk is a very difficult subject, and the state of the art (in my opinion) is covered over four chapters of our text. We will only be covering the first, which is about the valuation of individual defaultable claims. There are three main random objects in this domain -

- The default stopping time, τ
- The hazard rate process associated with this time, γ_t
- The impact of the default on the value of the claim, δ_τ

Keep in mind that there are fundamental (Knightian) uncertainties associated with the default event itself and the recovery process, probably involving commercial law (e.g. bankruptcy) and possibly systemic phenomena (e.g. bank runs), both entailing local idiosyncracies and agency effects.

Incomplete Markets

It is also important to recognize that most credit risk is only partially hedgeable.

Risk-Neutral Pricing

The theory of arbitrage concludes that the value of an arbitrage-free contingent claim is an expectation of the risk-free net present value under a risk-neutral measure $\mathbb{Q}$.

- But $\mathbb{Q}$ is only uniquely identified if there is a perfect hedge. With credit risk there is rarely a perfect hedge, so valuations in general are not unique.

The best you can expect from the theory then is a range of arbitrage-free prices and this has implications for liquidity risk. Unfortunately, this topic is beyond our current scope.

Instruments

The salient different between a bond and a loan is the existence of a liquid secondary market in the former from which price processes can be sampled. More technically, bonds usually possess identifiers issued by a national numbering agency recognizing them as securities subject to securities law and depository services, and they often possess ratings issued by one or more nationally recognized statistical rating organization. Loans in contrast are generally not individually rated or marketed, but may be combined into securitizations.

Over-The-Counter Instruments

Money market instruments, such as loans between financial institutions, are not registered securities, but are generally considered liquid. A number of benchmarks are available to assist with their valuations (at least in aggregate), such as LIBOR and OIS.

Instruments

You may be surprised to learn that often credit default swaps on major corporate or financial creditors are more liquid than the bonds they insure.

Credit Default Swaps

CDSs, which can be either exchange-traded and centrally-cleared or over-the-counter, have more standardized contract terms than bonds. At origination, the protection buyer agrees to make fixed, periodic payments for a fixed term on a fixed notional (the swap rate), and the protection seller agrees in exchange to purchase a particular bond or subset of bonds at face value in event of a default event.

Effectively, the purchaser of a fixed-coupon defaultable bond can eliminate default risk by also purchasing a corresponding CDS. (Presumably, the coupons less the protection payments are close to the coupons (i.e. yield) of a similar Treasury bond trading near par value.)

Quantitative Methodologies for Credit

There are historically three main approaches to developing market risk models for credit securities, driven by available data.

- Ratings Migration based on aggregate data from ratings agencies
- Corporate Structure based on quarterly financial filings and other accounting disclosures
- Hazard Rates based on historical price timeseries, e.g. for CDS rates
  - Stochastic Hazard Rate models borrow from the extensive literature on short-rate term structure models

N.B.: These models tend to neglect risky recovery, and there is some justification for this; but I want you to think about this in the assignment for this week.

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Ratings agencies are paid to rate securities, but for major creditors they also rate the obligor. Agencies also publish statistics about the conditional dynamics of obligor ratings, which some researchers have attempted to use to fit Markov-chain models. The goal of this is to illuminate the real-world dynamics of (aggregate) yield spreads and to forecast real-world default probabilities for long-tenor bonds.

Markov Processes

A Markov process is a discrete filtered random variable whose evolution is entirely described by its current state. Agency ratings do not seem to be well-described by Markov processes: both the direction (upgrade/downgrade) of the previous ratings change and the duration since that change seem to condition the next ratings transition. It may be possible to introduce additional "hidden" state variables to capture this, but we do not typically have the historical data to calibrate these expanded versions.

Structural Models

An alternate approach to measuring the expected default frequency is to model the firm's balance sheet (capital structure). The main assumption in the simplest version of this is that default is strictly a consequence of negative equity. We have already seen an alternate model (in assignments from the fall) in which a structural model is assembled around insolvency, which may be more useful.

Accessible Default

A problem with structural models based on continuous processes for assets is that the default event has no inherent randomness: We can observe the equity process transition continuously from positive to negative. This is termed an accessible event.

A consequence of this is that the implied credit spreads continuously go to zero as the tenor goes to zero, which is unrealistic. A jumpy model for the assets or a fuzzy boundary for the liabilities is a vital remediation.

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The main alternative to the structural model approach is the hazard rate approach, which is based on more actuarial concepts. Here, the default stopping time can be codified as the point at which $\int_0^\tau \gamma(t)\,dt = X \,|\mathcal{F}_0 \sim \exp(1)$ where $\gamma(t) > 0$ is the (deterministic) hazard rate function.

- If γ is very small, it might take a long time for the integral to accumulate to X.
- If X happens to be very small or very large, the magnitude of γ might not matter much at all.

Default here is an inaccessible event.

Evaluating Expectations

Conveniently, we can calculate simple expressions for $\mathrm{E}\,\mathrm{I}_{\{\tau \leq T\}}$, $\mathrm{E}\,e^{-r(\tau\wedge T)}$ and other useful expressions for valuing credit-risky cashflows with this model.

Stochastic Hazard Rates

A key feature of the hazard rate model, especially in light of the increasing availability of high-quality historical CDS spreads data, is that it continues to work even if we replace the deterministic $\gamma(t)$ with a stochastic process γ_t. We term the resulting default stopping time doubly stochastic. This works very well in the context of risk-neutral valuation, because of the parallel role played by the instantaneous risk-free rate r_t. For example, if $\tilde{X}$ is $\mathcal{F}_T$ measurable (e.g. a European-style payoff),

$$\mathrm{E}_0\left[\mathrm{I}_{\{\tau>T\}}\exp\left(-\int_0^T r_t\,dt\right)\tilde{X}\right] = \mathrm{I}_{\{\tau>0\}}\,\mathrm{E}_0\left[\exp\left(-\int_0^T\left(r_t+\gamma_t\right)\,dt\right)\tilde{X}\right]$$

Affine Models

Working with doubly stochastic default times puts us in a similar valuation framework as stochastic short-rate models, and we can borrow solution techniques from there. In particular, there is a class of stochastic processes r_t such that

$$\mathrm{E}_t\left[e^{-\int_t^T r_s\,ds}\right] = e^{\alpha(t,T)+\beta(t,T)r_t}$$

where α and β are solutions to particular linear ordinary differential equations.

- The ODE for β is a Riccati equation, and the ODE for α is essentially just integration.
- A famous example with an explicit solution is the Cox-Ingersoll-Ross model.

The class of stochastic processes for which this separation of variables can be achieved is called the affine class. This class is relatively rich, and includes mean reversion and jump diffusion features.