Basic Concepts in Risk Management

MFM Practitioner Module: Quantitative Risk Management

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Module Introduction

My goal is to provide you with a grounding in applied probability and statistics as it relates to financial risk management. Quizzes and assignments motivate the acquisition of vocabulary, financial and mathematical concepts, and scientific computing techniques. Projects provide exposure to the practice of professional research.

- `http://www-users.math.umn.edu/~dodso013/fm503/`
- module syllabus
  - schedule
  - evaluations & grading
- module text
  - McNeil-Frey-Embrechts
- module discussion & office hours
  - Piazza, real-time
  - Zoom, Sundays 7:00 PM

Outline

Financial Accounting

If you are going to work with bankers, traders, or investment managers, it is important for you to understand the language and concepts of accounting, commercial law, finance, and investment performance measurement.

Financial accounting

Financial accounting is contrasted with managerial accounting in that it is directed at outsiders. Consequently, its terms and concepts are highly standardized and its application is usually subject to audit.

Double-Entry Bookkeeping

Concepts

entity concept autonomy with rights and obligations

going concern concept assume that the entity will persist

balance sheet financial condition at a point in time

income statement financial activity over a period in time

account elements asset, expense; liability, revenue, capital

journal entry amount, debit account, and credit account

closing the books periodic adjustment of the balance sheet

accounting identity assets $=$ liabilities $+$ capital

N.B.: An entity's assets may include shares of other entities' debt and equity.

Financial Statements

Wells Fargo & Company

Income statement 2018 ($ billions)

net interest	50	*credit provisions*	2
commissions/fees	27	other expenses	56
other income	9	income taxes	6
		dividends	9
		retained earnings	13
total revenue	86	total	86

Balance sheet 12/31/2018 ($ billions)

cash	173	deposits	1,286
investments	648	short-term debt	184
loans	970	long-term debt	229
loan allowance	-10		
other assets	115	capital	197
total assets	1,896	total	1,896

Investment Accounting

Investment accounting uses single-entry bookkeeping on a mark-to-market basis with a daily net asset value

In place of the dual aspect accounting identity, we have

$$\text{net assets} = \text{net cash} + \sum_{i \in \text{holdings}} \text{price}_i \times \text{quantity}_i$$

Note the liquidity assumption: Unlike in normal microeconomics, price here does not depend on quantity.

- Cash enters and leaves the portfolio through subscriptions and redemptions or dividends.
- Cash also changes through transactions which create or modify holdings.
- Net cash is adjusted for unsettled trades, taxes payable, and accrued interest and fees.

Performance Measurement

- daily return is measured as

$$1+\text{daily return}_t = \frac{\text{net assets}_t - \text{subscriptions}_t + \text{redemptions}_t + \text{dividends}_t}{\text{net assets}_{t-1}}$$

- this may be interpreted as a weighted average

$$\text{daily return}_t = \sum_i \text{weight}_{i,t} \times \text{daily return}_{i,t}$$

where the (beginning) weights satisfy

$$\sum_i \text{weight}_{i,t} = 1 - \frac{\text{net cash}_{t-1}}{\text{net assets}_{t-1}}$$

- return over longer periods is measured "geometrically"

$$\prod_{t\in\text{period}} (1 + \text{daily return}_t) - 1$$

Securities

A security is a claim on future cashflows from its issuer

- U. S. Treasury
  - (discount, nominal, floating, indexed) bill/note/bond
- bank (SIFI: systemically important fin. instit.)
  - interbank loan/deposit, commercial paper
  - swap, over-the-counter derivative, currency contract
  - depositary receipt, exchange-traded note
- corporation
  - (common, preferred) equity share
  - (secured, senior, subordinated, convertible) bond
  - (short-term) commercial paper
- municipality
  - (revenue, general obligation) bond
- derivatives clearinghouse (SIFMU: SIF mkt. utility)
  - futures, option, credit default swap
- collective investments
  - (open-ended, closed-ended, exchange-traded) fund and unit trust

Capital

The term capital comes up in various contexts in economics, finance, and accounting and it has various meanings across these contexts. We will use the definition in QRM §2.1.3:

Capital

...items on the liability side of a balance sheet that entail no (or very limited) obligations to outside creditors...

Capital in this sense can be raised through a sale of equity shares, but it cannot be borrowed. The capital of a firm ultimately represents the invested wealth of the firm's owners.

- Capital is available to absorb losses;

but if the firm is incorporated as a limited liability entity, the owner's potential loss is limited by the value of his or her equity stake in the firm.

Loss Distribution

The first step to risk management is to determine what accounting metric is most representative of loss or potential loss to the firm's owners. Ideally this would relate directly to capital; but since our definition of capital is linked to the balance sheet and is subject to a complicated and relatively infrequent updating process (typically quarterly for public disclosure and monthly for private regulatory disclosure), it is typical to rely on an investment accounting proxy such as mark-to-market profit/loss on the trading book.

Projection models under a $\mathbb{P}$-type measure

A loss distribution presumes an analysis horizon $t+h$, typically a few days to a couple of weeks out from a well-defined present moment t. The projection model must define random variables for all relevant risk factors X_{t+h} under the sigma-algebra $\mathcal{F}_t \in \mathbb{F}$ and objective (public) or subjective (private) real-world probabilities.

Loss Distribution

In addition to projection models, in order to form a loss distribution we need to compose the risk factors into our chosen accounting metric, such as mark-to-market loss. If the holdings are assumed to be fixed over the horizon and we have a risk factor for each price, this may be a simple matter. If we have risk factors for interest rates or bond yields or implied volatilities, we will need additional models to value the holdings in terms of these risk factors.

Valuation models under a $\mathbb{Q}$-type measure

Generally these models will entail risk-neutral valuation, which is our only guarantee that the valuations will be free of arbitrage. The risk-neutral probability measure is generally not unique if markets are incomplete (which they are!).

Loss Distribution

Calculation techniques

Analytical method

Historically the first method to be popularized for calculating the loss distribution, under the J.P. Morgan Value-at-Risk model, made severe assumptions about the linearity of exposures and the normality of risk factors in order to arrive at an analytic description of the loss distribution. Semi-analytic methods, such as the Delta-Gamma model based on the Cornish-Fisher expansion have also been used.

Historical simulation method

Historical simulation is based on the applying the empirical distribution of past risk factor changes to the present holdings. It is also relatively easy to implement, but entails a fairly severe assumption about the nature of risk in the future being fully captured by the relatively recent past.

Monte Carlo method

A more accurate, but also more computationally intense, approach to calculating the loss distribution is to replace history with simulation to calculate an empirical distribution of arbitrary fineness.

- This requires fully parameterized projection models and tractable valuation models.

A typical Monte Carlo size is of order 10^4; but a much larger simulation may be required if precision is important.

Equilibrium calibration

We will be exploring this later this term, but financial timeseries tend to exhibit periods of low volatility and periods of high volatility. Nonetheless, over long horizons the distribution of residuals seems to be stationary. Depending on your purposes, this long-run stationary distribution might be more appropriate than a short-run distribution which might tend to promote pro-cyclical behavior.

Conditional calibration

If your goal is more concrete, to make the best possible estimate of the loss distribution for $t + h$, you can estimate econometric models for your risk factors that account for the conditionality in short-run volatility. Obviously, this will lead to more volatile risk metrics and possibly more reactionary behavior from users.

Risk Measurement

Methodology

Notional Exposure

For simple uni-directional (e.g. long-only) portfolios with mostly linear exposures to a small number of risk factors, a simple weighted Notional-at-Risk might be adequate for measuring risk. Traditional minimum capital requirements for banks is based on this approach.

Loss Distribution

Once you have a loss distribution, a natural metric is the quantile at some fixed confidence level α. This is the basis for J.P. Morgan's Value-at-Risk. Quantile-based risk measures do not work well for credit risks, and we will explore coherent alternatives extensively.

Stress Scenarios

Loss distributions encode a specific real-world probability measure $\mathbb{P}$. Financial history suggests that the most significant losses come from exceptional events that are not well foretold by history.

Risk managers are therefore encouraged to construct their own stressed probability measures $\mathbb{P}^\star$. Sometimes this is done by inflating the the parameters fit to historical data. Another approach is to define a set of generalized scenarios.

This also has the advantage of simplicity, but the potential for adverse surprises if the scenarios are not sufficiently comprehensive.

Risk Measurement

Value-at-Risk

When it was first introduced by J. P. Morgan, it was argued that $L = b'X$ with $X \sim \mathcal{N}(\mu, \Sigma)$ was an adequate description of exposure and risk in the market. We can recover a simple expression for the marginal decomposition.

$$\mathrm{VaR}_\alpha = q_\alpha(F_L) = b' \left(\mu + \Phi^{-1}(\alpha) \frac{\Sigma b}{\sqrt{b'\Sigma b}} \right)$$

A similar result holds generally even if X is not normal:

$$q_\alpha\left(F_{b'X}\right) = b' \mathrm{E}\left[X \,\middle|\, b'X = q_\alpha\left(F_{b'X}\right) \right]$$

It is easy to interpret this in a simulation setting.

1. sample the risk factors N times and evaluate the loss in each
2. sort them in descending order and isolate a range of results around $\alpha \times N$
3. the marginal value-at-risk for each position is the average in this range of the contribution

Beyond the normal approximation, the next most useful approximation to the quantile risk measure comes from the Cornish-Fisher expansion.

Cornish-Fisher Expansion

In general,

$$q_\alpha(F_L) = \mathrm{E}(L) + \mathrm{sd}(L)\left(z_1(\alpha) + \frac{z_2(\alpha) - 1}{6}\,\mathrm{sk}(L)\right) + \cdots$$

where $z_1(\alpha) = \Phi^{-1}(\alpha)$ and $z_2(\alpha) = z_1(\alpha)^2$.

A good risk measure should respect diversification, in the sense that if L_1 and L_2 are random variables for the loss associated with two investments, then

$$\varrho(L_1 + L_2) \leq \varrho(L_1) + \varrho(L_2)$$

If not, then application of the risk measure for allocation decisions may encourage concentrations.

Value-at-Risk

Value-at-risk is not necessarily subadditive. An example of the problem was popularized by Claudio Albanese. The value-at-risk of a diversified portfolio of loans can be reduced to zero by concentrating all of the investment into a single loan as long as the probability of default over the analysis horizon is less than the complement of the confidence level.

Risk Measurement

Coherence

- Monotonic

$$L_1 \leq L_2 \text{ almost surely} \implies \varrho(L_1) \leq \varrho(L_2)$$

- Translation Invariant

$$L_1 \text{ constant a.s.} \implies \varrho(L_1 + L_2) = \varrho(L_1) + \varrho(L_2)$$

- Positive Homogeneous

$$\lambda > 0 \implies \varrho(\lambda L_1) = \lambda \varrho(L_1)$$

If a risk measure is subadditive, monotonic, translation invariant, and positive homogeneous, it is termed coherent.

Risk Measurement

Expected Shortfall

The fact that value-at-risk is not generally subadditive has led to a modified definition: expected shortfall.

$$\mathrm{ES}_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_u(F_L)\,du$$

The marginal decomposition is similar to that of value-at-risk.

$$\mathrm{ES}_\alpha = p'\mathrm{E}\left(X \,\middle|\, p'X \geq q_\alpha\left(F_{b'X}\right)\right)$$

It is also subject to the same Cornish-Fisher expansion, with the modification

$$\tilde{z}_1(\alpha) = \frac{1}{1-\alpha}\int_\alpha^1 z(p)\,dp$$

$$\tilde{z}_2(\alpha) = \frac{1}{1-\alpha}\int_\alpha^1 z(p)^2\,dp$$

Risk Measurement

Cornish-Fisher Expansion

It is instructive to compare the Cornish-Fisher representations of value-at-risk and expected shortfall

α	$z_1(\alpha)$	$z_2(\alpha)$
0.95	1.65	2.71
0.99	2.33	5.41

vs.

α	$\tilde{z}_1(\alpha)$	$\tilde{z}_2(\alpha)$
0.95	2.06	4.39
0.99	2.67	7.20

We see that expected shortfall is more sensitive to skewness than value-at-risk.

Normal Loss

If the loss distribution is normal, value-at-risk and expected shortfall are equivalent, in the sense that $\text{ES}_\alpha = \text{VaR}_{\tilde{\alpha}}$ and there is a simple correspondence between α and $\tilde{\alpha}$ independent of L.

An alternate representation of expected shortfall is

$$\mathrm{ES}_\alpha = \sup_q \left(q + \frac{1}{1-\alpha} \mathrm{E} \left(L - q \right)^+ \right)$$

and, in fact, the optimum value of the argument above is

$$\mathrm{VaR}_\alpha = q^\star$$

This is an intuitive result, in that

$$\mathrm{ES}_\alpha = \mathrm{VaR}_\alpha + \frac{\mathrm{E} \left(L - \mathrm{VaR}_\alpha \right)^+}{\mathrm{P} \left\{ L > \mathrm{VaR}_\alpha \right\}}$$

But the presence of the sup in the dual representation is a useful feature in an optimization setting.