

A few people have misplaced their midterms and have asked for copies. In response I've posted copies of the problems here.

- Find the Cartesian equation for the plane which includes the points  $P(-1, 2, 0)$ ,  $Q(0, 3, 1)$ , and  $R(1, 2, 3)$ .
- Write an equation for the quadric surface which would most closely resemble the Earth. You should center the Earth at the origin. The distance from the center to the North (or South) Pole is about 3810 miles, while the distance from the center to any point on the equator is about 3822 miles.
- Let  $T$  and  $S$  be the following linear transformations:

$$T(x, y, z) = (2y, -3x + y, x + y + z, z - y)$$
$$S(x, y, z, w) = (w, y, z, x)$$

- (a) Write down the matrix which represents  $T$ . (b) Find the linear transformation  $S \circ T$ , or explain why the composition is not possible. Do the same for  $T \circ S$ . (You may do this with or without matrices, according to your preference.)
- Find a parametrization for the curve which is the intersection of the cylinder  $x^2 + y^2 = 3$  and the plane  $z = 4x - 5y + 12$ .
  - Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined as follows:

$$f(x, y) = \begin{cases} 1 - |y| & \text{for } |y| < |x| \\ 1 - |x| & \text{for } |y| \geq |x| \end{cases}$$

Find  $\frac{\partial f}{\partial x}$  at the point  $(x, y) = (0, 2)$ , or show that it does not exist.

- Find the angle between the following vectors in  $\mathbb{R}^5$ . Be clear if you are expressing your answer in degrees or radians.

$$\vec{a} = (1, 0, 2, 0, 3)$$
$$\vec{b} = (2, 0, 1, 1, 1)$$

- Let  $f(x, y) = (x - y, x^3, -2y)$ , and  $g(x, y, z)$  be a function such that the Jacobian of  $g$  is given by

$$J_g(x, y, z) = \begin{pmatrix} 1 & 3z^2 & 6yz \end{pmatrix}$$

Find the Jacobian of  $h = g \circ f$  at the point  $(1, 1)$ . Then write down the linear approximation of  $h$  at the point  $(1, 1)$ . (This was also called the equation of the tangent plane. You may leave the equation in a form which includes matrices and/or vectors, if you wish.) Assume that  $h(1, 1) = 5$ .

- Let  $f(x, y) = xe^y - y$ . Compute the directional derivative of  $f$  at the point  $(2, 0)$  in the direction of the vector  $\vec{v} = (3, 4)$ .
- Compute the integral  $\int_C F \cdot d\vec{x}$  where  $F(x, y) = (e^y, xe^y - 1)$  and  $C$  is the part of the parabola  $y = x^2$  from  $(0, 0)$  to  $(1, 1)$ .
- Consider the following triple integral:

$$\int_0^3 \int_0^{2-\frac{2}{3}x} \int_0^{1-\frac{x}{3}-\frac{y}{2}} f(x, y, z) dz dy dx$$

Draw a picture of the region of integration. Then rewrite this integral so the order of integration is  $dx dz dy$ .

- Let  $C$  be the counter-clockwise oriented boundary of the region  $R$ , which is bounded by the functions  $y = x^2$  and  $y = 3x$ . Let  $F(x, y) = (-4xy, 2y)$ . Evaluate the following integral using Green's Theorem.

$$\int_C F \cdot d\vec{x} = \int_C -4xy dx + 2y dy$$

- Compute the integral  $\int_C F \cdot d\vec{x}$ , where  $F(x, y) = (-y, 0)$  and  $C$  is the counter-clockwise quarter circle of radius 1 in the first quadrant.
- Exercise 6.3.1
- Exercise 5.8.6
- Find the absolute value of  $\frac{\partial(x, y)}{\partial(s, t)}$  for the transformation  $x = st$ ,  $y = s/t$ .
- Exercise 5.6.12
- Exercise 6.4.1
- Exercise 5.5.3, slightly modified so that  $x$  is between -2 and 3.