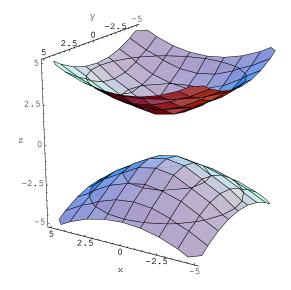
1.3, #13: We can begin by dividing through by -5 to get

$$-\frac{x^2}{5} - \frac{y^2}{5} + \frac{z^2}{(5/2)} = 1$$

and we see that this matches the format of a hyperboloid of two sheets, with  $a^2 = 5$ ,  $b^2 = 5$ , and  $c^2 = 5/2$ . Below is a graph of this surface using our good friend Mathematica; of course, your sketch doesn't have to look as good as this.



1.7, #24ac: For part (a), we first need to find the points in the plane (we're given points in the xy-plane, not the z = 5x + 2y plane) and then show they form a parallelogram. We're given a rectangle in the xy-plane, so we can use the x and y coordinates of that rectangle to find the portion of the plane lying above those x and y coordinates. We just plug them into the equation z = 5x + 2y, so the points we're interested in are:

These points form a quadrilateral in space, and to show that the quadrilateral is actually a parallelogram, we just need to show that its sides are parallel. We do this using vectors, of course. Label the points above A, B, C, and D respectively. We can find the vectors  $\vec{AB}$  and  $\vec{CD}$ :

$$\vec{AB} = (3 - 1, 1 - 1, 17 - 7) = (2, 0, 10)$$
  
 $\vec{CD} = (1 - 3, 2 - 2, 9 - 19) = (-2, 0, -10).$ 

Since  $\vec{CD}$  is a scalar multiple of  $\vec{AB}$  (the scalar here is clearly -1), they are parallel. You do the exact same thing to show that the vectors  $\vec{BC}$  and  $\vec{AD}$  are parallel.

Part (c) asks you to find the area of the parallelogram. That's given by the length of the cross product of two vectors that span the parallelogram (i.e., two non-parallel sides). For example, you could use the vectors  $\vec{BA}$  and  $\vec{BC}$ , find their cross-product (which is another vector), and then find the length

of that resulting vector. That length is the same as the area of the parallelogram. You should get  $\sqrt{120}$  as the area.

1.8, #10: To find the equation of a plane, we need three points in the plane that are not collinear. Here we're given an entire line (infinitely many points!) and a point not on the line (you can check that it's not on the line). So we just need two points from the line; we can find those points by picking any two values of t. To make our lives easy, let's use t = 0 and t = 1. From those, we get the points (-1, 7, 1) and (3, 10, -1).

Now we use those three points to find two vectors parallel to the plane. I used (-10, 0, 11) as my "base point" to get the vectors (9, 7, -10) and (13, 10, -12). If we take the cross product of those vectors, we will find a normal vector to the desired plane, so a normal vector is:

$$(9, 7, -10) \times (13, 10, -12) = (16, -22, -1).$$

From here, you can figure out the equation of the plane; it's 16x - 22y - z = -171.

A current version of these solutions is available at http://www.math.umn.edu/~drake Dan Drake <drake@math.umn.edu> September 30, 2003