1.9 #12: We can use the vector that goes from $(0,3,-2)$ to $(1,5,8)$ to describe this line segment. That vector is $(1,2,10)$, so we can parametrize this line segment with

$$(0,3,-2)+t(1,2,10)$$

for $0 \le t \le 1$. You can verify that this starts at $(0,3,-2)$ and ends at $(1,5,8)$.

1.10 #9: Given $\boldsymbol{x} = (e^t \cos t, e^t \sin t, e^{-t})$, we can find a tangent vector by taking the derivative. The derivative of a vector-valued function is just the derivative of each component, so we find that

$$\frac{d}{dt}\boldsymbol{x} = (e^t \cos t - e^t \sin t, e^t \cos t + e^t \sin t, -e^{-t})$$

Plugging in $t=0$, we see that a tangent vector to the curve (when $t=0$) is $(1,1,-1)$. We'll use this tangent vector as a direction vector for the line we're going to find.

Also, using the original function, at $t=0$ the curve is at the point $(1,0,1)$ in space. Now we know a point on the line and a direction vector for the line, so we can parametrize it:

$$(1,0,1)+s(1,1,-1).$$

2.2 #9: If A is a 3×3 matrix with

$$A\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 0 \end{bmatrix},$$

then we know that the first column of A must be

$$\begin{bmatrix} 3 & ? & ? \\ -2 & ? & ? \\ 0 & ? & ? \end{bmatrix}$$

because the vector ignores the second and third column (second and third entries are zero) and multiplies each entry of the first column by 2.

Similarly, given that

$$A\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \\ 7 \end{bmatrix},$$

we know that

$$A = \begin{bmatrix} 3 & ? & -1 \\ -2 & ? & 3 \\ 0 & ? & (7/3) \end{bmatrix}.$$

Finally, we're told what the second column is, so that when asked to find

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

we can just do normal matrix multiplication. Your answer will, of course, be one of the columns.

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