2.3 #21: In order to show that $T(\boldsymbol{x}) = \boldsymbol{a} \times \boldsymbol{x}$ (for a given $\boldsymbol{a} \in \mathbb{R}^3$) is a linear transformation, there's two things we can do:

(1) show directly that for any $\boldsymbol{x}$ and $\boldsymbol{y}$, we have $T(\boldsymbol{x} + \boldsymbol{y}) = T(\boldsymbol{x}) + T(\boldsymbol{y})$, and also that for any scalar c, we have $T(c\boldsymbol{x}) = cT(\boldsymbol{x})$. Or...

(2) show that $T(\boldsymbol{x})$ can be described by a matrix and appeal to the properties of matrices to insure that the transformation really does have the properties we listed in (1).

I'll do (2); only one person in my sections used the method in (1) and it would take a long time to write out.

Let's say that $\boldsymbol{a} = (a_1, a_2, a_3)$ and $\boldsymbol{x} = (x_1, x_2, x_3)$. The cross product of these two vectors is given by (you can use the determinant method to figure this out)

$$(a_2x_3 - a_3x_2,\ a_3x_1 - a_1x_3,\ a_1x_2 - a_2x_1)$$

and we can translate this into a matrix by examining the coefficients of the x_is:

$$\begin{bmatrix} 0 & -a_2 & a_3 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Since $T(\boldsymbol{x})$ can be represented by a matrix, it is a linear transformation.

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