**OPTIMAL EFFICIENT FRONTIER USING GENERALIZED HYPERBOLIC DISTRIBUTION DURING CRISIS**

**DISSERTATION**

Submitted in fulfillment of

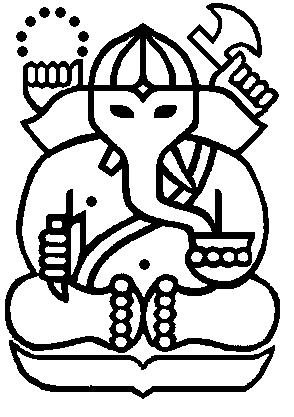
the requirements for Doctoral Degree

by

Colin Gan Cheong Kiat

39008306

(Doctoral Study Program of Science in Management)

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**INSTITUT TEKNOLOGI BANDUNG**

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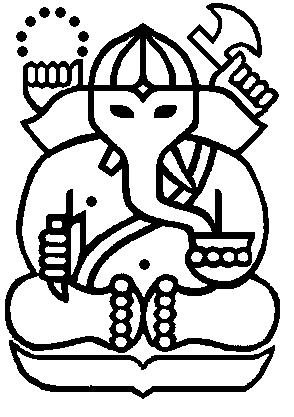
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**ABSTRACT**

**OPTIMAL EFFICIENT FRONTIER USING GENERALIZED HYPERBOLIC DISTRIBUTION DURING CRISIS**

by

Colin Gan Cheong Kiat

39008306

Portfolio management is a highly uncertain process. No one really knows what the outcome will be with certainty based on any investment or speculation decision taken to produce the propitious expected return wanted. This leads to risks. The risks are further exacerbated during crisis which cannot be averted because of disruptive financial jolts i.e. sudden and unprecedented events e.g. subprime mortgage crises in 2008 which are inimical. To find the certainty amidst the great uncertainty during crisis, risks needs to be quantified and managed efficiently and optimally to produce the propitious expected return wanted. Take care of the risks and the propitious expected returns will take care of itself. This dissertation is about quantification of market risks then managing them efficiently and optimally to produce the propitious expected return wanted using the efficient frontier created with generalized hyperbolic distribution during crises. This is tenable. The importance of this dissertation is demonstrating how to simplify the calculcation of risk factors by allowing them to be linearized. Practically, the results of this dissertation show that it is possible to obtain an optimal efficient frontier using generalized hyperbolic distribution during crisis. Theoretically, the results of this dissertation show that linearized risk factors fit the Generalized Hyperbolic distribution.Thecontributions from this dissertation are using the simplest implementation methods to test the formulated mathematical model. Inspired by Wolfram (2010), this dissertation is divided into four sections. First, posing the right questions on what is the appropriate probability distribution and risk measure to use during crisis discussed in the introduction. Second, the appropriate mathematical model on the appropriate probability distribution and risk measure is formulated for crisis situation discussed in section 4. Third, computation discussed in section 5. Lastly, verification of the formulated model to the crisis situation discussed in section 5.

**Keywords**: Portfolio management, efficient frontier, generalized hyperbolic distribution

**GUIDELINES FOR THE USE OF DISSERTATION**

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I would also like to express my special gratitude to my family especially mom, dad and brother for guiding, believing and praying for me to complete my PhD. My dad is also my role model as an exemplary scientist. A very special mention go to Yenny, my future wife, and Nuel, my very good friend for assisting me to complete my PhD. Last but not least, I am very grateful to the Almighty God for seeing me through the PhD journey.

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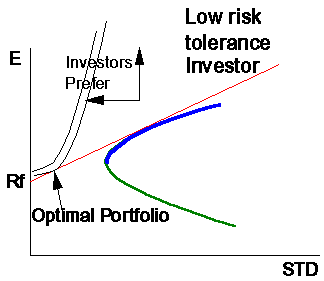
# INTRODUCTION

## I.1 BACKGROUND

Portfolio management is essentially about management of risk factors leading to portfolio loss over the pertinent time horizon. Risk factors are logarithmic price of financial assets, yields or logarithmic exchange rates. Traditionally, risk factors are modelled using the bell curve which shows how values fluctuate about the average because of random effects combining together. Bell curves are explained with two numbers: the average and the standard deviation. The first provides the value at which the bell curve peaks while the latter provides the spread of the values either side. Crises like the global financial crises which started in 2008 do not follow bell curves. They are well-represented by their special curve which is not symmetrical. Financial crisis can be applied to different situations in which some financial assets suddenly lose a large part of their nominal value detected through structural change testing (Li, 2012).

Portfolio development encompasses several steps. First, determine the asset classes to be part of it e.g. stocks, bonds, currencies etc. Second, determine the weightage of the asset classes given the risk and expected return agreed. Capital asset pricing model (CAPM) is one of the most well-known and commonly used models for portfolio construction based on Markowitz’s model of portfolio choice, also known as the mean-variance model. Two assumptions are made: investors are risk-averse; investors only care about the mean and variance during one period investment return. Two options result from that: knowing the expected return of the portfolio, minimize the variance; knowing the variance of the portfolio, maximize the expected return.

One of the most basic tenets of finance theory and practice is that returns above the　riskless rate come with increased risk. This is the basis of Markowitz’s portfolio theory (Markowitz, 1959) and of the Capital Asset Pricing Model (CAPM) (Merton, 1990). Investors want to be sufficiently compensated for taking on risk. They want to earn a high enough return to make them comfortable with the level of risk they are assuming. Thus, it is a fundamental premise of efficient markets that one cannot increase the return and lower the risk at the same time. This result originates simply from the linear (respectively quadratic) dependence of the average return (respectively variance) of a portfolio return on the weights of its constituting assets leading to a parabolic efficient frontier in the return-risk diagram. This is the meaning of mean-variance efficient.



**Fig. 1.1**: Mean-Variance Efficient Frontier

Source: <http://people.duke.edu/~charvey/Classes/ba350/control/opc.htm>

Fig. 1.1 depicts the mean-variance efficient frontier. Rf refers to risk-free rate from US t-bills. E refers to expected return. STD refers to standard deviation, measurement of risk. Higher risk will lead to higher return. The upper half of the efficient frontier is what investors strive for. A risk-loving investor will want to invest above the Rf. A risk-averse investor will want to invest along the Rf. For the portfolio to be mean-variance efficient, Sharpe (1964) and Lintner (1965) added the following assumptions:

1) complete agreement: the joint distribution of asset returns from t – 1 to t is true with the market clearing asset prices at t – 1. In this case, true means the joint distribution is used to test the model drawn.

This means the market is already efficient with the best price available.

2) borrowing and lending at the same risk-free rate for all regardless of amount borrowed or lent.

This means no arbitrage can be performed.

However, in the real world, the variance of portfolio returns provide only a limited quantification of incurredrisk, as the distributions of returns have “fat tails” (Gopikrishnan, Meyer, Nunes Amaral, & Stanley, 1998;Lux, 1996; Lux& Marchesi, 1999) and the dependences between assets are only imperfectly accounted for by thecorrelation matrix (Litterman & Winkelmann, 1998). Value-at-Risk (Jorion, 1997) andother measure of risk (Artzner, Delbaen, Eber, & Heath, 1996;Bouchaud, Sornette, Walter, & Aguilar, 1998;Sornette, 1998;Sornette,Simonetti, & Andersen, 1998) have been developed to account for the larger moves allowed by non-Gaussiandistributions. This is most prevalent during crises like collapse of Bear Stearns in 2008 (Sorkin, 2008).

Skoglund, Erdman, and Chen (2010)’sdemonstrated the unconditional coverage and ability of the model to produce independent VaR exceedances when deciding a sufficient model for VaR must be contemplated. They backtested the performance of VaR models to predict future losses of a portfolio with stocks, futures and options to verify the risk model.Historical stock data with ABB, Astra Zeneca, Ford, AIG, Microsoft, Ericsson and Volvo with the recent crisis period April 6 2001 to June 17 2009 to compare VaR models in matching the expected number of exceedances and spreading exceedances arbitrarily over time is used.Backtesting in this manner show that simple risk models like delta-normal and covariance simulation models underestimate risk exceedingly. This is because expected exceedances are rejected at low levels of significance.RiskMetrics covariance matrix for those simple models improves the performance with insufficient measurements for 99% and 99.5% VaR level.Historical simulation model has a better expected number of exceedances at 99.5% VaR level and undervalues risk exceedingly at 99% VaR level.More sophisticated copula dependence models using univariate GARCH perform quite well on expected exceedances. Normal variance mixture copula and GARCH normal residuals or empirical residuals with t tails models are accepted at significance levels for99% and 99.5% VaR confidence levels. T-distributed GARCH model residuals and normal, t10 or t5 copula models are accepted at normal significancelevels.GARCH volatility models are successful in creating independent exceedances spreading exceedances over time from independence and duration test.RiskMetrics or arithmetic average variance and covariance models cannot create independent exceedances.GARCH models are crucial in well-stated VaR models.GARCH models were necessary to create independence exceedances in the backtesting comparison. They are insufficient for creating both independent exceedances and matching expected VaR exceedances.Only a few copula models with GARCH variances are accepted at 5% level for unconditional coverage. They are the symmetric normal mixture copula, univariate t-distribution for the GARCH residual and normal copula and t5 and t10 copulas, empirical GARCH residual distribution and t-tails with symmetric normal mixture copula. Financial return commensurate with risk taken.During crisis, the financial risk escalates with diminishing returns and ballooning losses.Using the wrong distribution and risk measure can exacerbate the loss during crisis.It is even more important to determine the right distribution and risk measure during crisis.

First, I pose the right questions about the appropriate risks to be quantified and managed efficiently and optimally to produce the propitious expected return wanted during crises. Second, I attempt to capture the real world involved using the mathematical model formulated subjected to the appropriate assumptions and constraints. Third, I compute the formulated mathematical model. Fourth, I verify the formulated mathematical model against the real world.

This demonstrates that it is possibleto improve on the optimal portfolio by increasing the return by decreasing the market risks. This is related to minimizing the variance.

The background is subdivided into 1.2 MEASURING RISK WITH STANDARD DEVIATION, 1.2.1 MEASURING MARKET RISK, I.2.2 MEASURING MARKET TAIL RISK to help the reader understand why measuring risk alone with standard deviation during crises is both wrong and insufficient. Rather, risk is much better measured considering market risks.

## I.2 MEASURING RISK WITH STANDARD DEVIATION

The traditional and conventional measurement of risk is from volatility caused by standard deviationintroduced by Markowitz (1959) in prices of financial instruments like stocks, bonds, and so forth. It is assumed that the data is from a normal distribution.This is not the case during crises because of external shocks causing asymmetric distribution with the presence of fat tails i.e. concentration of bad risks.

The standard deviation is a poor and inaccurate measurement of risk during crises. Extremely favorable realizations and adverse ones are treated the same way.

It isthe old way of measuring risk.Rather, Value-at-risk and Expected Shortfall will provide a much better and more precise measurement of risk.This is a more comprehensive and accurate measurement of risk rather than depending only on standard deviation. Only market risk will be measured.

### I.2.1 MEASURING MARKET RISK

Dusak(1973)dissects the long-standing controversy over whether speculators in a futures market earn a risk premium within the context of the capital asset pricing model developed by Sharpe, Lintner, and others.In this approach, the risk premium required on a futures contract should depend not on the availability of prices but on the extent to which the variations in prices are systematically related to variations in the return on total wealth.This hedges the risk of agricultural cycles.The systematic risk was estimated with a sample of wheat, corn, and soybean futures contracts over the period 1952 to 1967 and found to be close to zero in all cases.Average realized holding period returns on the contracts over the same period were close to zero. However, the authors fail to explain whether this phenomenon is observable in other markets e.g. stocks, bonds etc.

### I.2.2 MEASURINGMARKET TAIL RISK

The Value at Risk (VaR) Model can be used to measure market tail risk.It allows the measurement of market risk with only one model used.Hsu and Kalesnik (2009) argue that successful risk managementneeds a detailed understanding of the distributions where random shocks to asset prices are drawn.Uncertainty is present in both the actual returns distribution and the parameters characterizing the distribution.

The uncertainty in estimating the distributional parameters will be focused on.They demonstrated how value at risk calculations is impacted by this uncertainty.Poor management results is obtained from some traditional and naïve methods for handling parameter uncertainty..Techniques for quantifying risk more accurately when distribution parameters have ow precision estimates or when there are disagreements over the parameter estimates are provided. In traditional VaR analysis, the parameters characterizing the probability distribution with great accuracy are assumed.This is counterintuitive because the exact probability distribution to model shocks to asset prices unabled to be determined.Levy, Cauchy, and other fat tail stable paretian distributions have been considered for modeling asset returns on top of classic lognormal distribution.The short time horizon over which the parameters estimates must hold true is estimating the mean and covariance of the return distribution ischallenging for VaR applications.Risk management is a short horizon activity whereas investing is a long horizon endeavor.VaR is not a coherent risk measure. Therefore, expected shortfall, a coherent risk measure, is used instead. However, the authors fail to provide the solution for risk management in both long and short horizon.

### I.2.3 SIGNIFICANCE

To find the certainty amidst the great uncertainty during crisis, risks needs to be quantified and managed efficiently and optimally to produce the propitious expected return wanted. This is most critical during crises e.g. Subprime Crises, Greece Fiasco, Black Monday, 1997 Asian Financial Crises, Long-Term Capital Management Crises.This dissertation is about managing risk appropriately and efficiently to generate the appropriate returns during crises.Only liquidity, market, and credit risk which are most common in stock, bond, currency indices and options on stock, bond and currency will be looked at.This dissertation considers these assets because they are most representative of a good and flexible portfolio allowing hedging to mitigate those risks. This dissertation provides a more comprehensive and accurate way of measuring risk.This is done by understanding and measuring the impact of credit, liquidity, and market risk.This dissertation hypothesized that it is possible to construct an optimal efficient frontier using generalized hyperbolic distribution during crisis.This will be proven theoretically and then empirically tested.Thus, constructing a better portfolio compared to current conventional models during crisis.

## I.3 RELATED OR PREVIOUS STUDIES

Related or previous studies most pertinent to my dissertation topic are discussed despite many papers written on efficient frontier and portfolio theory. Markowitz (1959) talks about the Markowitz's model, an investor selects a portfolio at time t ? 1 that produces a stochastic return at t. The model assumes investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that the portfolios 1) minimize the variance of portfolio return, given expected return, and 2) maximize expected return, given variance. Thus, the Markowitz approach is often called a "mean-variance model." The portfolio model provides an algebraic condition on asset weights in mean-variance-efficient portfolios. The CAPM turns this algebraic statement into a testable prediction about the relation between risk and expected return by identifying a portfolio that must be efficient if asset prices are to clear the market of all assets.Fama and French (2004) give credit to William Sharpe et al. for coming up with the capital asset pricing model (CAPM) in the 1960s. The markets compensate investors for accepting systematic or market risk, but do not discount for idiosyncratic risk can be eliminated through diversification. This affects decisions about hedging, which should be left to investors. The test on risk premiums is done by regressing an average asset returns cross section on estimates of asset betas. The model explains the intercept in these regressions is the risk-free interest rate, *Rf*, and the coefficient on beta is the expected return on the market in excess of the risk-free rate, *E(RM) -Rf*. This is not correct. First, estimates of beta for individual assets are imprecise because they creating a measurement error problem when they are used to explain average returns. Second, the regression residuals have common origins of variation. Positive correlation in the residuals leads to downward bias in the usual ordinary least squares estimates of the standard errors of the cross-section regression slopes. Thus, risk involved i.e. market risk cannot be measured accurately if the risk premium is measured wrongly. CAPM supports my dissertation topic because investors or speculators are rewarded according to the risk they take. However, CAPM fails to answer whether it is possible to have an efficient portfolio. This is the dissertation gap I intend to fill.

## I.4 STRENGTHS AND WEAKNESSES OF THE STUDY

The first strength of this study lies in coming up with a novel way to categorize risk to reduce the undervaluation or overvaluation of risk during crises.This study attempts to challenge the conventional way of managing the portfolio return by price fluctuation rather than managing the risk to manage the portfolio return. Take care of the risk and the profit will take care of itself. The second strength of this study is coming up with the appropriate and precise measures for portfolio returns to guide investors to invest in a more efficient portfolio. The weakness of the study is only static optimization is performed. In the real world, dynamic optimization is more realistic.

# PROBLEM STATEMENT AND OBJECTIVES

## II.1 INTRODUCTION

Portfolio management is a highly uncertain process. No one really knows what the outcome will be with certainty based on any investment or speculation decision taken to produce the propitious expected return wanted. This leads to risks. The risks are further exacerbated during crisis which cannot be averted because of disruptive financial jolts i.e. sudden and unprecedented events e.g. subprime mortgage crises in 2008 which are inimical. To find the certainty amidst the great uncertainty during crisis, risks needs to be quantified and managed efficiently and optimally to produce the propitious expected return wanted. Take care of the risks and the propitious expected returns will take care of itself. It is about quantification of market risks then managing them efficiently and optimally to produce the propitious expected return wanted using the efficient frontier created with generalized hyperbolic distribution during crises. This is tenable.

## II.2 DISSERTATION QUESTIONS/HYPOTHESES

I hypothesize that it is possible to construct a predictable optimal efficient frontier based on selected measurable risk factors to generate the propitious expected measuarable returns from asset classes like stocks, bonds, and currencies during crises using generalized hyperbolic distribution. The general dissertation question is as follows:

* 1. How to construct a predictable optimal efficientfrontier based on selected measurable risk factors to generate the propitious expected measurable returns from asset classes like stocks,bonds, and currencies during crisesusing generalized hyperbolic distribution?

## II.3 OBJECTIVES

1. Identify and measure market risks.
2. Identify and measure the appropriate lognormal returns.
3. To construct anoptimal efficient frontier using generalized hyperbolic distribution during crises.

## II.4 DISSERTATION CONTRIBUTION

According to Maginn, McLeavey, Pinto, and Tuttle(2007), to identify market risk, we need to know its definition:

* Market risk is the risk associated with interest rates, exchange rates, stock prices, and commodity prices;

First, to measure the portfolio returns appropriately and precisely to create a more efficient frontier. Lognormal returns are measured on stocks, bonds and currencies.

Black swan events (BSE)which are events mostly unlikely to happenaccording to Taleb (2007). However, when they do happen, the impact is devastating. Statisticians call them fat and skinny tails.

Table 2.1Summay of Dissertation Contribution shows the market risk measured in terms of risk factors.

**Table 2.1** Summary of Dissertation Contribution

| **No** | **Type of risk(s) measured** | **Risk(s) measurement methods** |
| --- | --- | --- |
| 1. | Market risk | Risk factors from loss distribution of stocks, bonds and currencies. |

# LITERATURE REVIEW

# (STATE OF THE ART)

## III.1 INTRODUCTION

The literature review will be divided into four sections. First, it will discuss about the best representation of financial data during crisis: gaussian versus non-gaussian distribution. Second, it will discuss the most appropriate measure of risks during crisis: standard deviation, VaR or expected shortfall. Third, it will discuss the most appropriate optimization during crisis: maximize return subject to minimum risk. Fourth, it will talk about portfolio performance through backtesting of portfolio return from portfolio optimization against the S&P 500 benchmark. Fifth, it will talk about structural change testing to detect the appropriate crisis financial data.

**III.2 GAUSSIAN VERSUS NON-GAUSSIAN DISTRIBUTION**

Mandelbrot (1963)’s research problem is Louis Bachelier is well mentioned in books on diffusion process.His early and groundbreaking contribution was building a random-walk model for security and commodity markets.It is the brownian motion but does not account for the abundant data around since 1900 by empirical economists because the empirical distributions of price variation are usually too peaked to be relative to samples from Gaussian populations.Histograms of price changes are unimodal and central bell reminds one of the Gaussian give.There are many outliers that gives fitted to price changes mean square are much lower and flatter than distribution of data themselves.Tails of price changes distributions are so extraordinarily long that sample second moments vary erratically.These facts show that a radically new way to problem of price variation is needed.A new model of price behavior will be presented and tested.The Gaussian distributions are replaced throughout by stable Paretian.The Gaussian is a limiting case of this new family with the new model a generalization of that of Bachelier in a complex way.

The key findings of his research are a principal attraction of modified Bachelier process is the logarithmic relative which is a Gaussian random variable for all value of T.The only entity changing with T is the standard deviation of L(t,T).A property expresses a kind of stability or invariance under addition fundamental in probability theory referred simply as stability.The Gaussian is only solution of stated equation for which the second moment is finite or for which the stated relation is satisfied.The stated equation has many other solutions when the variance is infinite.The Cauchy Law states that integral moments of all orders are infinite.Gaussian and Cauchy laws are stable and correspond to cases α = 2 and α = 1, β = 0 respectively.The α is index of peakedness varying from 0 (excluded) to 2 (included).β must disappear if α = 1.The α will be intimately related to Pareto's exponent.β is index of skewness varying from -1 to +1.

The stable densities are symmetric if β = 0.The familiar additivity property of Gaussian variance defined by mean-square is played by either γ or by a scale factor raised to power α.

The generalization of the classical "T1/2 Law" is carried out.Differences between successive means of Z(t) are observed.The densities of stable random variables follow a generalization of asymptotic behavior of Cauchy law excluding the Gaussian limit case.

The empirical tests of the stable paretian laws is applied to cotton prices.First, motivate and develop a model of variation of speculative prices based on stable Paretian laws already discussed from statistical economics perspective.Second, the theorems concerning sums ΣUn to build a new test of Pareto law from statistics viewpoint with theory of data analysis.

The law of pareto is introduced to represent price changes.The erratic behavior of sample moments is explained by assuming that the population moments are infinite.This hypothesis is the law of Pareto.Price increase over days, weeks, months and years would have the same distribution also ruling the fixed-base relatives.This leads directly to the probabilitists' idea of stability examined.Conclusions about estimation are made.Prediction is plausible only if the sample size is both very large and stationary, or if the sample size is small and the sample values are of comparable sizes.The unicity of the estimator is because of ignorance if the sample size is one for prediction.

The important gaps in knowledge and research are presumed the happening of Gaussian law in many practical applications relative to sums of variety of random effects.This is based on the central limit theorem with underlying normal or gaussian distribution.This seldom happens during crisis when the distributions can be asymmetric like generalized hyperbolic distribution.Large price changes is explained by causal or random contaminators.Large price changes commonly traceable to well-determined causes should be removed before one tries a stochastic model.Doing that will bring any distribution closer to the Gaussian.There will not be any observable discontinuity between outliers and rest of distribution and above censorship is indeterminate when one restricts to study of quiet periods of price change. It assumes that observations are generated by a mixture of two normal distributions.One of which has a small weight and large variance and is considered a random contaminator.It is essential to introduce a larger number of contaminators and simplicity of the model is destroyed to explain sample behavior of moments.Two samples from a stable paretian process is given.The maximum-likelihood procedure states that one should neglect one of the available items of information, any weighted mean of two recommended extrapolations being worse than either and nothing says which item one should neglect.The most likely value of δ is the most probable value in case of uniform distribution of a priori probabilities of δ.A priori probabilities are seldom uniformly distributed.They are usually very poorly determined.It is sufficient to choose the most likely of the two maximum-likelihood estimates.Abandon the hypothesis that δ is the same for both changes L(t, T/2) and L(t + T/2, T/2).One may be tempted to believe that the trend δ is parabolic if these changes are very unequal.Extrapolation would approximately amount to choosing among two maximum-likelihood estimates the one which is chronologically latest.It is an example of variety of configurations so unlikely in Gaussian case considered as non-random and of help in extrapolation.Their probability may be substantial in the stable Paretian case.Three samples from a stable paretian process is given.The indeterminacy of maximum likelihood can be lifted by one of the three methods discussed.The method of non-linear extrapolation will suggest a logistic growth if the middle datum only is large.One will try a parabolic trend if the data increase or decrease when taken chronologically.The probability of these configurations from chance under the author's model will be much bigger than in the Gaussian case.A large number of samples from a stable paretian process is studied.For a sum of random variables to follow a central limit of probability, each of the addends must be negligible relative to the sum.This is essential to investigate the predictions of the author's stable Paretian model.Limit laws hold only if the value of the sum is not controlled by any single addend known in advance.The following is observed for a process with independent stable Paretian L(t):

The contribution of the day of largest price change is likely to be non-negligible in relative value but will remain small in absolute value if α, β, γ, and δ are known.This will not change too much from Gaussian prediction even the largest addend is negligible for large and finite N.Suppose L(t, T = one month) is very large, the Paretian theory predicts the sum of a few biggest daily changes will be very close to total L(t, T).One should expect to find the law of L(t + τ, 1) possess a few widely outlying values if one plots the frequencies of various values of L(t, 1) conditioned by a known and very big value for L(t, T).The conditioned distribution of L(t + τ, 1) should depend little on the value of conditioning L(t, T) if outlying values are taken out.This is very well satisfied by prices.Implications about estimation are discussed.Given δ is unknown with a large sample of L(t + τ, 1), the estimation procedure for plotting the empirical histogram translates it horizontally until one has optimized its fit to the theoretical density curve.This best value will be very little influenced by the largest outliers.The rejection of outliers is fully justified.Path functions of a stable process in continuous time is discussed.It is universally assumed in mathematical models of physical or social sciences that all functions can safely by considered to be continuous and having as many derivatives as one may wish.Functions created by Bachelier are continuous with no derivatives and there is no need to be concerned because price quotations are always rounded to simple fractions of currency unit.In Paretian case, if the process is interpolated to continuous t, paths generated are discontinuous everywhere.Most of their variation is performed through non-infinitesimal jumps.The number of jumps are larger than u and positioned within a time increment T governed by the law C'T|d(u-α)|.Very small jumps of log\_e\_Z(t) are not perceivable because price quotations are expressed in simple fractions.There is a non-negligible probability that a jump of price is so large that supply and demand is not matched.The stable Paretian model may predict the occurrence of phenomena probably forcing the market to close.Such large variations are very extremely unlikely that the occasional closure of markets must be explained by non-stochastic considerations in a Gaussian model.There is a large probability of prediction for medium-sized jumps by stable Paretian model.They could be removed by market mechanisms like specialists activities if those medium-sized movements were oscillatory.Market specialists could at best change a discontinuity into a change that is fast and progressive if the movement is all in one direction.Very few transactions would be expected at the intermediate smoothing prices.A large number of intermediate prices are listed even if Z(t) performs a large jump in a short time and they are likely to be so fleeting, and to apply to so few transactions that they are irrelevant from enforcing a stop loss order of any type.When borrowings are oversubscribed, the market may resort to special rules of allocation in less extreme cases.The fairness of alexander's game is discussed.S. S. Alexander suggested the following rule of speculation: "if the market goes up 5%, go long and stay long until it moves down 5%, at which time sell and go short until it again goes up 5%."It is based on the random walk of price variations over time.Price series must be watched continuously in time and buy or sell whenever its changes reaches the defined value.This is possible only if the process Z(t) produces continuous path functions in the original Gaussian process of Bachelier.His procedure cannot be followed in the author's first-approximation model of price variation where there is a probability equal to one that the first move not smaller than 5 percent is bigger than 5 percent and not equal to 5 percent.

It can be proven that the stable Paretian theory predicts that this game is fair too.The evidence interpreted by Alexander suggest to go beyond the simple model of independent increments of price.Alexander's inference was based on discontinuous series from closing prices on successive days.He assumed that intermediate prices could be interpolated by some continuous function of continuous time.Price series produced by the author's process will have the price paid for a stock almost always bigger than that corresponding to a 5 percent increase.The speculator will almost always have paid more than assumed in Alexander's evaluation of returns.The price received will almost always be less than suggested by Alexander.At best, Alexander overestimates the yield related to his speculation method and at worst, the yield is positive may be a delusion because of overoptimistic evaluation of what occurs during the few most rapid price variation.A more refined model of price variation is discussed.A closer inspection of the author's model show that big price changes are not isolated between periods of slow change.They tend to be the result of several fluctuations some of them overshoot the final change.Price movements in periods of tranquility appear to be smoother than predicted by the author's process.It is mandatory to avoid removing of periods of rapid change of prices for correct estimation of α.It cannot be argued that they are casually explainable and should be eliminated before the noise is examined more closely.A Gaussian-like remainder devoid of any significance will be obtained if eliminating all large changes in this way is successful.The addition of successive daily changes of a price may be given the terms "aggregation in series" and "aggregation in parallel".This is borrowed from elementary electrical circuit theory and may not work for entities not possessing electrical circuit attributes.

The trends and themes of the researchare the facts about moments suggested a check whether the logarithmic price relatives for unsmoothed and unprocessed time series related to very active speculative markets are stable Paretian.Cotton provided a good example.

The theory also applies to many other commodities like wheat and other edible grains, to many securities such as those of the railroads in their nineteenth-century heyday and to interest rates like those of call or time money.There are many economic phenomena with much fewer outliers are observed although the available series are very long. It is natural in these cases to favor Bachelier's Gaussian model as a limiting case in the theory and its prototype.Pareto's graphical method is applied to cotton-price changes.The theoretical log Pr(U > u) relative to δ = 0, α = 1.7 and β = 0 is plotted on same graph for comparison.The various graphs should be horizontal translates of each other and a cursory examination shows data are in close conformity with predictions of the model if various cotton prices followed stable Paretian law with δ = 0, α = 1.7 and β = 0.The positive tails contain systematically fewer data than negative tails, β takes a small negative value.The negative tails begin by slightly overshooting their asymptotes, creating bulge that should be expected when α is greater than the critical value αo relative to one tail but not to the other.The graphical method is applied to study changes in distribution across time.The next test is considering monthly price changes over a longer time span.The actual changes between middle of one month to middle of the next.A longer sample is available when one takes the reported monthly averages of price of cotton.The graphs should be straight, parallel and uniformly spaced if cotton prices were generated by stationary stochastic process.Each of the 15-year subsamples contains only 200-odd months so separate graph graphs cannot be expected to be as straight as those relative to the usual samples of 1,000-odd items.The graphs are not as neat as those relating to longer periods but in the absence of accurate statistical tests, they are sufficiently straight and uniformly spaced except for period 1880-96.The process of generating cotton prices has changed only in scale with exception of Civil War and of periods of controlled or supported prices since 1816.Long series of monthly price changes should be represented by stable Paretian laws mixture. The effects of averaging is studied by the application of the graphical method.The empirical distribution of these variations does not differ significantly from distribution of changes between monthly means by averaging all daily closing quotations within months.It is a single average price for each month.The majority part of the averages distribution differs from that of actual monthly changes by a horizontal translation to the left.It is essential to rephrase it by replacing Z(t) by logeZ(t) throughout but the geometric and arithmetic averages of daily Z(t) do not change much in medium-sized over-all monthly changes of Z(t).The biggest variations between successive averages are smaller than predicted.

The relationships between key concepts are Levy has shown that tails of all non-Gaussian stable laws follow an asymptotic form of law of Pareto.Both tails are Paretian if |β| ≠ 1.

It is strong reason for replacing term "stable non-Gaussian" by less negative one of stable Paretian.The negative and positive tails decrease quicker than law of Pareto of index a in extreme cases where β = 1, C" = 0 and β = -1, C' = 0.It can be proven that it diminishes away even faster than Gaussian density so that extreme cases of stable laws are J-shaped. The generalization of the asymptotic law of Pareto under conditions of Pareto-Doeblin-Genedenko resolves the problem of existence of limit for A(N)ΣU\_n - B(N).The positive tail should always be bigger than negative.The shape of stable paretian distributions outside asymptotic range is governed by theory observing the following:

(a) densities are always unimodal;

(b) densities depend continuously on the parameters;

(c) the positive tail is fatter if β > 0 and it is greater than most probable value and greater than median if 1 < α < 2.

The symmetric cases, β = 0, are considered.The skewed cases are considered too.The joint distribution of independent stable paretian variables is examined.Isolines of small probability possess a characteristic plus-sign shape.The distribution of U1 when U1 and U2 are independent stable paretian variables and U1 + U2 = U is known. This conditional distribution can be obtained when the intersection between the surface represents the joint density p0(u1, u2) and plane u1 + u2 = u.It has two sharply distinct maxima located close to u1 = 0 and close to u2 = 0.Sample paths of Brownian motion look like empirical curves of time variation of prices or of price indexes noted by Bachelier and by several modern writers.Upon closer inspection, we see very well effect of abnormal number of large positive and negative changes of logeZ(t).The differences concern some of economically most interesting features of generalized central-limit theorem of calculus of probability with still closer inspection.Conditional distribution of a Gaussian L(t) with L(t, T) = L(t, 1) + … + L(t + T - 1, 1).μ is roughly uniformly distributed over T time intervals each contributing negligibly to the whole.Sufficiency of μ for estimation of mean drift μ from L(t + τ, 1) is discussed.δ has disappeared from distribution of any L(t + τ, 1) conditioned by value of μ.

The estimate of δ must be a function of μ alone whichever method of estimation favored. The only thing that can be explained in Gaussian case is mean drift interpreted as a trend and Bachelier's model assuming a zero mean for price changes can only represent price movements once the broad causal parts or trends have been deleted because the causes of any price movement can be traced back only if it is sufficient.Causality and randomness in stable paretian processes are discussed.An economic change must at minimal be large enough to allow the tracing back of the sequences it causes.The only causal part of a Gaussian random function is the mean drift δ.This also applies to stable Paretian random functions when their changes are roughly uniformly distributed.When logeZ(t) varies widely between the times t and t + T changing mostly during a few of the contributing days, they are sufficiently clear-cut and separated from noise to be traced back and explained causally as well as the mean drift.A careful observer of a stable Paretian random function can extract causal parts from it.There will be a large degree of arbitrariness in the distinction between causal and random if the total change of logeZ(t) is neither very big nor very small.We cannot tell whether the predicted proportions of two kinds of effects are empirically correct.The distinction between causal and random areas is sharp in Gaussian case and very diffuse in stable Paretian case.Causality and randomness in aggregation in parallel are discussed.The addition of successive daily changes of a price may be given the terms "aggregation in series" and "aggregation in parallel".Any occurrence of a large value for L(t, T) is traceable to a rare conjunction of large changes in all or most of L(i, t, T) in the Gaussian case.Large changes L(t, T) is traceable to one or a small number of contributing L(i, t, T) in stable Paretian case.A large L(t, T) is probably traceable to L(i, t + τ, 1) appears to be very big for one or a few sets of values of i and of τ.They would stand out sharply and be causally explainable.They should rejoin the noise made up by other factors after some time.The next fast change of logeZ(t) should be because of other causes.A contribution is trend-making for a large number of time-increments is doubtful to be under the same theory as fluctuations.Price variation in continuous time and theory of speculation are discussed.Certain systems of speculation would be advantageous if implemented cannot be followed in price series generated by a Paretian process.Infinite divisibility of stable paretian laws is discussed.Its predictions are varied by market mechanisms but are very illuminating.

The inconsistenciesare nonsense moments and periodicities in economic time series are found.The behavior of second moments and failure of least-squares method of forecasting is questionable.Cauchy's distribution was introduced in the study of the method of least squares.A method based on minimization of sum of squares of sample deviation cannot be appropriately used if expected value of this sum is infinite. Least-squares forecasting of the minimization of expected value of square of error of extrapolation is a problem.This expected value will be infinite for every forecast so that the method is extremely questionable in the stable Paretian case.A method of least ζ-power of forecasting error where ζ < α can be applied, however such a method would not have formal simplicity of least squares manipulation.The most hopeful case is ζ = 1 corresponding to minimization of sum of absolute values of errors of forecasting.The behavior of kurtosis and its failure as measure of peakedness is discussed.Pearson's index of kurtosis is defined.

Kurtosis is expected to become bigger without bound as N →∞.Things are less simple but quite similar for small N.Cootner's hypothesis is that prices change at random only if they do not reach either an upper or a lower bound considered by knowledgeable speculators to demarcate an interval of reasonable values of price.Operations of well-informed speculators will influence the price to be back within Taussig's penumbra when poorly-informed speculators allow the price to go too high or low.If contributing weekly changes were independent, price changes over periods of fourteen weeks should be smaller than would be expected. The theory is very appealing a priori but is not generally true because, with cotton, the facts do not support it.The observation that price changes of certain securities over periods of fourteen weeks have much smaller kurtosis than one-week changes from Cootner's justification.However, the sample only has 250-odd weekly changes and 18 fourteen-week periods.It is expected a priori to obtain a smaller kurtosis for longer time increment on general evidence concerning speculative prices.Cootner's evidence is not a proof of his theory and other methodologies must be used to attack the possible dependence between successive price changes.Spectral analysis of random time series is applied.Applied mathematicians are given the task of describing the stochastic mechanism for generating a given time series u(t) randomly.The theory of second-order random processes is applied to investigate what is obtained first.E(U) = 0 is assumed.

Population covariance, R(τ), is always assumed to be finite for all.Its Fourier transform produces spectral density of harmonic decomposition of U(t) into a sum of sine and cosine terms.The method has been very successful although many small-sample problems are unsolved.Its applications to economics is questionable in the large-sample case.The finding is not surprising in the context of the author's theory.2E[U(t)U(t + τ)] = E[U(t) + U(t + τ)]2 - E[U(t)]2 - E[U(t + τ)]2.The three variances are infinite for time series covered by the author's model, thus spectral analysis loses its theoretical motivation. Curves produced by stable Paretian processes present an even larger number of interesting formations than curves produced by Bachelier's Brownian motion.One expects to find that this change was performed mostly during several periods of especially high activity if price increase over a long time period occurs a posteriori to be commonly large in a stable Paretian market.The majority of contributing daily changes are distributed on a symmetric curve whereas a few especially high values fall well outside this curve.The only difference will be that daily changes will have no outliers if the total increase is the usual size.The results will be used to solve one small-sample statistical problem that of estimation of mean drift δ with other parameters known.There is no sufficient statistic for this problem and maximum likelihood equation does not have a single root.This has severe consequences from viewpoint of very definition of concept of trend.

The research designs or methods seemed inadequate because evidence is presented in a new way.It is shown that daily changes of cotton price strengthens concerns about monthly changes and conversely.The basic assumption is that successive daily changes of log price are independent.This argument have to be revised with assumption improved on.The population second moment of L(t) appears to be infinite and monthly or yearly price changes are patently not Gaussian. The problem of whether any limit theorem etc applies to logeZ(t + T) - logeZ(t) can be answered in theory by scrutinizing if the daily changes satisfy Pareto-Doeblin-Gnedenko conditions.It is impossible to ever obtain an infinitely big differencing interval T or to ever verify any condition relative to an infinitely big value of the random variable u.It must be considered that a month or year is infinitely long and the largest observed daily changes of logeZ(t) are infinitely big.Inference from aggregation is carried out.The cotton price data on daily changes of logeZ(t) appear to follow weaker condition of Pareto-Doeblin-Gnedenko. Inference from disaggregation is carried out.Data shows that price variation over weeks and months follow the same law up to a change of scale.It is a stable Paretian. The daily changes of log Z(t) must satisfy the conditions of Pareto-Doeblin-Gnedenko (P-D-G).The difficulties of disaggregation is discussed.The P-D-G conditions are weaker than asymptotic law of Pareto because they need limits exist for Q'(u)/Q"(u) and for [Q'(u) + Q"(u)]/[Q'(ku) + Q"(ku)] not for Q'(u) and Q"(u) taken independently.The functions Q'(u) and Q"(u) of daily variation must vary very little in regions of tails of usual samples if the limit is quickly achieved.The asymptotic law of Pareto must apply to daily price variations.

The difficulties of aggregation is discussed.The stable Paretian law should be increasingly accurate when T increases.It is senseless to consider values of T up to a century because one cannot get samples sufficiently long to have sufficiently inhabited tails.The year is acceptable for some grains only if the long available series of yearly prices are poorly known and variable averages of small numbers of quotations, not prices quoted on some market on fixed day of each year.There are two much more fundamental difficulties with very large T from economics viewpoint.First, the model of independent daily L's remove consideration of every trend excluding the exponential growth or decay because of a non-disappearing δ.Many trends are negligible on daily mode would be expected to be predominant on monthly or yearly mode.Second, it is in the linear character of aggregation of successive L's used in the model.A small logeZ(t + T) - logeZ(t) will be undistinguishable from relative price change [Z(t + T) - Z(t)]/Z(t).Adding small L's is related to principle of random proportionate effect.Stochastic mechanism of prices readjusts itself immediately to any level that Z(t) may have achieved.This assumption is quite usual and very strong.It is very likely that it changes little from change relative to single day of most rapid price variation if it is found that log Z(t + one week) - log Z(t) is very large.This conclusion only holds for independent L's.The greatest of N successive daily price variations will be so big that one may question both the use of logeZ(t) and independence of L's.Sample from a process of independent stable paretian increments is studied.The unbiased estimate of δ is L(t, T)/T and the maximum likelihood estimate matches the observed L(t, T) to its a priori most probable value.The bias of maximum likelihood is given by an expression of form γ1/αf(β) and function f(β) is to be determined from numerical tables of stable Paretian densities.β evaluation needs very large samples and quality of one's predictions will depend greatly on quality of one's knowledge of the past because β is mostly manifested in relative sizes of tails.The bias of a maximum likelihood estimate is mainly from an underestimate of size of changes so large as to be catastrophic.The forecaster should treat such changes separately and take account of private feelings about many things not included in the independent-increment model.

The opinion regarding the quality and importance is the quality of the predictions of the model concerning the distribution of changes of cotton prices between fifteenth of one month and fifteenth of the next is good.The negative tail has the expected bulge and most extreme changes are precise extrapolates from rest of the curve.Depression and similar periods would not affect general results very much.It suggested that support prices around 1950 varied less than their earlier counterparts.The apparent number of trading days per month was smaller than the actual number.

The topic should be further studied because having the right probability distribution with the right risk measures is crucial during crisis for portfolio optimization.

Gnanadesikan and Kettenring (1972)’s research problem is the overview of concepts and techniques relating to:(i) robust estimation of multivariate location and dispersion;(ii) analysis of two types of multi-dimensional residuals i.e. those that occur in context of principal component analysis and more familiar residuals connected with least squares fitting; (iii) detection of multiresponse outliers.Methods for informal exploratory analysis and coverage is a survey of current techniques and attempt to propose tentatively several new methodology needing further investigation and development.Usage of the methods are included.

The key findings of their research show that the study of residuals in a variety of ways especially through plotting is one of the most insightful processes for exposure in uniresponse data analysis.Researchers have worked on value of analyzing univariate residuals.It is an expository overview of concepts and techniques pertinent to multiresponse robust estimation and to detection of multivariate maverick observations.

It is a survey of existing work.It is concerned with some new methods.They are proposed tentatively as possibly useful methods and need further theoretical investigation and more extensive use.However, they are informal exploratory techniques and not formal confirmatory ones like tests of specific hypotheses.Robust estimation of multiresponse location and dispersion are discussed.Preliminary findings of a limited Monte Carlo study of the estimates are revealed.Methods for analyzing multidimensional residuals are discussed.Two types of residuals are defined then methods for studying them are explained.Residuals associated with principal component analysis and usual type of residuals from least squares fitting are discussed.Detection of multivariate outliers is further discussed.Concluding comments are made.Limitations of present study and to effect of finite sample sizes cause the Monte Carlo results to exit from the above asymptotic outcomes.y\*\_M is the least efficient, bivariate midmean y\*\_T(.25) is better and y\*\_HL and y\*\_T(.1) are very similar and most efficient among all.The two covariance estimators are very similar and disparity between relative efficiencies for α = 0.05 and α = 0.10 increases with ρ.The indications are for large ρ, ρ\*\_12 is more efficient than r\*\_12 with neither efficient enough when α = 0.10 considering α = 0.05 may be too high.A few isolated cases of |r\*\_12| > 1 happened for ρ = 0.9 but the frequency was never bigger than 3%.Overall results for Winsorized estimators were not different enough for discussion except for n = 20, ρ = 0.9 and α = 0.10, thirteen incidents of |r\*\_12| > 1 were discovered in 100 computed correlations.

The important gaps in the knowledge arestatistical estimation of parameters from data affects summarization and for uniresponse situation, problems and methods of robust estimation have received considerable attention.Univariate work has been largely concerned with obtaining and studying estimates of location insensitive to outliers.Alternatives to normal distribution are always symmetrical with heavier tailsthan the normal to compare relative efficiencies and other formal properties of the proposed estimates.Little attention is given to the robust estimation of higher order parameters like scale and shape parameters more susceptible to effects of outliers in the univariate case.The development of robust estimates and methods for analyzing residuals and for detecting outliers in multivariate situations is very difficult but more needed than in uniresponse case.

The interrelated character of outlier effects on location, scale, and orientation may not always be able to define or delineate outliers for one of these purposes, for example, location without considering some of the others, for example, dispersion, is an inherent difficulty of the multivariate case.An iterative way is an integral part of the requirements for the multivariate situation.Multiresponse models or modes of summarization are inherently more complicated and variation in which the data can deviate from the model are infinitely more varied than in the univariate case.Thus, it is even more necessary to have informal, informative summarization and exposure procedures.Common estimate of covariance matrix is sample covariance matrix, S.Direct method of getting a robust estimate of covariance matrix is constructing robust estimates of variances and covariances from methods of previous subsections.

The trends and themes of the research is the multiplicative constant is an interesting property of estimating correlation coefficient mandatory for getting rid of biases involved in trimmed or winsorized variances cancels out because it is in the numerator and denominator of defining equations.The trimmed or winsorized variances provide bases for getting corresponding robust estimator of correlation coefficient between variables, r\*\_12, and ρ\*\_12 to be used directly without any multiplicative constant for any sample size.Corresponding robust estimator of correlation coefficient between variables, r\*\_12, and ρ\*\_12 are not implied to be unbiased estimators of population correlation. Trimmed and winsorized versions with α = 0.05 and 0.10 of robust estimator of covariance between Y1 and Y2 and covariance estimator for n = 20 and corresponding robust estimator of correlation coefficient between variables, r\*\_12, and ρ\*\_12.Consequences of having defective responses in a multivariate sample are more complicated than in the univariate sample.First, a multivariate outlier can distort location, scale and orientation measures.

Second, it is much more difficult to characterize the multivariate outlier.Third, a variation of multivariate outliers may arise i.e. a vector response may be faulty due to gross error in one of its components or systematic mild errors in all its components.Tailor detection procedures to protect against specific kinds of situations, for example, correlation distortion, thus building up techniques with different sensitivities.The result of selective segregation of outliers should be a more efficient and effective use of available data if several analyses are to be performed on the same sample.The procedures should be computationally inexpensive enough for routine screening of large data sets.Those which can simultaneously reveal other features of data like distributional peculiarities have added economic appeal.

The relationship between key concepts are the emphasis is on techniques of developing summary statistics robust to possible outliers in multiresponse data.The estimation of only relatively simple characteristics,namely location and dispersion, is considered and concerned as in the univariate case with protection against outliers considered to be in the tails of the data distribution.The possibilities of other kinds of outliers exist in multiresponse data and the issues of multivariate outliers will be more fully discussed.Estimators have been proposed to handle robust estimation of multivariate location.Robust estimation of multivariate dispersion through scale i.e. variance and orientation i.e. correlation is neglected.Estimators described looks promising but further study is required.

A very limited Monte Carlo study's preliminary results are included.The sample mean vector is the usual estimator of location.The following robust estimators may be considered:

(i) the vector of medians of observations on each response;

(ii) the vector of Hodges-Lehmann estimators defined as the median of averages of pairs of observations for each response;

(iii) the vector of α-trimmed means defined as the mean of the data remaining after excluding a proportion α of the smallest and of the largest observations for each response.

Trimmed and winsorized means for the univariate case has been advocated. General efficiency characteristics of trimming and winsorizing appear comparable.The impact of extreme observations is greater on the winsorized than on the trimmed ones.This paper does not go after the use of a vector of winsorized methods in any detail.Estimating multivariate location by minimizing pth power deviations (1 < p < 2) is not discussed and described.Very simple extensions are the only ones agreed for present purposes.All the mentioned estimators are univariate estimators vectors obtained by analyzing observations on each response separately.A different multivariate procedure belong to the general class of robust estimators used in analysis with a combined manipulation of observations on different responses.tr is independent of those correlations and is like univariate relative efficiency when variables have equal variances compared to det which is quite sensitive to correlations between elements of y\_ and of y\*.Third measure depends on correlation approaching unity is not influenced by difficulty unlike det and tr2.The positive definiteness of the discussed estimators is equal to positive definiteness of compatible estimators, R\*\_1 and R\*\_2 of the correlation matrix.Each off-diagonal element of R\*\_2 lies in the span [-1, +1] does not imply positive definiteness of R\*\_2 excluding the bivariate case.Positive definite robust estimators of covariance matrices is on a sufficiently big number, v, of observations i.e. v > p subselected from total sample to make estimate robust to outliers.They are all on a combined scrutiny of both scale and orientational sides compared to S\*\_1 and S\*\_2 constructed from separate scrutiny of these sides.Developing suitable compounding matrices for a squared distance function in an internal comparisons technique for analyzing a collection of single-degree-of-freedom contrast vectors gives rise to the first method for ensuring a positive definite robust estimator of covariance matrix.First step of procedure rank multiresponse observations using Euclidean distance or squared Euclidean distance from some robust estimate of location.Second step is choosing a subset of observations whose ranks are smallest 100(1 - α)% to compute the sum-of-products matrix.Limited experience of the authors with this method suggest that unless α, β, and n are moderately large namely α and β ≥ 0.2 and n ≥ 50 unless underlying correlation structure for observation is nearly singular.A vector of multivariate residuals exists between data and fit to give some summarizing fit to multiresponse data with importance of how to express them.There are two broad classifications of statistical analyses of multiresponse problems:

i. internal structure analysis and

ii. superimposed or extraneous structure analysis.

The first classification includes techniques like principal components, factor analysis and multidimensional scaling useful for learning about internal dependencies and for reduction of dimensionality of response.The second classification includes multivariate multiple regression and multivariate analysis of variance are classical techniques for investigating and specifying dependence of multiresponse observations on design characteristics or extraneous independent variables.Each category of analysis can produce multivariate residuals.Linear principal components analysis is viewed as fitting a set of mutually orthogonal hyperplanes to the data to minimize sum of squares of orthogonal deviations of the observations from each plane in turn at each stage.Residuals that are perpendicular deviations of data from fitted hyperplane are present at any stage.The well-known least square residuals = observations - predictions from a least squares fit is obtained from analyzing superimposed structure by multivariate multiple regression.Least squares residuals as input to principal component analysis would lead to orthogonal residuals mentioned for data analysis.Augmenting multivariate multiple regression fitting by principal components transformation of residuals from fit can be helpful in describing statistical correlations in errors of combined original variables, or in showing insufficiencies in fit of response variables by design variables.

The inconsistencies or contradictions areonly data representation is studied by this paper.

Risk and performance measurements of investment portfolio are missing.

The research designs or methods seemed improper, insufficient, or inadequate because statistical data analysis is for extracting and explicating informational content of data.That involves summarization in terms of a statistic e.g. correlation coefficient undergirded by some tightly specified model or in terms of a simple plot e.g. a scatter plot.That involves exposure i.e. the presentation of analyses to facilitate the detection of both anticipated and unanticipated characteristics of the data.The role and value of the twin-pronged process of summarization and exposure in data analysis were discussed by previous researchers.Statistical methodology for summarization is usually developed without specific attention to the exposure value of the techniques.The goal of summarization has sometimes been artificially separated from the objective of exposure even with awareness of possible inadequacy or inappropriateness of certain assumptions.One may overlook the desirability of having techniques for delineating outliers if they exist in the data with the use of certain robust estimates of location designed to be insensitive to outliers.The goals of summarization and exposure must be viewed as two sides of the same coin from a data-analysis standpoint.There is insufficient affine commutativity of robust estimators of multivariate location against the usual mean vector not possessing this property.A location estimator is affine commutative if affine transformation operations and estimate formation can be interchanged without affecting the result. This may be viewed based on degree of commitment to coordinate system for observations.There are two extremes to consider.

First, interest may be confined fully to observed variables and thus commutative issue would be distant. Second, the location problem is inherently affine commutative and all estimators should possess this property.A more limited commutativity than the very general affine commutativity as an intermediate position is sought.Proposed location estimators using 100 Monte Carlo realizations of Y, 2 X n matrix, columns are a sample of size n from a bivariate normal distribution with zero means, unit variances, and correlation ρ.Independent samples were formed for n = 20, 40, and 60 for ρ = 0.0 and altered by substituting the second row of each Y with ρy\_1i + (1 - ρ2)1/2 X y\_2j, j = 1,…,n to construct additional data for ρ = 0.5 and 0.9.The comparisons for fixed n are not independent though the results for different sample sizes are independent.To compute the Monte Carlo mean vector and var(1), var(2) and cov(1, 2), the associated correlation, corr (1,2) and eigenvalues, c\_min and c\_max, of Monte Carlo covariance matrix of y\_\*\_M etc respectively.The Monte Carlo experiment's scope was not big enough to establish properties like efficiencies to a satisfactory degree of precision.Few general indications can be inferred from the tables:

(i) Every location estimator is reasonably unbiased.

(ii) There are considerable differences between the efficiencies excluding Y\_HL except Y\_\*\_HK and Y \* T(.1) behaving much alike;

(iii) The correlations between robust estimator’s elements especially y\_\*\_M are less than the correlations for sample mean vector for ρ = 0.5 and 0.9.

Three multivariate measures of relative efficiencies of y\*'s with respect to y\_ computed with given information provided to us for further comparisons among robust estimators.Efficiency is measured by square root of product, sum, and square root of sum of squares of eigenvalues of Monte Carlo covariance matrices of y\_ and y\*.The relative efficiencies are corresponding ratios of values for y\_ to values for y\* referred to as det, tr, and tr2 respectively.det is selected as measure of asymptotic relative efficiency in the study of y\*M and y\*HL.det = 0.96, 0.94 and 0.92 for large n and ρ = 0.0, 0.5 and 0.9.Two aspects of multivariate dispersion depend on the scales of the response (i.e. variances) of the responses and concerned with orientation (i.e. inter-correlations among the responses).Sometimes, consider robust estimation of each of these aspects separately whereas for other purposes a combined view may be in order.For the first approach, it has the advantage of using all available and relevant information for each estimation task whereas the latter would involve retaining only observations pertaining to both aspects simultaneously.The latter approach may be computationally simpler and more economical.The problem of robust estimation of variance of a univariate population is considered not as intensive or extensive as the location case.Certain conflicting aims seem to appear in estimating higher order characteristics of a distribution when one leaves the location case.A possible conflict can exist between the desire to protect estimate from outliers and information for estimating the variance depends more heavily on the tails.The routine use of robust estimation procedures for these higher order characteristics in relatively small samples is questioned.

The use of a 10% trimmed sample for a sample size of 10 to provide a robust estimate may lead to an estimator with efficiency unacceptably low when close enough to the normal.

It may be both expedient and wise to study the observations more closely, removing only clearly shown outliers or changing the observations to make them more nearly normally distributed.The usual unbiased estimator of variance for ith response (i = 1, …, p) on n observations may be denoted s\_ii and a corresponding robust estimator, s\_\*ii, may be developed by using any of the following three methods:

(i) trimmed variance from α-trimmed sample;

(ii) winsorized variance from an α-winsorized sample;

(iii) slope of lower end of a χ\_1\_2 probability plot of 1/2n(n - 1) squared differences between pairs of observations.

An estimate of location and a direct suggestion would be to use a trimmed mean for trimmed variance and a winsorized mean for winsorized variance with regards to the first two methods.To get the winsorized variance a trimmed mean is used.This may be appropriate for t-statistic kinds of considerations related with the trimmed mean.It is advisable to use a smaller proportion of trimming (or winsorizing) for variance estimation than for location estimation in samples as large as 20.Multiplicative constants are required to get unbiased estimates from a trimmed or winsorized variance.They are based on moments of order statistics of normal distribution with assumption that the 'middle' of the sample is sufficiently normal.A table of required constants for small (n ≤ 15) samples and tables together with tabulation of expected values of cross-products and squares of normal order statistics may be used for calculating the necessary constant for samples of sizes up to 20.Asymptotic results do not appear to be sufficient at n = 20 and more work is required on developing the needed multiplicative constants for larger values of n.A positive definite estimate of covariance matrix is necessary to analyze multiresponse data when the underlying distribution is not singular.Inverse of covariance matrix estimate is utilized to scrutinize the sample's configuration based on generalized squared distance of the sample centroid observations.The usual estimator, S, is positive definite with probability 1 if dimensionality, p, is not more than the number of independent observations namely n - 1 for an unstructured sample.Neither of estimators, S\*\_1 and S\*\_2, defined above is surely positive definite.This may be a poor estimator when a large fraction of the observations are outliers.Multiplicative constants k and k' are not as straightforward conceptualized or computed as constants used in trimmed or winsorized variances and covariances.Estimating these constants through scale parameter of approximate gamma distribution of quadratic forms estimates should be pursued.They involve a type of trimming of multivariate sample and amount of trimming namely values of α and β is expected to depend on both n and p.Developing bases for recommending reasonable values of α and β will need more work.Appeal of a winsorizing-type of scheme may be stronger because of issue of discarding observations is quite uneconomical in multiresponse situation where n may not be much greater than p.Weighted Euclidean metric developed iteratively may be especially important with variables possessing markedly different scales and high correlations.

Analogously, an approach with pairwise differences of observations avoiding location estimation is possible.Results of calculating S\*\_3 with Y\*\_HL and α = β = 0.10 from n = 20 and ρ = 0.5 Monte Carlo data are presented to show how the estimators might work.Principle component analysis begins with a set of n p-dimensional observations, columns of p X n matrix Y for analysis as an unstructured multivariate sample.Such observations may either be original or derived data like outputs of other analyses, for example, least square residuals from a multivariate analysis of variance.It is a method of fitting linear subspaces, or a statistical technique for finding and describing possible linear singularities in the data through projections of the data onto principal component coordinates corresponding to small eigenvalues.Studying the sum of squared lengths of projections of observations on the last few principal component coordinates is one method to detect lack of fit of individual observations. These might include:

(i) Two- and three-dimensional scatter plots of bivariate and trivariate subsets of last few rows of Z with points named in various ways like by time if it is a factor.

(ii) Probability plots of values within each of last few rows of Z.

These values may be more nearly normally distributed than the original data and normal probability plotting providing a reasonable starting point for analysis because of the linearity of transformation involved.This may assist in pinpointing specific smallest principal component coordinates on which the projection of an observation may look abnormal.

(iii) Plots of values in the last few rows of Z against distances in space of first few principal components.

It may be informative to plot the projections on each of the three remaining principal component axes against the centroid distance of each of the projected points in the two-dimensional plane associated with two largest eigenvalues.A certain type of multidimensional insufficiency of fit if magnitude of residuals in coordinates associated with smaller eigenvalues related to clustering of points in two-dimensional space of two eigenvectors corresponding to largest two eigenvalues may be shown.Similar concepts and approaches in generalized principal components for detecting and describing nonlinear relationships among responses has to be developed.Care is needed to define, compute and express statistically the perpendicular deviations of the observations from the fitted function.

Robustness of the suggested analyses is important.Upon detecting an aberrant observation, it should be excluded from the initial estimate of S (or R) and the process of getting and analyzing the principal component analysis is repeated.A robust estimate of the covariance (or correlation) matrix for the initial analysis should be used so that aberrant observations would become even more noticeable in subsequent analysis of residuals.Describing multivariate multiple regression require separate regressions of each multivariate response on a common design or regressor matrix producing a matrix of estimated regression coefficients possessing certain joint statistical properties.Step-down analysis is used when there is a natural ordering among the responses.At each stage, a univariate analysis of a single response using all responses which have been analyzed at preceding stages as covariates is done.Step-down residuals are obtained and studied by any available techniques for analyzing univariate least squares residuals.Partial residuals and how to use them are proposed.Completely analogous definitions of multivariate partial residuals and how to analyze them are suggested.The entire collection of residuals already defined as unstructured multivariate sample should be analyzed.At times, subsets of residuals may be more appropriate for such a view.An aberrant observation can yield a residual where the associated quadratic form value would be unduly large causing a departure of corresponding point from linearity of other points on gamma probability plot.Heteroskedasticity would be indicated by a piecewise linear configuration with points corresponding to residuals derived from observations with same covariance structure belonging to same linear piece.The shape parameter was estimated by maximum likelihood based on 50 smallest values of quadratic form as the 50 smallest order statistics in a random sample size of 100.A possibly aberrant residual has been spotted and verified.There are at minimum two sources of statistical difficulties in analyzing residuals.First, constraints on subsets of residuals imply correlations among residuals.The singularities among residuals is very critical when numbers of levels of factors involved are small.Second, outliers might seriously bias usual effects subtracted from an observation to mask local effect of an outlier on corresponding residual.This could be important when the factors each have a moderate number of levels.

The extreme outlier may so badly biased the mean vector for the first row that all residuals = observation vector - row mean vector in that row have been unduly biased.A method is required to insure against masking effects of outliers on the residuals if the outlier is extreme enough for this to happen.This is done by combining the ideas and methods of robust estimation with desirability of analyzing the residuals combining summarization and exposure.Modified residuals is mainly used to accentuate the presence of outliers.Internal analysis techniques like principal component analysis are appropriate for examining unstructured samples of data.It is a fundamental method of showing multivariate data uncovering outliers through two- and three-dimensional plots of original and principal component variables.The first and last few principal components are the most interesting.

The first ones are very sensitive to outliers inappropriately inflating variances and covariances or correlations.Probability plots and standard univariate outlier tests can be made on the rows of either Y or Z axes.Easily calculated univariate statistics measuring contribution of individual observations or combinations of observations to specific multivariate effects are useful.Generated values can be studied graphically with probability plot despite minor difficulties because of correlations among the statistics.External analysis techniques like canonical correlation analysis are applicable in presence of some superimposed structure.It is the discriminant analysis of two or more groups of observations and canonical analysis of two or more sets of variables.Canonical analysis involves formation of one or more stages of canonical variables selected to capture linear relationships among sets of variables.The canonical variables are unit-variance linear compounds of different sets and at each stage one canonical variable is selected from each set.Valuable insight can be gleaned from two- and three-dimensional displays of discriminant and canonical variables showing relative size, shape, and location of groups and peculiarities in positions of individual points.Discriminant analysis can be preceded by internal analyses of individual groups for outliers to make the dispersions within the individual groups similar for validity of standard multigroup discriminant analysis procedure.Remaining observations can be used to derive discriminant coordinates but visual displays include positions of all data in transformed space.Canonical variable plot can reveal outliers inducing an artificial linear relationship among sets.Plots of principal components or other suitable linear functions of canonical variables are alternative summaries with special appeal when the number of sets is large.Normal probability plots and univariate outlier procedures can be applied to canonical variables or to their linear functions, and to discriminant variables making a different plot for each group.The slopes of the configurations on the last-mentioned of these plots give a partial check on homogeneity of dispersion among groups.Iteration or robustification can be used to refine all of the procedures.

The opinions regarding the quality and importance arethis paper only used location and dispersion parameters to detect outliers.Skewness, scale and shock factor for skewness and scale are not handled in investment portfolio.

The topic should be further studied because first advantage of the third method mentioned above is that location estimate is not needed.Second advantage is adjustment type given by multiplicative constant in trimmed and winsorized variances is in the probability plot itself namely the abscissa (or quantile axis) to scale the ordinate for determining the slope, an estimate of twice the variance.Third advantage is looking at 1/2n(n-1) parts of information with some redundant because of statistical correlations and error configuration on X\_1\_2 probability plot may often be indicated more stable than by a normal probability plot of n observations.Fourth advantage is its exposure value in assisting the detection of unanticipated peculiarities in data.Disadvantage of the technique is it may not be useful and even be misleading for estimating the variance in situations where a large proportion of observations may be outliers.Previously generated data were used by the first two methods.

The results for the trimmed variance in a typical case, n = 20 and ρ = 0.5, are shown.A suitable trimming percentage for location estimation namely α = 0.10 may be too high for variance estimation from the empirical evidence.Corresponding results for winsorized variance are not included although they are similar.Robust estimators are decided with no deliberations of fulfilling the Cauchy-Schwarz inequality relationship between covariance and variances.Comparable robust estimator of correlation coefficient between variables may not be in the admissible range, [-1,+1], for the correlation coefficient.Modification is suggested to ensure an estimate of correlation coefficient in the valid range and remaining with the above approach of obtaining the covariance estimate as difference between two variance estimates.Risk and performance measurement of investment portfolios should be studied too.

Barndorff-Nielsen (1977)’s research problem shows that continuous type distribution is a hyperbola e.g. logarithm of probability density function.It is a hyperboloid in several dimensions and is investigated.Such distributions is a mixture of normal distributions. The focus is on the mass-size distribution of Aeolian sand deposits referring to the findings of R. A. Bagnold.

The key findings of their research show that this distribution family is applicable to empirical distributions pertinent to size distributions e.g. of mined diamonds or of personal incomes.When studying size phenomena, the sizes of the single items cannot be observed directly but the data is made up of grouped moment distributions.When working with moment-size distribution, the sizes of the single items sampled is not completely independent random variables.A plot of histogram of mass-size distribution with logarithmic scales will strongly suggest a theoretical curve which first increases almost linearly and after asmooth transition decreases almost linearly.The hyperbola is the simplest mathematical curve representing such histogram.The fitted curve usually lies below the observed points in both tails of the distribution and above the highest observed point(s) if the model gives a good description.Theoretical probability distribution commonly used in connection with empirical particle size or mass-size distribution is the log-normal.The mean value and variance occurs with a brownian motion with drift. This is the clue to the derivation of the theoretical distribution through the stochastic process model. Pareto’s Law states that how people are distributed based on income, of firms according to size and so forth has a strong tendency to decrease in the upper tail.It is not mentioned whether the hyperbola distribution more suitable during crisis situation or not.

The important gaps in the knowledge are we are far from fully understanding and modeling the dynamical processes resulting in the regularities in the distribution based on the size of the particles of windblown sands.For a size distribution, if the n observations is considered as independent, then the procedure will coincide with that of maximum likelihood. However, independence is not always a tenable assumption in size distribution investigations.Maximum likelihood estimation assumes that the parameters can be expressed as logarithmic returns. In the real world, this may not always be so.

The trends and themes of the research is related to gaussian versus non-gaussian distribution because logarithm of probability density function during crisis is hyperbolic.This is a non-gaussian distribution.

The relationship between key concepts are the mean and variance with a drifted brownian motion allows the derivation of the theoretical distribution using the stochastic process model.

Possible contradictions are the maximum likelihood estimation assumes that the parameters can be expressed as logarithmic returns. In the real world, this may not always be so.

The research designs or methods seemed inadequate because the maximum likelihood estimation should be tested on logarithmic of probability density function.

The opinion regarding the quality and importance is the quality of the data used can be improved if applied to financial market returns from stocks, bonds, currencies and commodities.

Further studies should be done to show how such distributions fit financial market returns better during crisis.

Barndorff-Nielsen, Kent, and Sorensen (1982)’s research problem is about general properties of normal variance-mean mixtures including various new results are surveyed.The class of self-reciprocal normal variance mixtures is rather wide.Some tauberian results are established where relationships between the tail behavior of a normal variance-mean mixture and its mixing distribution may be deduced.The generalized hyperbolic distribution and modulated normal distributions give examples of normal variance-mean mixtures whose densities can be in terms of well-known functions. It is proven that the z distributions including hyperbolic cosine and logistic distributions are normal variance-mean mixtures.Z distributions are the class of distributions from beta distribution through logistic transformation after using location and scale parameters.

The key findings of their research show that the general properties of normal variance-mean mixtures, generalized hyperbolic and modulated normal distributionsare surveyed. This delineates a broad class of self-reciprocal distributions.Elementary results about normal variance-mean mixtures is stated. R-dimensional generalized hyperbolic distributions is a quite wide class of normal variance-mean mixtures.It can be the reciprocal of a gamma distributed random variable, r-dimensional t, gamma, McKay's Bessel function, skew Laplace, normal Laplace, generalized inverse Gaussian and normal distributions because of a number of useful mathematical and statistical properties.The family of generalized hyperbolic distributions is closed under margining, conditioning with marginals and affine transformations.Variance-mean mixture distributions are self-decomposable.Generalized hyperbolic distributions are not self-decomposable because of the restricted, homothetical definition of self-decomposability of multivariate distributions.All generalized hyperbolic distributions are infinitely divisible.An intuitive argument is given on why normal variance mixtures are capable of describing the changes in many real data sets.It connects the mixing process to a number of active elementary errors randomly.The argument generalizes to an argument for normal variance-mean mixtures use if the elementary errors are allowed a nonzero mean.The modulated normal distributions which are examples of normal variance mixtures are studied as an example.There are two types of modulated normal distributions: I and II.Type II modulated normal distribution is also self-decomposable.Mixtures of normal distributions are limiting distributions in generalizations of central limit problem to nonindependent summands.The (unconditional) limiting distribution of the maximum likelihood estimator in nonergodic stochastic processes is a normal mixture usually.

The important gaps in knowledge are x is a random vector of normal variance-mean mixture distribution where Δ is a symmetric, positive-definite r x r matrix with determinant one.|Δ| = 1 is imposed to avoid an unidentifiable scale factor.This means it is possible to get an unidentifiable scale factor.The value of λ in Theorem 5.1 is unique but the function L(u) is unique up to any asymptotically equivalent function.The description in Theorem 5.1 is sufficient for most purposes and L(u) is often asymptotically equal to a constant.The conditions of Theorem 5.2 appear very mild but can be difficult to check in practice.

The trends and themes of the research is the focus is on normal variance mixtures with a continuous type mixing distribution.This is a special type of normal distribution.Z distributions are surveyed and new properties from them are presented.Z distributions are normal variance-mean mixtures.Their mixing distributions are specified.Z distribution has log linear tails.When the density function is plotted on a logarithmic scale for the ordinate axis, the lower tail is a asymptotical straight line with slope α/σ but the slope of the asymptote of the upper tail is -β/σ.The distribution is symmetric when α = β.It is negatively skewed when α > β or positively skewed when α < β.The full location-scale class considered that several common distributions or transformations of such distributions are included in the class with limits tending to infinity to discriminate between them.The log gamma model, logistic, generalized logistic and hyperbolic cosine distributions are all classes of z distributions.Z distributions are useful as logarithmic income distributions.The Champernowne distributions has only two points of intersection with z distributions e.g. logistic and hyperbolic cosine distributions.Z distributions are proven they can be represented as normal variance-mean mixtures.Z distributions belong to the extended Thorin class and therefore are self-decomposable.Z distributions are infinitely divisible previously established for the hyperbolic cosine and logistic distributions.The extended Thorin class is closed under weak limits, the log gamma model belongs to the extended Thorin class.It can be directly proven that the log gamma distribution and therefore the z distributions belong to the extended Thorin class.This can be easily deduced from product representation of the characteristic function of the log gamma distribution.

The relationship between key concepts are in the theory and practice of statistics, mixtures of normal distributions are important.They are typical limit distributions in asymptotic theory for dependent random variables.They are used in data analysis for various heavy-tailed and skewed empirical distributions.Infinite divisibility of mixing distribution F implies that distribution P is infinitely divisible.Self-decomposability of mixing distribution F is not sufficient to ensure self-decomposability of distribution P. If F is self-decomposable, then P is self-decomposable only if β = 0.It is deduced that P belongs to the extended Thorin class.Self-reciprocal distributions are those whose density and characteristic functions are proportional.They are normal, hyperbolic cosine and r-dimensional generalized hyperbolic distributions with λ = r/4, μ = 0 and k = δ.The class of self-reciprocal normal variance mixtures with Δ = I and absolutely continuous mixing distribution can be generated.The extended function is integrable and can be normalized to integrate to 1 to get the probability density function.The normal variance mixture with structure matrix I and the probability density function as mixing distribution are self-reciprocal.Self-decomposability is a special statistical interest because only self-decomposable distributions can occur as one-dimensional marginal distributions of stationary autoregressive schemes.

It is an inconsistency where no conclusion and further research are given.

Research designs or methods seemedinadequate. Study the way how the tail behaviour of a normal variance-mean mixture depends on the tail behaviour of the mixing distribution in the one-dimensional case.No empirical proof is provided.

The opinion regarding the quality and importance is empirical proof should be carried out to support the theoretical proof given.

The topic should be further studied to give the empirical proof to support the theoretical model proposed.

Jarque and Bera (1987)’s research problem are the tests for normality of observations and regression disturbances are obtained from the Lagrange multiplier procedure or score test on the Pearson family of distributions.The suggested tests possess optimum power properties and good finite sample performance.They should prove to be useful tools in statistical analysis.

The key findings of their research show statisticians' interest in fitting curves to data goes back in history.Tests for the normality of observations are developed to differentiate normal and non-normal observations.Skewness, kurtosis and omnibus tests, analysis of variance tests and coordinate-dependent and invariant procedures have been developed.Testing normality of (unobserved) regression disturbances is also popular.The linear regression model is used with the necessary variables defined with the probability density function assumed to be normal probability density function.Outcomes of violating the normality assumption have been studied.Ordinary least-squares estimator known to be efficient under normality may be very sensitive to long-tailed distributions.The usual t and F-tests for inferential procedures are used with their sensitivity to nonnormality determined by numerical values of regressors.Some adjustment in degrees of freedom of these tests may be required to get the desired significance level.The distribution of s2 = (y-Xb)'(y - Xb)/N is studied and shows that significance level of usual χ2 test of hypothesis σ2 = σ2\_0 is not asymptotically valid in nonnormality.Homoscedasticity and serial independence tests for normal disturbances may lead to incorrect conclusions under nonnormality.Violating the normality assumption may result in suboptimal estimators, invalid inferential statements and to inaccurate conclusions.This emphasizes the importance of testing validity of the assumption.

The important gaps in knowledge/research/dataare the Lagrange multiplier test is based on normal distribution which is very unlikely during crisis.The linear regression model will not work optimally if the underlying distribution is not normal which is very likely during crisis.No risk measures for crisis with tail risks is mentioned for portfolio optimization.

Tail risks are best captured with Value-at-Risk (VaR) and Expected Shortfall (ES).

The trends and themes of the research is statistical tests have been developed to fit curves to data based on normal distribution during normal times.Non-normal distributions are prevalent during crisis.Appropriate statistical testings are required to fit such data to the curves.

The relationships between key concepts are ordinary least-squares residuals meet the condition u\_1+…+u\_N = 0 in linear models with a constant term.LM\_N is drastically simplified with u\_1 = 0.The statistic is very simple to compute needing only first four sample moments of ordinary least-squares residuals regardless of existence or not of a constant term in regression.Results of a Monte Carlo study is carried out to compare the power of various tests for normality of observations and regression disturbances is presented.Simulations for small and moderate sample sizes are carried out with N = 20, 35, 50, 100, 200 and 300.The four distributions from the Pearson family are considered: normal, gamma(2,1), beta(3,2) and Student's t with 5 degrees of freedom and one nonmember: the lognormal.Such distributions were chosen because they cover a wide variety of values of third and fourth standardized moments.Subroutines on a UNIVAC 1100/42 are used to generate pseudo-random variants u\_i from these and other distributions throughout the study.Each of the five random variables mentioned above was standardized to have zero mean.v\_i = u\_i since μ = 0.The tests considered for normality of the observations u\_i have the following:

(i) skewness measure test;

(ii) kurtosis measure test;

(iii) D\* test;

(iv) R test;

(v) W test;

(vi) W' test;

(vii) LM test.

Distance tests like Kolmogorov-Smirnov, Cramer-von Mises, weighted Cramer-von Mises and Durbin tests are not included because for a wide range of alternative distributions, the W test, was superior to these.N pseudo-random variants from a given distribution are generated.Values of √b\_1, b\_2, D\*, W, W' and L\_M are computed.Whether H\_0 is rejected by each individual test is checked.This is done for every experiment in the simulation study.250 replications are carried out and power of each test by dividing by 250 the number of times H\_0 was rejected is estimated.The power calculation obtained is presented.Only numerical results for sample sizes N = 20 and N = 50 sufficient for purpose of arriving at conclusions are fully reported.The power calculations reported are accurate to only two decimal points with a 90% confidence coefficient for the number of replications.

The theoretical results shared previously justify using LM test if we have large samples and are considering members of the Pearson family.The preferred tests would probably be W, LM then W' for finite sample performance.LM had the highest power for all distributions for N = 100, 200 and 300 whereas differences with the W and W' tests were small.Power for LM was 0.964, 0.856, 1.0 and 1.0 compare to 0.944, 0.848, 1.0 and 1.0 for W' respectively for beta, Student's t, gamma and lognormal densities for N = 200.LM may have good relative power even when the distribution does not belong to the Pearson family.

LM is preferred followed by W and W' which dominate other four tests.LM has an advantage over W and W' for its computation because neither ordered observations which may be expensive to get for large N nor expectations and variances and covariances of standard normal order statistics other than power considerations.Simulation results showed together with its proven asymptotic properties suggest the LM test may be the preferred test in many situations.It is worthy to carry out extensive simulations to obtain under normality finite sample significance points for LM.10,000 replications are carried out and presented with significance points for α = 0.10 and 0.05 for a range of sample sizes.The power of tests for normality of unobserved regression disturbances is studied.The tests considered are the same as those described and computed them with estimated regression residuals rather than true disturbances u\_i.The first six are modified large-sample tests discussed.

The seventh test is one already suggested.The modified Shapiro-Wilk test, ^W, has been reported to be superior to modified distance tests.A linear model with a constant term and three additional regressors with K = 4 and utilize ordinary least-squares residuals u\_i to compute the modified tests.The specific values of means and variances of these regressors have no effect on the simulation results.Utilize the same significance points as described except for W, W' and LM\_N for which points corresponding to α = 0.10.W(25) is 25th largest of the values of W in 250 replications under normal disturbances as significance point of W.The estimated power of each test for N = 20 and 50 is given.The best tests were LM\_N and W followed by W' for N = 20, 35 and 50.LM\_N had highest power for all distributions and W and W' performed quite well for N = 100, 200 and 300.Power for LM\_N was 0.928, 0.828, 1.0 and 1.0 which compares with 0.924, 0.820, 1.0 and 1.0 for W' respectively for beta, Student's t, gamma and lognormal densities for N = 200.The ranking of the tests in both tables was approximately the same for a given N and a given distribution.

The true and modified statistics are computed for their correlation to obtain a measure of closes between them.The power of the tests for different values of N using K = 4 and generating the regressors have been studied.The experiments have been repeated generating the regressors in a different way.All correlations decreased for each data set as K increased regarding the correlations between the true and modified statistics.The correlation between D\* and D\* for the normal was equal to 0.640 with K = 4 and to 0.329 with K = 10 for data set 1.The ranking among the tests remained the same.LM\_N performed with good relative power for all the forms of matrices Q\_X studied.This was true for both small N = 20 and large N = 300 encouraging the use of LM\_N when testing for normality of regression disturbances.The statistic LM\_N is simple to compute and in any regression problem an approximation to its finite sample distribution under H\_0 by computer simulation may be easily obtained.It should not be a serious problem with fast speed and increased availability of modern computers.Comparison can be made with the critical point from a χ2\_(2) if one has large samples.

The contradictions areLaGrange multiplier test is developed based on linear inequalities.This will not work with non-linear inequalities during crisis.

The research designs or methods seemed inadequate becauseProcedure presented for construction of specification tests is made up of usage of score test on a general family of distribution.It is the Lagrange multiplier test. First, it is asymptotically equivalent to the likelihood ratio rest, thus it has the same asymptotic power characteristics including maximum local asymptotic power.Second, it is computed with estimation under null hypothesis.Estimation under null hypothesis is easily carried out.

Thus, the test is computationally attractive compared to other asymptotically equivalent procedures e.g. likelihood ratio and Wald tests for inferential problems studied.The LaGrange multiplier method has been applied recentlytoo many econometrics problems. The procedure for construction of specification tests uses this principle too.However, the formulation of l(θ) is distinct.It is assumed that the true probability density function for u\_i belongs to a general family e.g. the Pearson family of which the distribution under the null hypothesis is a particular member.ui or transformation of ui is not assumed to have a particular probability density function.The LaGrange multiplier principle to test null hypothesis within this general family of distributions is then applied.The tests obtained are noted to have optimal large sample power properties for general family members specified.

This does not mean they will not have good power properties for nonmember distributions.

The tests can perform with extremely good power for distributions not belonging to the general family from which the test was derived.The Lagrange multiplier method is used to derive an additional test for normality of observations simple to compute and asymptotically efficient.Different tests for normality of observations are available.Tests can be based on either of the quantities defined.They possess optimal properties for large sample if departure from normality is from either skewness or kurtosis.Omnibus tests can be based on joint use of quantities defined.R test is a good example.Other tests only stated the expression of statistic and was asymptotically distributed as chi-square under normality.Its large or finite sample properties was not studied.The expression is a score or LM test statistic.A principle that proves its asymptotic efficiency has been uncovered.The study of its finite sample properties is encouraged.This highlights the difficulty of analytically getting the finite sample distribution of LM test statistic under the null hypothesis.Resorting to computer simulation is an alternative.LM is invariant to scale parameter i.e. the value of LM is the same if computed with vi/σ instead of vi for all finite σ > 0.A good approximation of the distribution of LM may be obtained with a large enough n to determine the critical point of test for a given significance level α or probability of a type I error for computed value of LM from a particular set of observations.Computer simulation is used to present a study comparing the finite sample power of LM with that of other tests for normality and a table of significance points for α = 0.10 and 0.05.The procedure utilized here may be applied in a similar fashion to other families of distributions.Gram-Charlier (type A) family and derived the LM normality test obtaining the same expression as for Pearson family.The approach may also be used to test hypothesis that f(u) is any particular member of e.g. Pearson family.The regression model defined earlier is considered.Testing the normality of the disturbances is equivalent to testing H0: θ\_2 = 0.

The resulting test statistic is written with a suffix N to indicate disturbance normality test.LMN is under H0, asymptotically distributed as χ2(2) and asymptotically efficient.It appears to be intractable when trying to obtain the finite sample distribution of LMN by analytical procedures.Using computer simulation to generate ui from a N(0, 1) for a given matrix X.LMN is invariant to scale parameter σ2.Computer simulation is used to study the finite sample power of LMN.

The opinion regarding the quality and importance is todoportfoliooptimization successfully, having both the right probability distribution and risk measures are crucial.This paper only focus on selecting the appropriate probability distribution.Nothing is mentioned about the risk measures.

The topic should be further studied because the development of score or Lagrange multiplier procedure to test nonlinear inequalities is an area for further research.Work should be done to determine the right risk measures to match the right probability distribution during crisis.

Madan and Seneta (1990)’s research problem is a new stochastic process, V.G. (Variance Gamma) process, is proposed as a model for uncertainty underlying security prices.The unit period distribution is normal on a variance distributed as a gamma variate.Its advantages include long tailedness, continuous-time specification, finite moments of all orders, elliptical multivariate unit period distributions and good empirical fit.The process is pure jump approximable by a compound Poisson process with high jump frequency and low jump magnitudes.Applications to option pricing display differential effects for options on the money, compared to, in, or out of the money.

The key findings of their research are introducing a continuous-time stochastic process, V.G. (Variance Gamma) model, to model underlying uncertainty determining stock market returns. They provide a practical and empirically related alternative to Brownian motion, as martingale component of motion in log prices.A stochastic process not just a distribution for unit period returns is crucial to European call option pricing that computes risk-neutral expectations but account for risk aversion through identifying an explicit different measure.

The empirically relevant properties searched in proposed process include:

(1) long tailedness relative to normal for daily returns with returns approaching normality over the long run;

(2) finite moments for minimum the lower powers of returns;

(3) consistency with underlying, continuous-time stochastic process with independent stationary increments and with distribution of any increment belonging to same simple family of distributions irrespective of length of time to which increment corresponds permitting sampling and analysis through time in straightforward fashion;

(4) extension to multivariate processes with elliptical multivariate distributions to maintain validity of capital asset pricing model.

Some of the probability distributions for market returns are:

normal distribution, symmetric stable distribution, compound events model combining normally distributed jumps at Poisson jump times;

Brownian motion does not adhere to property 1.Symmetric stable does not adhere to properties 2 and 3.Using V.G. model, long tailedness is present for daily returns whereas monthly returns tend to be normally distributed.Processes b(t), G(t), Y(t), U(t) and W(t) are examples of Levy processes which is a natural continuous-time analog of sequence of partial sums of independently and identically distributed random variables.Central to properties of V.G. processes are those with stationary independent gamma increments because all of G(t), U(t) and W(t) are of this form.

The important gaps in knowledge and researchare brownian motion does not adhere to property 1.Symmetric stable does not adhere to properties 2 and 3.Praetz t distribution does not adhere to property 3 because it is impossible to construct a stochastic process with property 3 and distributions of any increment being a t distribution regardless of time interval length considered as sum of independent t-variables is not a t-variable.Generalized beta does not adhere to property 3.Proposed V.G. model has further advantage as a pure jump process of a large number of small jumps although the compound events model of Press has all four properties described.Multivariate model has a weakness in that ν is same for all marginal distribution thus having identical kurtosis.

The trends and themes of the research are normal distribution with standard deviation is usually used in portfolio optimization during normal times.During crisis, a different distribution with appropriate risk measure should be used.Generalized hyperbolic distribution with expected shortfall or value-at-risk is suggested.

The relationships between key concepts areshown that V.G. model is a limit of a particular sequence of compound events models where arrival rate of jumps tends to infinity while magnitudes of jumps are progressively focused near the origin.V.G. model adheres the intuition underlying sample path continuity of Brownian motion as a model.The effect of increasing the degree of long tailedness increase the probability near the origin to increase tail probabilities at expense of probability in intermediate range.The t distribution densities do not belong to same simple family for increments over intervals of arbitrary length.On the contrary, gamma density uses the process of independent gamma increments whose structure is well known in probability literature to form consistency of unit period returns with easily describe underlying continuous-time stochastic process.

The inconsistency is the V.G. may not work well during crisis without the appropriate risk measure like expected shortfall and value-at-risk compared to standard deviation.

The research designs or methods seemed inadequate are the V.G. model is a good model for describing daily stock market returns.It was compared with normal, stable, press compound events model (ncp) using a chi-squared goodness-of-fit statistic on seven class intervals for unit sample variance data on 19 stocks listed on the Sydney Stock Exchange.The class intervals used were -∞, -1, -0.75, -0.25, 0.25, 0.75, 1.0, ∞.The chi-squared statistics possessed 6 degrees of freedom.Minimum chi squared was attained by V.G. model compared to ncp, stable and normal processes for 12 of 19 stocks studied.Remaining seven cases were best characterized by ncp for five cases, stable for two cases and none for normal distribution.

The opinions regarding the quality and importance are there are infinitely many jumps in any interval and characteristic of jumps vanishing in size shared by process of independent stable increments which is a property of the V.G. process Y(t).Such a description is more realistic than that by a diffusion process given stock prices are constrained to move in multiples of one cent.Using the right probability distribution and risk measure during crisis is crucial to get the right portfolio optimization.

The topic should be further studied because the effect of increasing the kurtosis is to initially lower option price, but as kurtosis is further increased, the V.G. option price rises above Black-Scholes value.The option price did not fall as quickly and rose above Black-Scholes sooner when kurtosis was increased for both in-the-money and out-of-the-money options than was for on-the-money options.V.G., a new stochastic process for underlying uncertainty driving security prices was proposed. Its distribution is normal conditional on variance distributed as a gamma variate. It is shown to be long tailed relative to normal for motion over smaller time intervals and tends to normality for motions over longer time intervals.The stochastic process is pure jump which is approximable by a particular compound Poisson process with high jump frequency and jump magnitude focused near the origin. The joint distributions of jumps in order of magnitude were identified.The unit period distributions had finite moments of all orders and have good empirical fit.A generalization to a multivariate stochastic process was made had elliptical unit period distributions consistent with capital asset pricing model.Applications to option pricing were made and a differential effect was observed on options on the money compared to in or out of the money. Increasing kurtosis have a greater upward effect on options either in or out of the money compared to those that are on the money.Selecting appropriate probability distribution with the appropriate risk measure is especially important during crisis.

Merton (1990)’s research problem is about the study of international finance and comparative financial systems is proposed.The posited structure of organizations and their functions is modeled after US system.Institutitonal discussions surrounding those analyses abstract for special issues from intersystem transactions crossing sovereign borders.General equilibrium analysis follows traditional line of separation between macroeconomics and finance and excludes public sector component from its model of financial system.This does not explicitly capture effects of central bank and other government activities on financial economy.The foundation of modern finance theory is based on perfect-market paradigm of rational behavior and frictionless, competitive, and informationally efficient capital markets.The base of the theory should be labeled super perfect-market paradigm with the future assumption of continuous trading.Such conditions are not literally satisfied in the real world.Its accuracy as a useful approximation to the real world changes greatly across time and place.The practitioner should apply continuous-time theory only tentatively, assessing its limitations in each application.The researcher should treat it as departure point for problem finding and problem solving. The keyfindingsof their research are the main function of financial intermediaries is to be principals to create financial instruments because of scale and terms specificity cannot be efficiently supported by direct trading in organized financial markets.The focus is on the economic function of financial intermediaries and their products not their particular institutional form.Financial intermediaries raise capital for operations through stock and debt issuance to investors.The theory assumes that managers of intermediaries and business firms share same primary objective of maximizing interests of current stockholders.Intermediaries hold only financial assets and create new liabilities whenever they sell their products unlike business firms.The capital market is collection of organized financial markets for trading standardized securities like stocks, bonds, futures contracts, and options.It is the central external environment linking the financial activities of households, business firms, and intermediaries.Firms raise capital necessary for investment and households deploy savings for future consumption by transacting in the capital market.

Function of securities traded in capital market is to provide households with risk-pooling and risk-sharing opportunities to facilitate efficient allocation of resources.The capital market is an important function as key source of information that helps coordinate decentralized decision-making in different sectors of the economy.Information influences the requirements on design of financial instruments and organizations.Portfolio-selection problem in classical static framework is formulated and solved based on household's consumption decision.The focus is on derivations of spanning and mutual fund theorems, Ross Arbitrage Pricing Theory (APT) Model, and Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM).The key of the classic CAPM is the Security Market Line, a linear equation relating equilibrium expected return on each asset (or portfolio of assets) to a single identifiable risk measure.Reduced form equations of this genus are among the most commonly and often used analytical tools in both applied and empirical finance.

The important gaps in knowledge and research are a study of how to best allocate and utilize resources across time in an uncertain environment and how economic organizations facilitate these allocations.The key organizations in finance are households, business firms, financial intermediaries, and capital markets.Neoclassical economics is about taking existence of households, their tastes, and their endowments as exogenous to the theory.On the contrary, this tradition does not extend to other economic organizations and institutions.

They are considered as existing primarily because of functions they serve and thus endogenous to theory.Therefore, optimal financial behavior of households is taken from individual and exogenously specified preference functions rank-ordering alternative programs of lifetime consumption and inheritances for each household.Optimal management decisions for business firms and financial intermediaries are taken from criteria determined by functions of those organizations in the financial economic system in contrast.

Analysis emphasizes demand side of capital markets and treats as exogenous the dynamics of supply curve for securities.Thus, the model does not have all structural equations of endogenous behavior necessary for a full equilibrium analysis of the system.Modigliani-Miller theorem usually fails in an environment of corporate taxes and deductibility of interest payments.Modifications to the pricing model that are sufficient to accommodate effects of taxes on financing option is discussed.This concludes with a survey of applications of CCA to corporate finance issues ranging from investment and financing decisions to evaluation of corporate strategy.The Breeden Consumption-Based Capital Asset Pricing Model (CCAMP) is obtained under more restrictive conditions.Equilibrium expected returns can be expressed in terms of single risk measure in this important version of intertemporal model.This risk measure is different from the one in static CAPM.Further results on investor hedging behavior and mutual-fund theorems add important detail to product identification part of financial intermediation theory. The model does not explicitly include either derivative security markets or much of intermediation sector, a simplification justified by quasi dichotomy findings. The analysis is on demand side of capital markets and treats as largely exogeneous dynamics of supply curves for securities in development of original CAPM.It does not provide all structural equations of endogenous behavior needed for a full equilibrium analysis of the system.The design of public pension plans is discussed.Pension annuities should be indexed to protect retirees against inflation suggested by academics and practitioners.However, such annuities do not provide protection against real gains in standard of livings.A new kind of plan indexing benefits to aggregate per capita consumption is investigated as possible solution to protect pensioners against both of these risks.A simple model for mortality with standard assumption that individual mortality risk is diversifiable across the population is posited.The equilibrium prices of consumption-indexed annuities are derived from applying competitive arbitrage techniques. The optimal consumption-saving model is used to determine required contribution rate to pension plan ensuring an optimal level of retirement benefits under condition that household preferences satisfy Life Cycle Hypothesis.The feasibility of implementing such a plan is discussed.Proper risk measure and probability distribution is needed during crisis.

The trends and themes of the research related to the topic are anexplosion of theoretical, applied and empirical research on option pricing occurred after the publication of Black-Scholes model in 1973.Its impact has been felt beyond the borders of economics.Renewed interest in estimation of variance rates for stochastic processes is created in applied statistics.Extensive new research on numerical integration of partial differential equations that must be solved to determine Black-Scholes option prices is stimulated in numerical methods.There are three topics without which the most fundamental presentation of modern option pricing theory would be conspicuously incomplete.They are the Cox-Ross risk-neutral pricing methodology, the binomial option pricing model, and basic theory for pricing options on futures contracts.Contingent-claims analysis (CCA) combines dynamic portfolio theory with Black-Scholes option pricing model to create one of the most powerful tools of analysis in modern finance.CCA is for studying a big range of topics in theory of corporate finance and financial intermediation.Unified theory for pricing corporate liabilities continues the study of financial instruments started.One-period utility-based theory of pricing to intertemporal theory based on conditional arbitrage is provided.

The pricing of general capital assets and determination of term structure of interest rates topics is introduced.

The relationships between key concepts are the mathematics of continuous-time processes include Ito's calculus and stochastic differential equations for mixtures of diffusion and Poisson-driven random variables is introduced.The basic two-asset version of lifetime consumption and portfolio-selection problem is resolved with stochastic dynamic programming.A discrete-time formulation of intertemporal model is started with continuous-time formulation as a limiting case derived.A risky asset with log-normally distributed returns and a riskless asset with constant interest rate are assumed.An explicit optimal consumption and portfolio rules for households exhibiting either constant relative risk aversion or constant absolute risk aversion.The intertemporal age dependent behavior of optimal consumption is consistent with Modigliani-Brumberg Life-Cycle Hypothesis.

The derived optimal portfolio rules have same structure as those in Markowitz-Tobin mean-variance model.This model is expanded to include wage income, uncertain lifetimes and several assets with more general probability distributions of returns.Ito's lemma is introduced to analyze dynamics of asset prices, wealth, and consumption.The derived structure of each household's optimal demands for assets is all optimal portfolios can be generated with simple combinations of just two portfolios in the prototypal case of joint log-normally distributed asset returns.This mutual-fund theorem is identical with separation theorem of static mean-variance model.Closed-form solutions for optimal consumption functions are determined for members of family of utility functions with hyperbolic absolute risk aversion.It is shown that these are the only time-additive and independent preference orderings leading to optimal consumption functions linear in wealth.The analyses performed do not explicitly impose feasibility constraints that neither consumption nor wealth can be negative in determining these optimal policies.The models posit a single consumption good and assume that preferences are time additive with no intertemporal complementarity of consumption.The robustness of the derived results with respect to relaxation of these assumptions is explored.The important Cox-Huang application of martingale theory to lifetime consumption and portfolio problem as a solution is introduced.

TheCox-Huang technique is especially well suited for incorporating these particular nonnegativity constraints as an alternative to stochastic dynamic programming.It shows that with mild regularity conditions on preferences the optimal consumption and portfolio strategies of households never risk personal bankruptcy unlike the unconstrained case.The fundamental spanning and mutual-fund theorems of preceding unconstrained analyses are left unchanged although details of optimal strategies are affected by these constraints.This same preservation of necessary structure of optimal portfolio demands is shown to obtain for households with preferences depending on other variables in addition to current consumption and age.Specific cases examined take into account multiple consumption goods, money-in-the-utility function and preferences with nonzero intertemporal complementarity of consumption.Pricing of financial instruments traded in capital markets is carried out after developing optimal investment behavior of households.Warrant and option pricing theory is the earliest example of continuous-time analysis application.These highly specialized securities have become increasingly more important in real world with creation and successful development of organized option markets during the last 15 years.

Neither history nor commercial success is reason for extensive treatment of option pricing.

Option analysis comes from option-like contracts found in almost every sector of economy.

Options serve as simplest examples of securities with nonlinear sharing rules in prototypal structure.One-period preference-based models of warrant and option pricing is connected to dynamic arbitrage-based pricing models.Development of modern option pricing theory starts with derivation of price restrictions necessary to rule out arbitrage opportunities.Such restrictions are insufficient to determine a unique set of option prices. The seminal Black-Scholes model applies and a unique set of option prices can be derived from arbitrage considerations alone conditional on twin assumptions of continuous trading and continous sample-path stochastic process for underlying stock price dynamics.This concludes with applying conditional arbitrage model to pricing of several kinds of options and warrants.Option pricing is examined when discontinuous changes or gaps in underlying stock price are possible.The conditional arbitrage argument to derive Black-Scholes model is no longer valid with such jumps in prices a possibility.An equilibrium model of option price is derived with further assumption that discontinuous components of stock price changes are diversifiable risks.The most popular separation of a firm's capital structure is between debt and equity.The pricing of corporate debt and levered equity, starting with simplest, nontrivial capital structure of a single homogeneous zeron-coupon bond issue and equity is investigated.Corporate debt can be represented functionally as combination of default-free debt and unlevered equity as shown by the analysis.The risk structure of interest rates uses default risk to differentiate among promised yields like term structure uses maturity to differentiate among bond yields.The derived pricing model for corporate bonds is to define risk structure and comparative statics are applied to show the effects of parameter changes on that structure.This is extended to include corporate bonds.The important Modigliani-Miller theorem states that the value of firm is invariant to choice of debt-equity mix in the presence of bankruptcy possibilities.A model for pricing general contingent claims or derivative securities is constructed using continuous-time dynamic portfolio theory.The Modigliani-Miller theorem is proven for firms with general corporate liability structures with frictionless markets and no taxes.The theory of financial intermediation with focus on risk-pooling and risk-sharing services provided by intermediaries is discussed.This is divided into three categories of contributions of continuous-time analysis to theory and practice of financial intermediation: product identification, product implementation and pricing, and risk management and control for an intermediary's whole portfolio.The theory claims that financial intermediaries and derivative-security markets are redundant organizations in idealized environment where all investors pay neither transaction costs nor taxes, have same information, and can trade continuously.The posited environment must include some kind of transaction or information cost structure where financial intermediaries and market makers have comparative advantage with respect to some investors and corporate issuers of securities.A simple binomial model of derivative-security pricing with transaction costs is formally analyzed.It shows that bid-ask spreads in these prices can be substantial and significant economic benefits can accrue from efficient intermediation.The theory of intermediation using a continuous-time model where many agents cannot trade without cost but lowest-cost transactors, financial intermediaries, can.This provides the raison d'etre for derivative-security products and allows standard CCA to determine costs for intermediaries to produce them.The theory of optimal consumption and portfolio choice is to identify customer demands for list of intermediary products from generic multipurpose mutual funds to custom-designed financial contracts to fit each investor's specific needs.The theory of product implementation with the production technologies and costs for intermediaries to manufacture the products is specified from the analysis.The same CCA and dynamic portfolio-selection tools are used to examine the problem of overall risk management for an intermediary to prepare for equilibrium analysis.The role of an efficient financial intermediation system in justifying models assuming dichotomy between real and financial sectors of the economy is discussed.Several observations on policy and strategy issues drawn from continuous-time theory of intermediation is discussed.Separate investigations of main organizations comprising financial economy are brought together to give an equilibrium analysis of the whole system.Optimal financial behavior of individual agents and organizations is given by a set of contingent decisions or plans depending on current prices and probability beliefs about future evolution of the economy.Conditions satisfied by an equilibrium set of prices and beliefs are all optimal plans can be implemented at each point in time and resulting ex post time path of economy is consistent with ex ante probability assessments.It is assumed that all agents and organizations are price-takers in capital markets and financial intermediation is a competitive industry.A standard Walrasian setting is posited to be the mechanism to clear markets at each point in time.Establishing similar kinds of necessary conditions to be satisfied by intertemporal equilibrium prices of capital assets is done.Equilibrium expected returns are linearly related to vector of risk measures in the intertemporal Capital Asset Pricing Model.Security Market Hyperplane is the reduced-form equation of continuous-time model.The continuous-time model is reformulated to framework of Arrow-Debreu model of complete markets for general equilibrium study of financial economy.An Arrow-Debreu pure security gives a positive payoff at only one point in time and in only one state of the economy.Pure securities are traded for each possible state of the economy at every future date in an Arrow-Debreu economy.The continuous-time version of an Arrow-Debreu economy needs a continuum of such securities.Financial intermediaries and other zero-cost transactors can synthetically make a complete set of pure securities by trading continuously in a finite number of securities.The continuous-time model provides a solid demonstration of Arrow and Radner observation that dynamic trading in securities can be a substitute for full set of pure security markets.Explicit formulas for trading technologies and production costs needed to make pure securities are derived building on theory of intermediation.The possibility of feasible approximations in real-world financial markets is unrealistic of complete markets in Arrow-Debreu world from the specificity of these theoretical findings.The lifetime consumption allocation problem for households is solved using static optimization techniques as in original Arrow-Debreu analysis.Such state-contingent optimal demand functions are used to derive equilibrium allocations, prices, and rational expectations for pure-exchange economy.Production and optimal investment behavior of business firms are added in second competitive equilibrium model.More specific version is from prescriptions of original CAPM in an intertemporal general equilibrium model with production.A Ramsey-Solow macro model is created to analyze dynamics of long-run economic growth in an economy with uncertainty about either demographics or technological progress.The focus is on biases introduced in long-run forecasting and planning by neglecting uncertainty.Closed-form solutions for steady-state probability distributions of relevant economic variables are obtained for Cobb-Douglas production function and constant proportional savings function.This concludes with examination of stochastic Ramsey problem of central planning.The evaluation of loan guarantees is crucial in both private and public finance.

CCA is used to develop generic models for pricing these guarantees and apply them to analysis of deposit insurance.Third-party guarantees of financial performance on loans and other debt-related contracts are often used throughout USA and other well-developed economies.Parent corporations usually guarantee debt obligations of their subsidiaries.

Commercial banks give guarantees on a broad spectrum of financial instruments from letters of credit to interest rate and currency swaps in return for fees.More specialized firms sell guarantees of interest and principal payments with tax-exempt municipal bonds.On the contrary, the largest provider of financial guarantees is the federal government either directly or through its agencies.It has guaranteed loans to small businesses and on occasion, with Lockheed Aircraft and Chrysler Corporation, for very large businesses.The United States Synthetic Fuels Corporation was empowered to grant loan guarantees to help financing of commercial projects involving development of alternative fuel technologies started in 1980.The government gives limited insurance of corporate pension-plan benefits through Pension Benefit Guarantee Corporation.The examples of noncorporate obligations that the government has guaranteed are residential mortgages and farm and student loans.

The most important of its liability guarantees economically and politically is deposit insurance.The Federal Deposit Insurance Corporation (FDIC) and Federal Savings and Loan Insurance Corporation (FSLIC) insure deposits of commercial banks and thrift institutions up to a maximum of USD 100,000 per account.The economic responsibility for monitoring a bank's activities transfers from its depositors to insurer.The insurer continuously monitor the values of those assets in relation to deposits in a frictionless world and only liquid assets and no surveillance costs.Any losses could be avoided by forcing liquidation of assets before insolvency point is arrived.Most bank assets are illiquid and surveillance costs are present and the models for determining cost of deposit insurance considers both potential losses from insolvency and monitoring cost.Evaluation of deposit insurance is further complicated in real world because deposit insurance obligations are not traded in markets.The government must estimate actuarial cost of giving this insurance without benefit of market prices.The implementation of CCA usually does not need historical price data on security to be priced.This evaluation technique is a well-suited appraisal tool for costs estimation of deposit guarantees.

The inconsistencies are comparative statistics analysis show that little can be derived about structure of optimal portfolio demand functions unless further restrictions are carried out on class of investor's utility functions or class of probability distributions for securities' returns.

This spanning property leads to a collection of mutual fund or separation theorems crucial to modern financial theory. A mutual fund is a financial intermediary holding as assets a portfolio of securities and issues as liabilities shares against this collection of assets.The portfolio of securities held by a mutual fund need not be an efficient portfolio unlike optimal portfolio of an individual investor.The question is not addressed what happens if the mutual fund is part of the optimal portfolio of an individual investor.For the theorem stating indifference, with information-gathering or other transaction costs and economies of scale, investors would prefer mutual funds whenever M < N.The spanning property is used to derive an endogenous theory for existence of financial intermediaries with functional characteristics of a mutual fund.A theory for their optimal management can be derived from these functional characteristics.The minimum number M\* of funds needed for spanning must be much smaller than number of N of available securities for mutual-fund theorems to have serious empirical content.The investor's portfolio-selection problem can be divided into two steps when such spanning obtains.First, individual securities are mixed together to form M\* mutual funds.Second, investor allocates his wealth among M\* funds' shares.Formal separation property will have little operational significance if M\* funds can be constructed only if fund managers know the preferences, endowments, and probability beliefs of each investor.We do not know if this is always possible.The determination of conditions under which nontrivial spanning will obtain is a subset of traditional economic theory of aggregation.All important models of portfolio selection show nontrivial spanning property for efficient portfolio set.Most macroeconomic models have highly aggregated financial sectors where investors' portfolio options are constrained to simple combinations of two securities: bonds and stocks.The distribution of wealth, or joint probability distribution for outstanding securities, models in which market portfolio can be proved to be efficient are more likely to produce testable hypotheses because market portfolio can be made without knowledge of preferences.The efficiency of market portfolio gives a rigorous microeconomic justification for use of representative of derives equilibrium prices in aggregated economic models.Market portfolio is efficient if and only if there is a concave utility function so that maximization of its expected value with initial wealth equal to national wealth can lead to market portfolio as optimal portfolio.On the contrary, general necessary and sufficient conditions for market portfolio to be efficient are not derived yet.

The equilibrium expected return on market portfolio is more than return on riskless security in all portfolio models with homogeneous beliefs and risk-averse investors.Every efficient portfolio and every optimal portfolio can be a simple portfolio combination of market portfolio and riskless security with a positive fraction allocated to market portfolio.The only way is relative proportions are identical with those in market portfolio if all investors want to hold risky securities in same relative proportions.The Security Market Line derived by Sharpe is a necessary condition for equilibrium in mean-variance model of Markowitz and Tobin when investors possess homogenous beliefs.It is central to most empirical studies of securities' returns published during last two decades.The Arbitrage Pricing Theory (APT) Model gives an important class of linear factor models generating at least approximate spanning without assuming joint normal probability distributions.Returns on all efficient portfolios need not be perfectly correlated unlike in mean-variance model.The model is appealing because the equilibrium structure of expected returns and risks of securities can be derived without explicit knowledge of investors' preferences or endowments.Early empirical studies of stock-market securities' returns seldom found more than two or three statistically significant common factors.Empirical foundation for assumptions of APT model exist because there are tens of thousands of different corporate liabilities traded in US securities markets.The number of common factors may be bigger and serious questions are raised on prospect for identifying factors by using stock-return data alone.How the firm finances its investment will not affect the market value of the firm unless the option of financial instruments changes the return distributions of efficient portfolio set for a given investment policy.An alternative way to development of nontrivial spanning theorems is to derive a class of utility functions for investors such that with arbitrary joint probability distributions for available securities, investors within the class can produce their optimal portfolios from spanning portfolios.

The research designs or methods seemedinsufficientbecausebasic investment-option problem for an individual is to figure out the optimal allocation of his or her wealth amidst available investment opportunities.Portfolio-selection theory is the solution to general problem of choosing the best investment mix.This starts with its classic one-period or static formulation.It is assumed that investor selects at the start of a period that feasible portfolio allocation maximizing the expected value of a von Neumann-Morgenstern unity function for end-of-period wealth.The utility function is represented by U(W) with Was end-of-period value of investor's wealth measured in US dollars.It is assumed that U is an increasing strictly concave function on range of feasible values for W and U is twice continuously differentiable.The strict concavity assumption implies that investors are everywhere risk averse.The strictly convex or linear utility functions on whole range can indicate behavior that is grossly at variance with observed behavior.This also rules out Friedman-Savage kind utility functions whose behavioral implications are reasonable.This implies U'(W) > 0 ruling out individual satiation.The assumptions made are:

1) Frictionless markets

2) Price-taker

3) No-Arbitrage Opportunities.

4) No-Institutional Restrictions

Nonsingularity condition on distribution of returns removes redundant securities, that is, securities whose returns are the exact linear combinations of returns on other available securities.Any one of securities is a riskless security is ruled out.Variance-covariance matrix of risky security is nonsingular and interior solution exists is assumed.Purpose of constraint is reflecting institutional restrictions designed to avoid individual bankruptcy is too weak because probability assessments on Z\_i are subjective.The optimal demand functions for risky securities and resulting probability distribution for optimal portfolio will depend on risk preferences of investor, his initial wealth, and joint distribution for securities' returns.von Neumann-Morgenstern utility function can only be calculated up to a positive affine transformation.An investor is more risk averse than second investor if for every portfolio, certainty-equivalent end-of-period wealth for first investor is less than or equal to certainty equivalent end-of-period wealth related to same portfolio for second investor with strict inequality holding for at least one portfolio.A feasible portfolio will be an efficient portfolio only if there is no other feasible portfolio less risky than it is.Inefficient portfolios are those portfolios which are not efficient.No two portfolios in efficient set can be ordered with respect to one another from the definition of efficient portfolio.All risk averse investors will be indifferent between choosing their optimal portfolios from set of all feasible portfolios and choosing their optimal portfolios from set of efficient portfolios.All optimal portfolios are assumed to be efficient portfolios without losing generality.A second definition of increasing risk is introduced more suitable for the risk of security.This second measure will not generally provide the same orderings as the Rothschild-Stieglitz measure. The ordering of securities by their systematic risk relative to a given efficient portfolio will be identical with their ordering relative to any other efficient portfolio.The systematic risk of a portfolio is weighted sum of systematic risks of its component securities.The Rothschild-Stiglitz definition measures total risk of a security by comparing the expected utility from holding a security alone with expected utility from holding another security alone.It is the suitable definition for identifying optimal portfolios and determining efficient portfolio set.However, it is useless for defining risk of securities often because it does not account investors can mix securities together to form portfolios. The manifest behavioral characteristic by all risk-averse utility maximizers is to diversify one's wealth among many investments.The benefits of diversification in reducing risk depends on degree of statistical interdependence among returns on available investments.The greatest benefits in risk reduction is from adding security to portfolio whose realized return is higher when the return on the rest of portfolio is lower.Noncyclic securities have returns orthogonal to portfolio return.The procyclical investments with returns higher when portfolio return is higher and lower when portfolio return is lower.A natural summary statistics for this characteristic of a security's return distribution is its conditional expected-return function conditional on portfolio realized return.The conditional expected-return function gives considerable information about a security's risk and equilibrium expected return.

The opinions regarding the quality and importance are investment decision by households consist of two parts:

(a) consumption-saving option where individual decides how much income and wealth to assign to current consumption and how much to save for future consumption;

(b) portfolio-selection option where investor decides how to assign savings among available investment opportunities.

The two decisions cannot be made independently.Many of important findings in portfolio theory can be more derived easily in one-period environment where consumption-savings assignment has little substantive impact on results.The formulation and solution of basic portfolio-selection problem in static framework given the individual's consumption decision is discussed.The necessary conditions for static financial equilibrium that are used to determine restrictions on equilibrium security prices and returns are derived.Such restrictions are used to derive spanning and mutual-fund theorems providing a support for elementary theory of financial intermediation.

The topic should be further studied because first, virtually all spanning theorems need generally implausible assumption that all investors agree on the joint probability distribution for securities, it is not unreasonable when applied to theory of financial intermediation and mutual-fund management.In financial world where economic ideas of labor division and comparative advantage as content, it is quite reasonable to expect efficient allocation of resources lead to some individuals gathering data and actively estimating joint probability distributions and the rest by buying this information directly or delegating their investment decisions by agreeing with fund managers' estimates.Second, it is about riskless security. Riskless security existence is assumed.Almost all derived theorems can be proven to be valid in absence of riskless security although some specifications will change slightly. Existence of riskless security greatly simplifies many proofs.We need to understand whether formulating optimal portfolio selection as an economic problem involving production and consumption is appropriate during crisis.

Eberlein and Keller (1995)’s research problem talks about distributional assumptions for the returns on underlying assets is key in theories of valuation for derivative securities. They investigate the distributional form of compound returns from daily prices of 30 DAX shares over three years.Some of the standard assumptions cannot be justified after conducting some statistical tests. They introduce class of hyperbolic distributions which fitted to empirical returns with high accuracy. They discussed two models built upon hyperbolic levy motion. They derive a valuation formula for derivative securities after studying Esscher transform of the process with hyperbolic returns.The results shows correction of standard Black Scholes pricing especially options close to expiration.

The key findings of their research show that the distributional form of returns of underlying assets is key in valuation theories for derivative securities in finance. They introduce a model which fits the data with high accuracy and conclude about option pricing after investigating classical assumptions especially the normality hypothesis.The Black Scholes formula is multiplicative and complete.The complete property allows duplication of cash flow of derivative securities.The valuation of these products by arbitrage is done.The qualitative picture stays the same if the time scale is changed because of the self-similarity of Brownian motion as the source of randomness.Real stock price paths change greatly if examined on different time scales.Thus, discrete models with price changes at equidistant discrete time points is needed.This is a first approximation of reality where price changes at random time points.The correct return distributions for discrete models have to be determined.The assumption of normality fails when tests are applied to the real data.Hyperbolic distributions can fit the empirical distributions with great accuracy.

The important gap in data starts with daily KASSA prices of 10 of 30 stocks from german stock index (DAX) during 3 year period (2 October 1989 to 30 September 1992). Time series of 745 data points for each return is obtained.They are corrected for dividend payouts.Dividends are paid once per year for german stocks.Two unusual price changes occurred during the period: crash on 16 October 1989 and drop because of Moscow coup on 19 August 1991.The 10 stocks were chosen because of their large trading volume and of specific activity of the company to get a good representation of the market.Markets from US and Asia should be considered to give a comprehensive coverage on whether hyperbolic distributions work well for them or not.

The trends and themes of the research is related to gaussian versus non-gaussian distribution is normal distribution is a poor model for stock returns, in this case, BASF and Deutsche Bank, especially during crisis.Quantile-quantile plots is used to test the goodness of fit for the stock returns distribution.Deviation from straight line representing normality is obvious.Corresponding normal and empirical density plots are completed too.There is more mass around the origin and in the tails than standard normal distribution can cover.Χ-square test for normality is done.Three different estimation procedures were done to avoid problems from partition sensitivity.Compute certain functions of the moments of the sample data then compare them with expected values for a normal population.Kurtosis and skewness are used because of scale and location invariance to test the composite hypothesis.They are both zero under assumption of normality. The hypothesis is rejected at 1% level for all stocks.The studentized range test is for testing normality of return distribution too.It is rejection at smallest level α = 0.005.Symmetric stable pareto distribution is represented by SP(α) where α is the characteristic component is considered too.α = 2 provides the normal distribution and α = 1 provides the cauchy distribution.When α < 2, stable distributions are more peaked around the center than normal ones and have arbitrarily heavy tails.When α < 2, variance is infinite.When α ≤ 1, the first moment does not exist.Models of stock returns for blue chips should have finite moments.The observed daily price changes are less than 20% for them, thus their variables are bounded.The stability under addition property is used to test the stable hypotheses because of the analytic properties of this class of distributions.The return values are split into groups of increasing size with each group is summed.The characteristic exponent is estimated for each resulting distribution.The stable hypotheses are rejected if the estimated characteristic component increases with each increasing sum size... The serial correlation problem between successive returns is overcome with the data randomized before building groups.If there is serial correlation, a higher kurtosis of monthly returns is induced.The value of the estimated alpha reaches 2 or is close to 2 for most of the shares concludes that gaussian distributions for monthly returns is apt.

The relationship between key concepts are the standard continuous time model for stock prices is described by geometric brownian motion.It is what the Black Scholes formula is built upon.Returns from the geometric brownian motion are brownian motion process increments.They are independent and normally distributed.Hyperbolic distributions have hyperbolic log densities whereas normal distribution has parabolic log density.The hyperbolic distributions can be expressed as normal, symmetric and asymmetric Laplace, generalized inverse gaussian and exponential distributions.α and β shapes the distribution.δ and μ are scale and location parameters.A maximum likelihood estimation is performed assuming independent and identically distributed variables.Kolmogorov Smirnov test was carried out with values between 0.70 and 1.20 for all.Hyperbolic distributions are infinitely divisible.

It is a contradiction with the hyperbolic distribution on top of gaussian distribution because financial data distribution is non-gaussian during crisis.

Research designs or methods seemedinadequate. Focus on empirical study to determine the correct distributions for the financial returns.Examined the normal, stable Pareto, student and finite discrete mixtures of normal distributions.

The opinion regarding the quality and importance is the study is important and the quality of the study can be improved by extending the data during crisis.

The topic should be further studied by applying it to data during crisis to capture loss distribution accurately.

Barndorff-Nielsen (1997)’s research problem shows that the normal inverse Gaussian distribution is a variance-mean mixture of a normal distribution with inverse Gaussian as the mixing distribution.It determines a homogeneous Levy process which is representable through subordination of brownian motion by inverse Gaussian process.The Levy type decomposition of the process is determined.The relationship of the normal inverse Gaussian to classes of generalized hyperbolic and inverse Gaussian distributions is reviewed briefly.A discussion on the potential of normal inverse Gaussian distribution and Levy process for modelling and analyzing statistical data with specific reference toextensive observations from turbulence and from finance.The need to extend the inverse Gaussian Levy process to account for certain, frequently observed, temporal dependence structures.Extensions of the stochastic volatility type are built through an observation-driven method to stage space modelling.Generalizations to multivariate settings are shown.

The key findings of their research show that a normal variance-mean mixture distribution (normal inverse Gaussian distribution) is to construct stochastic processes of interest for statistical modellingespecially in turbulence and finance.Univariate normal inverse Gaussian distribution create homogeneous Levy processes.The normal inverse Gaussian Levy process may be represented through random time change of a Brownian motion because of the mixture representation of the normal inverse Gaussian distribution.The normal inverse Gaussian Levy process is a subordination of Brownian motion by the inverse Gaussian Levy process.Levy decomposition analysis proves that the processes can be viewed as a superposition of weighted independent Poisson processes, weights of all numerically small sizes occurring.The small jumps are dominating the behavior of the normal inverse Gaussian Levy process.The norming constant is to ensure the probability density function sums up to 1.The normal inverse Gaussian and hyperbolic distribution are related to λ = -1/2 and λ = 1 respectively.The autocorrelations of an observed series from financial asset returns are 0 but the squared series have positive autocorrelations decreasing slowly to 0.

The important gap in knowledge is the model presented is analogous to ARCH models but the conditional law of the volatility at time t with observations from previous time points is here non-degenerate.

The trends and themes of the research is related to gaussian versus non-gaussian distribution because normal inverse Gaussian distribution approximates most hyperbolic distribution very closely.It can describe considerably heavier tail behavior than log linear rate of decrease characterizing hyperbolic shape.This is especially so during financial crisis.Normal inverse Gaussian distribution has more tractable probabilistic properties than hyperbolic distribution.

The relationship between key concepts are financial crisis is represented with a lot of jumps during statistical modelling.These jumps are best modelled with Levy processes.Levy decomposition analysis proves that the processes can be viewed as a superposition of weighted independent Poisson processes, weights of all numerically small sizes occurring.The small jumps are dominating the behavior of the normal inverse GaussianLevy process.

It is a contradiction to build the normal inverse gaussian distribution on top of gaussian distribution because financial data distribution is non-gaussian during crisis.

Research designs or methods seemedinadequate because only theoretical proof is given for the model described.

The opinion regarding the quality and importance is normal inverse gaussian distribution plays an important role in modelling stochastic volatility.Statistical significance testing like log-likelihood and akaike information criteria should be given to determine how well the normal inverse Gaussiandistribution fit the real financial data obtained during crisis.

Whether normal inverse gaussian distribution can model stochastic volatility precisely during crisis should be further studied to understand risk taken commensurate with the return wanted.

Gopikrishnan, Meyer, Nunes Amaral, and Stanley (1998) research problem is the probability distribution of stock price variations is studied by analyzing the Trades and Quotes Database documenting all trades for all stocks in three major US stock markets for a two year period from Jan 1994 to Dec 1995. A 40 million data points sample is extracted substantially larger than studied until now. An asymptotic power-law behavior for cumulative distribution with an exponent α ≈ 3. It is very much outside the Levy regime (0 < α < 2).

The key findings of their research are asymptotic behavior of increment distribution of economic indices has been of avid interest to others. Conclusive empirical results are hard to get because a proper sampling of tails requiring huge quantity of data is required. Database documenting every trade in three major US stock markets, New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotation (NASDAQ) for January 1994 to December 1995 (2-period) is analyzed. A sample of approximately 4 x (107) data points much larger than studied until now is extracted. 1000 time series Si(t) with Si as the market price of company i, that is, share price x number of outstanding shares, i = 1…1000 is rank of company according to its market price on January 1 1994. Relative price change is the basic quantity of the study with a time lag of 5 min. The increments are normalized with the volatility of the company i measured by standard deviation and time averaged. 20,000 normalized increments per company per year giving about 4 x (107) increments for 1000 largest companies in the time period studied are obtained. The cumulative probability distribution is obtained with the data fitted well by a power law with exponents from two to hundred standard deviations. The inverse of the local logarithmic slope is calculated to test this result. The asymptotic slope is estimated by extrapolating the inverse of the local logarithmic slope as a function set to 0. Negative and positive tail each using all increments larger than 5 standard deviations results are shown. Extrapolation of linear regression lines yield different values of α for both positive and negative tails. This method is tested by analyzing two surrogate data sets with known asymptotic behavior using two different independent random variables. The method gives the correct results. Several additional calculations are done to test the robustness of inverse cubic law α ≈ 3. First, the time increment in steps is changed to 120 minutes from 5 minutes. Second, S&P 500 index over the 13 year period (January 84 to December 96) is analyzed using the same methods mentioned before. Third, the definition of volatility is replaced by other measures. The results all appear consistent with α = 3.

The important gaps in knowledge/research/data are data of three selected markets may not be a true and accurate representation of stock price variations in the whole world. Standard deviation does not capture tail risks unlike Value-at-Risk (VaR) and Expected Shortfall (ES). Tail risks have low probability of occurrence but high impact. Tails of probability distributions are best represented by its skewness. Standard deviation assumes normal distribution. The data is fitted to the power law defined with two variables: g and exponent α. α = 3.1 +/- 0.03 covers the positive tail and α = 2.84 +/- 0.12 covers the negative tail from two to hundred standard deviations described by g. Any tails outside these range will not be covered. Using the right probability distribution alone without the right risk measure is insufficient to study stock price variations. You need both the right probability distribution and risk measure to study stock price variations. Kurtosis will also affect the skewness of the probability distribution which is not studied here. The power law defined does not take into account the kurtosis of the probability distribution.

The trends and themes of the research are common measure of risk like standard deviation is for common time. Uncommon measures of risks like VaR and ES are for uncommon times like crisis with tail risks etc. Uncommon times like crisis etc will produce probability distributions with greater skewness and kurtosis. Appropriate probability distribution and risk measure are both necessary to study stock price variations.

The relationships between key concepts are proposals have included (i) Gaussian distribution, (ii) Levy distribution, and (iii) truncated Levy distribution where the tails become approximately exponential. Inverse cubic law is different from all three proposals.

It has a diverging higher moments (larger than 3) unlike (i) and (iii). It is not a stable distribution unlike (i) and (ii).

The contradictions are using standard deviation with assumed normal distribution is inappropriate to capture tail risks. Tail risks are best represented by generalized hyperbolic distributions and measured with VaR and ES.

The research designs or methods seemed inadequate are the power law defined is naïve when it comes to capturing tail risks. It does not reflect all the skewness of the probability distribution.

The opinion regarding the quality and importance are the probability distribution based on the power law defined is both inappropriate and limited whereas generalized hyperbolic distribution is appropriate and extensive. Standard deviation assumed on normal distribution does not capture tail risks appropriately whereas VaR and ES are both the appropriate measures.

The topic should be further studied because using the right probability distribution and risk measures for portfolio management and optimization is crucial to assist investors, fund managers, speculators etc to achieve the expected returns wanted based on the risks taken.

This will minimize the occurrence of financial crises similar to the nature of subprime mortgage defaults etc.

Sornette, Simonetti, and Andersen (2000)’s researchproblem show that the research gives a precise analytical characterization of returns distribution for a portfolio made up of assets whose returns are given by an arbitrary joint multivariate distribution.A non-linear transformation maps the returns onto gaussian variables whose covariance matrix gives a new measure of dependence between non-normal returns generalizing the covariance matrix into a non-linear fractional covariance matrix.This nonlinear covariance matrix is shaped to specific fat tail structure of underlying marginal distributions ensuring stability and good-conditioning.The portfolio distribution is constructed from solution of mapping to a qi field theory in particle physics offering extensive treatment using Feynman diagrammatic techniques and large deviation theory illustrated in details for multivariate Weibull distributions.The main result of the authors' theory is minimizing portfolio variance i.e. relatively small risks may often increase large risks measured by higher normalized cumulants.Extensive empirical tests presented on the foreign exchange market validate the theory satisfactorily.The authors show that a sufficient prediction of risks of a portfolio depends much more on correct description of tail structure rather than on their correlations for fat tail distributions.

The key findings of their research are a representation as parsimonious as Gaussian framework but capturing non-Gaussian fat tail nature of marginal distributions with non-linear dependence existence.A non-linear transformation maps returns onto Gaussian variables with covariance matrix providing a new measure of dependence between non-normal returns, generalizing covariance matrix into non-linear fractional covariance matrix.

The linear matrix calculation of standard portfolio theory is supplanted by Feynman's diagram calculations and large deviation theory because portfolio wealth is a nonlinear weighted sum of Gaussian variables.All results can be derived analytically and fully controlled.The focus is on the risk component of the problem assuming symmetric return distributions.The average returns are close to zero that fluctuations governs.This approximation gives a precise representation of real data at sufficiently small time scales with Sharpe ratios less than one even if it is not true in reality.This is reasonably accurate for non-stock market assets with short-term trend.Empirical data from foreign exchange is used to test and validate the authors' theory.Future work is extension of their theory to cases where average return cannot be ignored.A novel full analytical characterization of portfolio returns distribution is made up of assets with arbitrary multivariate distribution is presented.A new dependence measure between non-normal variables generalizing the covariance matrix is introduced.The proposed nonlinear covariance matrix is shaped to specific fat tail structure of underlying marginal distributions providing stability and good-conditioning.The multivariate distribution is normal with reduced variables set that are natural quantities to work with. The calculation of returns distribution for an arbitrary portfolio can be formulated with functional integral approach.An extensive synthesis of tools to determine analytically full returns distribution and calculations for multivariate Weibull distributions are provided.The information captured by arbitrary portfolios returns distribution is compared to their moderate versus large risks.Analysis for uncorrelated assets with fat tails to compare different portfolios like those minimizing the variance, excess kurtosis or higher normalized cumulants is developed.These portfolios are compared to those determined from maximization of expected utility.Minimizing the portfolio variance, that is, relatively small risks, may often increase large risks measured by higher normalized cumulants.Practical implementations and comparisons with data from foreign exchange market are offered.The performance of proposed nonlinear representation is validated clearly.The comparisons between the theoretical calculations and empirical estimations of cumulants of portfolio return distributions are good.A good representation of portfolio risks tracking correlations between assets is much less important than precisely accounting for fat tail structure of distributions.Future extensions are discussed.The synthesis is discussed:

1. Minimizing variance compared to minimizing the normalized higher-order cumulants λ\_2m:portfolio with minimum variance may have smaller cumulants than that with minimum normalized cumulants but may have larger normalized cumulants characterizing the large risks.Therefore, minimizing small risks (variance) may increase large risks.

2. Minimizing variance compared to minimizing the non-normalized higher-order cumulants c\_2m:portfolio with minimum variance may have larger non-normalized and normalized cumulants of order bigger than two than that with minimum non-normalized cumulants. Again, minimizing small risks (variance) may increase larger risks.

3. Minimizing variance compared to benchmark portfolio:portfolio with minimum variance has smaller non-normalized higher-order cumulants than benchmark portfolio, though it may have smaller or larger higher-order normalized higher-order cumulants.The situation is similar to the first case where it is preferable to minimize the variance than taking the benchmark though the resulting portfolio distribution is less Gaussian.

The important gaps in knowledge and researchare problemsin finance like risk management, optimal asset allocation and derivative pricing, needs an understanding of volatility and correlations of assets returns.Volatility and correlations are usually estimated from historical data and risk dimension is displayed by variance or volatility for single asset and by covariance matrix for assets set.Variance of returns distribution is minimized by inverting the covariance returns matrix in portfolio optimization.On the contrary, covariance matrix is usually ill-conditioned and unstable. This unstable behavior is because of several reasons.First, the non-normality of asset returns, that is, large price changes are more likely than extrapolated from normal estimation.Second, the covariance matrix is not defined for Levy or power law distributions with index less than two.Option pricing and hedging depends on risk representation by single scalar measure, volatility.It has the complexity of time-dependent fluctuating surface as function of strike price and time-to-maturity.This complexity arises from fat tail structure of underlying distributions and their non-stationary. Whether volatility is better represented as vector measure providing more information is not investigated.Models beyond Gaussian paradigm is needed by practitioners, regulatory agencies and advocated in academic literature.Non-normal multivariate distributions to better assess factor-based asset pricing models is proposed.Some factor models are strongly rejected when depending on presumption that returns or models residuals are independent and identically distributed multivariate normal but no longer rejected when fatter tail elliptic multivariate distributions are used.It is impossible to quantify risk only by W'ΣW.The marginal variance of one of the variables will depend on number N of assets in the portfolio.It is an unwanted property for chosen multivariate distribution.This inconsistency does not occur for special cases like multivariate t-distribution or for mixtures of normal distributions.The origin of the problem is clarified and shown that for arbitrary elliptic distributions, the portfolio return distribution is based on asset weights P not only through the quadratic form W'ΣW.The weights W control the shape as a whole of portfolio return distribution.Minimizing variance W'ΣW of portfolio can distort portfolio return distribution to increase probability weight more and far in the tail causing increased large risks.Calibrating portfolio risks by a single volatility measure is dangerous.Empirical tests is carried out.Multivariate and marginal return distributions of six currencies is examined.Contour lines are used to represent different probability levels.A given level represents the probability that an event (r1, r2) falls in the domain constrained by corresponding contour line.Three probability levels correspond to 90%, 50%, and 10% shown for each pair excluding the pair RUR-JPY where only 50% and 10% levels are shown.Only one fourth of the data points randomly chosen are displayed to differentiate the contour lines from data points.The fat tail nature of these pdf's is very obvious and quantitatively reflected by exponents c's of Weibull pdfs are all found of order or less than one while a Gaussian pdf corresponds to c = 2.Goodness of the nonlinear transformation and statistical tests is determined.Variance and excess kurtosis is calculated.Fraction of wealth invested in a given currency is constant.Friction and transaction costs involved in dynamical reallocation is not addressed, rather the focus is on testing the theory.Excess kurtosis and sixth-order normalized cumulants quantify deviation from Gaussian distribution and measures degree of fatness of tails, that is, measuring large risks.Minimizing portfolio variance is unoptimal with respect to large risks.The strength of this effect depends on relative shape of the distributions of assets in the portfolio and their correlations.Excess kurtosis and sixth-order normalized cumulant have a direct or inverted S-shape with rather steep dependence or narrow well as function of asset weight p whereas the variance shows a smoother and rather slower dependence.Investor will be well-inspired to use additional information given by higher-order cumulants to control portfolio's risks.Empirical exponents c are much less than 2 for which the calibration of exponents c plays a much more critical role in determination of high-order cumulants that correlation.The set of portfolios with small excess kurtosis and small large risks and still reasonable variance, small risk, is identified.The importance of such precise analytical quantification to increase robustness of risk estimators: historical data becomes very unreliable for medium and large risks for lack of suitable statistics.

The trends and themes of the research related to the topic are non-gaussian nature of empirical return distributions has been first addressed by generalizing the normal hypothesis to stable Levy law hypothesis.The generalization of Markowitz's portfolio theory naturally is possible because Gaussian and Levy laws are stable distributions under convolution and enjoy simple additivity properties.A further generalization has been carried out to situations where marginal distributions of asset returns may possess different power law behaviors for large positive and negative returns with arbitrary exponents plausibly bigger than 2, that is, unstable in Levy laws but only in a large deviation theory.This stable or quasi-stable property by Levy and power laws are influential in generalization of Markowitz's theory with rather stringent restrictions.Such laws is the only solution observing the consistency property where any marginal distribution of random vectors whose distribution is a specific family belonging to the same family.This consistency property is crucial for independence of marginal variances on order of multivariate distributions.Generalization of portfolio theory from Gaussian to Levy and then to power laws depends fundamentally on consistency property.Elliptical distributions give a priori perhaps the simplest and most natural hope for describing fat tails and for generalizing portfolio theory to non-normal multivariate distributions, only on measure of covariance matrix.They are defined as arbitrary density fucntions F of quadratic form (X - <X>)'Σ-1(X-<X>) with X as unit column matrix of N asset returns and Σ is a N x N dependence matrix between N assets proportional to covariance matrix when it exists.The results of CAPM extends to such elliptical distributions because any linear transformations of an elliptical random vector is also elliptical, the risk measure W'ΣW is positively proportional to portfolio wealth variance with W, the column vector of asset weights.Ranking of portfolios by risk adverse investors appear to keep their ordering.Generally, the density of a marginal distribution does not have the same shape as function F.It is an additional misspecification of risk because it should be captured both by covariance matrix Σ and shape and tail structure of distribution.

The relationships between key concepts are performed non-linear change of variable from return δx of asset over unit time scale τ on a daily scale onto the variable y(δx) so that the distribution of y is normal.This is always possible for marginal distributions.A single security case is considered.It starts with the non-centered case. The modified Weibull distribution is the example considered.Stretched exponential pdf's give a parsimonious and accurate fit to full range ofcurrency price variations at daily intermediate time scales. Change of variable maps precisely the modified Weibull pdf onto Gaussian pdf.This property is not holding precisely for standard Weibull distribution.The modified Weibull distribution with c = 2 is the Gaussian distribution.It retrieves precisely the Normal law as one of its member.Weibull distribution is unstable under convolution, that is, distribution of weekly returns is not precisely the same form as daily returns distribution.The centered case is considered. The general case of several assets is considered.Non-linear mapping is carried out.The symmetric case is discussed.The nonlinear covariance matrix is calculated.Multivariate representation of joint distribution of asset returns is carried out. Approximation will be done.Marginal distributions of assets are characterized.New variables in which marginal distributions are Gaussian are defined.Covariance matrix of these new variables as new measure of dependence between asset returns is calculated. Nothing imposes a priori that multivariate distribution of the variables is also a multivariate Gaussian distribution if marginal distributions in terms of new variables y(j) are gaussian.

It is only guaranteed that projections of multivariate distribution onto each y(j) variable are Gaussian.However, a standard theorem from Information Theory states that, conditioned on the only knowledge of covariance matrix, the best representation of a multivariate distribution is Gaussian.This means the multivariate normal distribution has the least possible a priori assumptions in addition to covariance matrix and vector of means when they are non-zero, that is, the most likely representation of data.The non-linear change of variable ensures fat tail structure of marginal distribution is fully described and leads to a novel more stable measure of covariances.The normal structure of multivariate distribution in y variables does not lead to Gaussian multivariate distribution for returns.This nonlinear covariance approximation is exact for uncorrelated variables distributions where V = I.It is exact for Gaussian distribution modified by monotonic one-dimensional variable transformations for any number of variables or equivalently, multiplication by non-negative separable function.The multivariate distribution obeys automatically condition that corresponding marginal distributions are of same analytic form.This corresponds to consistency condition discussed in introduction.This approach has been introduced independently for analysis of particle physics.Multivariate Weibull distributions is applied.

The distribution of Ps(δS) of portfolio wealth variations is discussed.The theoretical formulation is discussed.Only symmetric multivariate distribution will be focused.All odd moments, especially the first moment which is the expected return, and all odd cumulants are disappearing.The focus is on risk embedded in variance and in all higher even order cumulants.No trade-off between risk and return exist because the expected return is zero.

The risk perspective of the portfolio is focused exclusively.

The contradictions are proper probability distribution functions from generalized hyperbolic distribution and risk measures like expected shortfall or value-at-risk maybe more appropriate during crisis.It is unknown whether excess kurtosis as risk measure and Weibull distribution for probability distribution function is general enough for crisis situation.

The research designs or methods seemedinadequate are the analysis is restricted to Weibull case because it is one of the simplest non-trivial situation allowing one to make apparent power of this method.The Weibull representation gives a very reasonable description of fat tail structures of empirical distributions through extended comparisons with empirical data.

Portfolio wealth is a nonlinear function of variables y changed from asset returns and its distribution expressed as a non-Gaussian multivariate integral a priori not obvious to estimate.It is the same as partition function of ϕq field theory in particle physics with N components and imaginary coupling coefficients ikw\_i.y is like a Gaussian field with interactions by second nonlinear term in the exponential.ϕ3 theory is the simplest non-trivial case leading to fat tails.All cumulants with order larger than two are zero for Gaussian distributions.The cumulants are convenient ways to quantify deviation from normality.They are equivalent to knowledge of full distribution given mild regularity conditions hold.All odd-order cumulants are disappearing and even-order cumulants are sufficient to calculate because only symmetric distributions are focused on.The Gaussian case c = 2 is explored.That is when the multivariate asset return distribution is Gaussian. The distribution of portfolio wealth variations δS is uniquely and fully characterized by the only value of its variance c2 in covariance matrix of asset price returns and their weight W in portfolio, c2 = W'VW.The case of uncorrelated assets is considered.Absence of correlations between assets occur where covariance matrix is diagonal. The portfolio distribution is only sensitive to intrinsic risks presented by each asset.Cumulants is discussed.Tails of the portfolio distribution for c < 1 is discussed.Extreme tails of wealth variations distribution of total portfolio with N assets is characterized.Results where all assets have the same exponent are presented.Dependence of c\_i = 2/q\_i in the formulas is reintroduced because it will be essential when comparing to empirical data given later.Two cases are to be differentiated:

1. q > 2: C(m,q) increases faster than exponentially for large cumulant orders 2m.Thus, the tail of distribution P\_S(δS) is controlled by large cumulants.

2. q ≤ 2: C(m,q) decreases with m and the structure of the tail from large cumulants cannot be derived.

The high-order cumulants of portfolio distribution are influenced by single asset that maximizes product w2\_id\_iq of square of weight with variance of the return.This is known as maximum w2\_maxdq\_max.Such formulation of tail of distribution P\_S(δS) of portfolio wealth δS allows the characterization of risk by single scale parameter χ.This is analagous to portfolio theory for power law distributions with all different risk dimensions are embedded by scale parameter of power law distribution of portfolio wealth variations. Tails of the portfolio distribution for c > 1 is discussed.The above derivation does not hold for q < 2, that is, c > 1.Instead, extreme deviation theorem is used to determine the shape of extreme tail of portfolio wealth distribution.It is only applicable for exponent c > 1 and stretched Weibull case is excluded from this analysis.Minimization of risk in tail is resolved completely by locating the weights wi that minimize χ.The proposed theory is more complete because the determination of all cumulants produces a characterization of complete distribution not only of its tail.The general case of correlated assets is discussed.

Cumulants of portfolio distribution with N assets is determined.It is assumed that all assets are characterized by Weibull distribution with same exponent c = 2/q.The characteristic function is computed by perturbation analysis and functional generator.Portfolio with two assets is discussed.Portfolio optimization for uncorrelated assets is discussed.All cumulants and variance of portfolio wealth variations distribution must be considered as risk measures for non-Gaussian distributions.Variance and higher order cumulants depend on weights wi of assets comprising the portfolio with different functional forms.It is crucial to determine relative variation of cumulants when weights wi are changed.It will be shown that portfolio minimizing variance, that is, relatively small risks, can often larger risks measured by higher normalized cumulants.Minimization of variance is discussed.The focus is on all assets with same exponent c = 2/q for notation simplicity.Reintroducing the asset dependence of q's in formulas is simple.Weights change with order r of the cumulant that is minimized.It is impossible to minimize simultaneously all cumulants of order larger than two.This contrasts with normalized cumulants result to be discussed.The weights that minimize very large r → +∞ order cumulants reaches asymptotically values that minimize all normalized cumulants of order larger than two.Conclusions obtained for normalized cumulants carry out for cumulants in limit r → ∞.Comparing values used by cumulants for weights that minimize variance with values taken by cumulants for weights that minimize cumulant of given order 2r. Values of cumulants for weights that minimize variance higher than those for weights that minimize cumulant of given order 2r > 2.Minimizing small risk increases large risks.Minimization of excess kurtosis and higher normalized cumulants is discussed.Normalized cumulants give a better measure of large risks than non-normalized cumulants.Fourth normalized cumulant λ4 is known also as excess kurtosis k.It is identically zero for Gaussian distributions.Quantitative deviation of a distribution from normality is denoted with normalized cumulants λ2m.Higher order cumulants of portfolio which minimize excess kurtosis are always bigger or equal to cumulants of portfolio that minimizes variance for any other set of nonlinear covariance matrix.Comparison between excess kurtosis of minimum-variance portfolio and benchmark wi = 1/N is carried out.Excess kurtosis of the benchmark is smaller or larger than excess kurtosis of portfolio with minimum variance.Determining the portfolio with minimum variance may either increase or decrease its excess kurtosis compared to the benchmark. A portfolio with two uncorrelated assets is such that excess kurtosis of benchmark and of optimized variance portfolio are the same.The higher-order cumulants of portfolio with minimum variance are smaller than the benchmark.The expected utility approach is discussed.The investor starting with initial capital W0 > 0 is assumed to have preferences rational in von-Neumann-Morgenstern sense with respect to end-of-period distribution of wealth W0 + δS.His or her preferences are represented with a utility function u(W0 + δS) depending on wealth variation δS at end-of-period.The expected utility theorem states that investor's problem is to maximize E[u(W0 + δS)] where E[x] describes the expectation operator.Maximizing expected utility obtained minimizing portfolio variance in limit of small risk aversion.Higher-order cumulants have non-negligible contributions for larger risk aversions.Only first few cumulants are considered in the sum because others are weighted by negligible factor for large initial wealth W0 compared to one-period price standard deviations.The best portfolio obtains the optimal variance portfolio in this limit.E[u(W0 + δS)] has non-negligible contributions from higher-order cumulants in the other limit where initial wealth W0 is not big compared to one-period price standard deviations.The best portfolio is a weighted compromise between different risk dimensions given by different cumulants.Optimal portfolio depends on initial wealth W0 ceteris paribus is the important insight from the analysis.This is from existence of many different risk dimensions given by different cumulants each weighted by appropriate power of initial wealth.The initial wealth quantifies relative importance of higher-order cumulants.A small risk aversion is maybe unwillingly implicitly expressed using the standard variance minimization method.All information on large-order cumulants controlling the large risks is dropped.However, full treatment incorporating higher cumulants as relevant measures of risks allows response to much larger spectrum of trader's sensitivity and risk aversions.

My opinion regarding the quality and importance is a novel and general methodology for multivariate distributions with non-Gaussian fat tails and non-linear correlations is presented.This is done by projecting the marginal distributions onto Gaussian distributions via highly nonlinear variable changes. The covariance matrix of these nonlinear variables permits defining a novel measure of dependence between assets, nonlinear covariance matrix, specifically adapted to be stable in non-gaussian structures presence.The formulation of corresponding portfolio theory requiring performing non-Gaussian integrals to get full distribution of portfolio returns is presented.A systematic perturbation theory with technology from particle physics of Feynman diagrams to calculate the cumulants of portfolio distributions with Weibull marginal distributions is developed. Minimizing the portfolio variance may increase the large risks quantified by higher-order cumulants. Detailed empirical tests on a six currencies panel confirm relevance of Weibull description allowing making precise comparisons with the theoretical predictions.Valid determination of large risks quantified by excess kurtosis is much more sensitive to correct Weibull exponent of each asset than to their correlation appears almost negligible for fat tails distributions.

The topic should be further studied because assets with different exponents c have been treated only for uncorrelated assets and corresponding problem of heterogeneous c's in correlated case relevant for a precise comparison with empirical data.Assets with large exponents c ≥ 1.5 have not been studied.Relevance of correlations increases with increasing c and a precise determination of correlation matrix to become more important as c → 2 is expected.Analysis on risk dimension of problems by studying symmetric distributions, that is, assets not expected to have long-term trends is focused on.A natural and relevant extension of the theory to treat the case where mean return is non-zero and different from asset to asset.The next level of complexity is having non-symmetric distributions with variable Weibull exponents c with correlations.Perturbation theory with Feynman diagrams can be used for other distribution classes and explore in details other useful classes can be interesting.Nonlinear covariance matrices are stationary is assumed and verified reasonably.No conceptual difficulty in generalizing and adapting the ARCH and GARCH models of time-varying covariance to the formulation in terms of effective y variables is present.Empirical tests have been performed on small portfolios with two and six assets for pedagogical exposition and easier tests for the theory.Extending the work to larger and more heterogeneous portfolios is worthwhile.

Hu (2005)’s research problem shows that financial quantities distributions are known to exhibit heavy tails, skewness and other non-Gaussian attributes.The multivariate generalized hyperbolic distributions (GH) includes and generalizes the Gaussian and Student t distributions and skewed t distributions is studied.Numerical difficulty of calibrating the distributional parameters to the data discourages the use of such distributions.

A novel method to stably calibrate GH distributions for a wider range of parameters than previously available is reported.A modified version of EM algorithm for calibrating GH distributions is developed.It extends the stability of calibrating procedure to a variety of parameters including parameter values maximizing log-likelihood for real market data sets.

Certain GH distributions can be used for modeling when previously they are numerically intractable for the first time.New uses of GH distributions in three financial applications is possible because of the algorithm.Forecast univariate Value-at-Risk (VarR) for stock index returns and demonstrate out-of-sample backtesting GH distributions outperform Gaussian distribution.An efficient frontier for equity portfolio optimization under skewed-t distribution using Expected Shortfall as risk measure is calculated.Gaussian efficient frontier is impossible if skewed t distributed returns are used.An intensity-based model to price Basket Credit Default Swaps through calibrating the skewed t distribution directly without separate calibration of the skewed t copula is build.This is the first use of skewed t distribution in portfolio optimization and in portfolio credit risk.

The key findings of their research show that financial returns distributions are usually not normally distributed and the technical difficulties of dealing with non-normal distributions stood in the way of using them in financial modeling.Address some of the problems in context of generalized hyperbolic (GH) distributions, a parametric family with some useful properties including many familiar distributions like Student t, Gaussian, Cauchy, variance gamma and so forth.Generalized hyperbolic distributions in the study of grains of sand was introduced.A computer program, HYP, was developed to fit hyperbolic distributions using a maximum log-likelihood method.It is impossible to calibrate the hyperbolic distribution if dimension is greater than or equal to four.Detailed derivations of the derivatives of the log-likelihood function for generalized hyperbolic distributions and applied HYP to calibrate the three dimensional generalized hyperbolic distributions.Univariate hyperbolic distribution was fitted to return series of German equities and a high accuracy fit was obtained.Takes a lot of time to calibrate multivariate generalized hyperbolic distributions if log-likelihood is directly maximized leading to very few applications of multivariate generalized hyperbolic distributions.The calculation is simplified by considering the simpler symmetric generalized hyperbolid distributions.Calibration of generalized hyperbolic distributions has been difficult.EM(expectation maximization) algorithm is a powerful way of calibration in different contexts.EM algorithm can be used to obtain maximum likelihood estimates.EM algorithm and its extensions, ECME(expectation conditional maximization either) and MCECM (multi cycle expectation conditional maximization), are used to calibrate the Student t distribution.EM algorithm was used to calibrate the λ fixed multivariate generalized hyperbolic distributions by maximizing the augmented log-likelihood.Multivariate generalized hyperbolic distributions when dimension is bigger than three is calibrated.Five dimensional normal inverse Gaussian(NIG) distribution to return series on foreign exchange rates is fitted.EM algorithm on NIG and skewed t is used to model financial returns.It is based on a particular parameterization of GH family because the parameterization is implemented in the HYP program.It is consistent with GH description as normal mean variance mixtures where the mixing distribution is a generalized inverse Gaussian distribution (GIG).The parameters of the underlying GIG distribution are not preserved by linear transformations under this standard parameterization.An important innovation was made with a new parameterization of GH and EM algorithm under this parameterization.Linear transformation of generalized hyperbolic distributions remains in the same sub-family characterized by parameters of GIG under this new parameterization.The algorithm in this dissertation solves the stability problem for calibrating the whole family of generalized hyperbolic distributions.A fast algorithm for subfamilies of GH like normal inverse gaussian (NIG), variance gamma (VG), skewed t, and Student t.Expected shortfall, not standard deviation and VaR, is a coherent risk measure.Minimizing any translation invariant and positive homogenous risk measure will have the same portfolio composition as Markowitz portfolio composition for a certain return under elliptical distribution no matter what the confidence level is.Different portfolio compositions is from difference in distribution.Heavy tailed distributions should be chosen to model portfolio risk.Filtered i.i.d five dimensional returns series is fitted with multivariate generalized hyperbolic distributions.Skewed t possesses the largest log likelihood with fewest number of parameters among the four tested generalized hyperbolic distributions.Student t possesses the second largest log likelihood.Skewed t is not elliptical while student t is.The normal, Student t and skewed t efficient frontier under risk measure ES is obtained.Different distributions produce different portfolio compositions at a given return.Different confidence levels result in the same portfolio compositions for normal or Student at frontier.Different confidence levels result in slightly different portfolio compositions for skewed t distribution.Normal frontier is unreachable in Student t distribution and student t frontier is unreachable in skewed t distribution under 99% ES.

Kendall's τ remains under monotone transformations thus copula and distribution can be used to model correlation of default times by correlation of underlying equities.Single name and portfolio credit risk modeling are used to price the basket credit default swaps.Student t copula is commonly used to price basket credit default swaps for its lower tail dependence.It is necessary to specify the marginal distributions first then calibrate the marginal distributions and copula separately.There is no good method to calibrate the degree of freedom ν.The calibration is very slow.A fast EM algorithm for Student t and skewed t distributions.All parameters are calibrated together.The author is the first to introduce distribution to price basket credit default swaps to his knowledge.Student t copula results in higher default probabilities and spread price of basket credit default swaps for last to default (LTD) and lower default probabilities and spread price for first to default (FTD) than those from Gaussian copula.Both Student t and skewed t distributions result in higher spread prices of basket credit default swaps for LTD and lower spread prices for FTD than those from Student t copula.

The important gaps in research are density of generalized hyperbolic distributions is defined directly.Application of multivariate generalized hyperbolic distributions become inconvenient because some important characterizing parameters are not invariant under linear transformations.Introducing the generalized hyperbolic distributions from generalized inverse Gaussian distributions by mean-variance mixing method allows the linear transformations of generalized hyperbolic random vectors to remain in the same sub family of generalized hyperbolic distributions characterized by some type parameters.Little work has been done about multivariate generalized hyperbolic distributions although univariate generalized hyperbolic models are applied widely in the financial data modeling.Maximizing log-likelihood of multivariate generalized hyperbolic distributions is not very tractable.The conventional HYP program cannot deal with calibration when dimension is bigger than three because there are too many parameters.Some can only calibrate the symmetric generalized hyperbolic distributions at any dimension using HYP.Mean-variance representation of generalized hyperbolic distributions has a great advantage because the EM algorithm can be applied to it.The generalized hyperbolic random variable can be represented as a conditional normal distribution with most of the parameters (Σ,μ,γ) calibrated like a Gaussian distribution if other three type parameters (λ,χ,ψ) are already estimated or with some values.The existing EM algorithm framework for generalized hyperbolic distributions is followed.An algorithm is provided by the author for generalized hyperbolic distributions with large |λ| naming it the ψ algorithm and χ algorithm.

The author's special algorithms for limiting or special cases: VG, skewed t, NIG and Student t distributions are provided.The framework is in the usual definition of GH.That algorithm is unstable when λ is small. The existing algorithm is a special case of the author's algorithm when λ is bigger than a certain number.The author's algorithm can calibrate all sub-family defined by λ ϵ R.The author is the first who can calibrate the generalized hyperbolic distribution when |λ| is large.|λ| is set less than 10 because when |λ| is bigger than a very bigger number say 100, the Bessel function is non tractable.The original log-likelihood function given current estimates can be maximized to obtain a fast algorithm called ECME algorithm.The MCECM algorithm recalculates δ, η and ξ.This technique for variance gamma distribution and skewed t distribution is not used because the calibration of those two distributions need to calculate the extra ξ lowering the calibration speed.

The trends and themes of the research are Student t distribution is popularly used in modeling of univariate financial data to model the heaviness of the tail through the control of the degree of freedom, ν.It is used in the modeling of multivariate financial data because EM algorithm can be used to calibrate it.It is commonly used to model dependence by creating a Student t copula from Student t distribution.Student t copulas are popularly used in modeling of financial correlations because they are upper tail and lower tail dependent and very easy to calibrate.They are symmetric and bivariate exchangeable.Symmetric and exchangeable copulas should not be used because financial events tend to crash more than boom together.All copulas made from symmetric generalized hyperbolic distributions are tail independent.A limiting case of symmetric generalized hyperbolic distributions is the student t distribution.Creating the skewed t copula from skewed t distributions may be interesting.More discussions about symmetry and exchangeability of Student t copula and its extension, skewed t copula can be found.One dimensional generalized hyperbolic distributions are noted for their better fit to univariate financial data especially the hyperbolic and normal inverse Gaussian distribution.Heavy, or at least semi-heavy tailed distribution is provided.Tails of cauchy distribution exist where the distribution has no mean and variance.Tail of stable distribution follows the power function.Tails of gaussian distribution is an exponential function of square function where both tails are very thin.Tails of student t distribution follows a power function.Tails of generalized hyperbolic distributions is the product of a power function and an exponential function.Generalized hyperbolic distributions are semi-heavy tailed distributions.Tails of skewed t distribution decay as power function and is heavier than student t distribution.Log density of hyperbolic distribution is a hyperbola whereas log density of gaussian distribution is a parabola.Hyperbolic and normal inverse gaussian have heavier tails than gaussian distribution.NIG has slightly heavier tails than hyperbolic distribution.The tails are heavy when |λ| is small and the tails become thinner when |λ| becomes much bigger.Symmetric generalized hyperbolic distributions appear like Gaussian distribution when |λ| is very big.

Variance gamma distribution is limiting distribution of generalized hyperbolic distributions when χ approaches 0 and variance gamma has the heaviest tail among the tested distributions.Skewed t distribution is the limiting distribution of generalized hyperbolic distributions when ψ approaches 0 and λ = - ν/2 and it has the heaviest tail among the tested distributions.μ is location parameter, σ is scale parameter and γ is skewness parameter.

Location and scale parameters are common for all distributions.Skewness parameter is commonly used in modeling financial return data.

The relationship between key concepts is generalized hyperbolic distributions is applied to risk management, portfolio optimization and portfolio credit risk pricing.For risk management, Value at Risk (VaR) from normal distribution is the standard risk measure since J.P. Morgan released RiskMetrics in 1994.The Basel Committee on Banking Supervision proposed using the 10 day VaR at 99% level.However, financial return series is heavy tailed and leptokurtic with large losses occurring far away from VaR based on normal distribution.VaR at 99% level based on normal distribution underestimates the true risk for financial return series.VaRα depends only on the underlying distribution option.

Semi heavy tailed GH is an alternative.Univariate generalized hyperbolic distributions with usual parameterization to model the Brazilian data to get more accurate VaR measurements are applied.The new parameterization of generalized hyperbolic distributions to fit the univariate return series and calculate the related VaRα is used.A GARCH(1,1) filter with Student t or Gaussian innovations is used to filter the data because financial return series usually is not independently and identically distributed (i.i.d). This is done before modeling returns with a particular distribution.NIG, VG, skewed t and hyperbolic distributions from the general GH family are calibrated after obtaining the i.i.d. return series.A VaR backtesting procedure is then implemented.GH not Gaussian distributions allows better VaR forecasts.For portfolio optimization, it is from trading off risk and return.Efficient frontier, portfolios with minimum risk for given return, is constructed from two inputs: risk measure option and probability distribution to model returns.A precise concept of risk and accurate description of returns distribution are crucial.Standard deviation of portfolio return as risk measure and normally distributed returns constitutes the efficient frontier of fully invested portfolios with minimum risk for a given specified return.Standard deviation as risk measure has the drawback of insensitivity to extreme events. VaR describes more about extreme events but cannot aggregate risk sub additive on portfolios.A coherent risk measure, expected shortfall (ES), addresses the difficulty. The optimized portfolio composition with a certain return will be the same as traditional markowitz style portfolio composition when the underlying distribution is Gaussian or elliptical, no matter what positive homogeneous and translation invariant risk measure and confidence level.Only the distribution option will affect the optimized portfolio.Financial returns series deviation from multivariate normal distribution cannot be neglected.Heavy tailed elliptical distributions like Student t and symmetric generalized hyperbolic distributions and non-elliptical distributions like skewed t distribution is used to model financial return series.Minimization of ES will not require knowing VaR first and construct a new objective function.VaR and ES are obtained after minimizing this new objective function.This is carried out and Monte Carlo simulation to approximate the new objective function by sampling the multivariate distributions is done.Efficient frontiers for various distributions can be constructed.The usual Gaussian efficient frontier is unreachable for Student t or skewed t distributed returns.For correlations, copulas and credit risk, copulas is a popular method to describe and construct multivariate distributions dependence structure.Credit events usually happen together because of business connections.Tail dependence is used to model co-occurrence of extreme events relating to heavy tail property in univariate distributions.Student t copula is tail dependent whereas Gaussian copula is tail independent.Student t copula calibration is separate from marginal distributions calibration.

Empirical distributions to fit the margins is generally suggested.Most important issue is correlations between default obligors in pricing of multigame credit derivatives like basket credit default swaps (CDS) and collateralized debt obligations (CDO).Copulas can model these correlations by using correlations of corresponding equity prices.Kendall's τ correlation is invariant under monotone transformations.This forms the foundation of modeling the correlation of credit events by using correlation of underlying equities through copulas although nobody mentions this correlation invariance property.Use multivariate distribution of underlying equity prices instead of using copulas to model correlation of credit events with correlation of underlying equities by using this correlation invariance property.The calibration procedure differs between a copula and distribution approach.

The calibrations of marginal distributions and copula are separate for copula approach whereas the two are jointly calibrated for distribution approach.There is still no good method to calibrate the degree of freedom ν except by direct search for Student t copula.Calibrating Student t copula require days while calibrating skewed t or Student t distribution require minutes.Student t and skewed t distributions are used to model correlations of default obligors.Gaussian distributions and Student t distributions are closed under linear transformations.Generalized hyperbolic distributions are also closed under linear transformations.The method used in portfolio risk management using Gaussian or Student t distribution can also be used in generalized hyperbolic distributions.Gaussian distributions and Student t distributions are elliptical whereas generalized hyperbolic distributions are not, capturing more attributes about financial data series like asymmetry which do not incur more difficulty in application.This allows the calculation of portfolio risk like standard deviation, VaR and ES without Monte Carlo simulation.Marginal distributions are obtained automatically once the multivariate generalized hyperbolic distributions are calibrated.Credit risk modeling has two approaches: structural and reduced form.Default is modeled as first passage time across a barrier, the default time τ is predictable so that short credit spread tends to zero which is inconsistent with market observations with structural approach.τ is unpredictable and short credit spreads tends to intensity in reduced form or stochastic intensity modeling.The stochastic intensity can be modeled like interest rate process.The reduced form approach to credit risk models is focused in the author's dissertation for these reasons.The most important issue is correlations between default obligors in pricing of multigame credit derivatives like basket credit default swaps (CDS) and collateralized debt obligations (CDO).Defaults seldom occur.Copulas can be used to model these correlations using correlations of corresponding equity prices.Kendall's τ correlation is invariant under monotone transformations.This lies the foundation to model correlation of credit events by using correlation of underlying equities through copulas.Default times are increasing functions of copula uniform random variableswhicharecumulative distribution function (CDF) transformations of corresponding equity prices so that default times are increasing transformations of corresponding equity prices under the author's setting of portfolio credit risk.Thus, Kendall's τ correlations of default times are same as the correlations of corresponding equities.The pricing of basket CDS and CDO via copulas have been discussed by researchers.Multivariate distribution of underlying equity prices to model correlation of credit events with correlation of underlying equities using this correlation invariance property instead of using copulas.Calibration procedure is different for a copula versus distribution approach.

Calibrations of marginal distributions and copula are separate for copula approach.Calibrations of marginal distributions and copula are jointly calibrated for distribution approach.Multivariate distributions or copulas have many applications.Copulas are used to minimize expected shortfall (ES) modeling operational risk.Copulas are used in the portfolio optimization of CDS.Student t copula and transition matrix with gamma distributed hazard rate and beta distributed recovery rate to get the efficient frontier for credit portfolios by minimizing ES.Generalized hyperbolic distributions are used to model multivariate equity returns with EM algorithm.Skewed t has better performance with fast algorithm compared to other generalized hyperbolic distributions.Highly simplified formulas and faster algorithm for Student t distribution compared to skewed t distribution.No good method to calibrate degree of freedom ν except by direct search for Student t copula.Thus, calibration of student t copula takes days whereas calibration of skewed t or Student t distribution takes minutes.Student t and skewed t distributions are used to model correlations of default obligors.

The inconsistencies are the author claims that financial data exhibits tails.It is unknown whether the presence of tails can lead to higher default intensities of obligors.He also claims that such defaults are seldom observed.However, during crisis, risks can exist throughout the probability distributions not just at the tails.Table 5.3 is the spread price for k-pth to default using different models.Lower tail dependent copula results in higher default probability for LTD and lower probability for FTD, however this does not result in higher spread price for LTD and lower spread price for FTD.Student t possesses almost the same log likelihood and spread price for k-pth to default as skewed t distribution.They do not result in higher spread price for LTD and lower spread price for FTD.

Research designs or methods seemedinadequate because the identification problem causes a lot of trouble in calibration with redundant information causing the algorithm to be unstable.

Many people set the determinant of Σ to be 1 when fitting the data calling this new dispersion matrix Δ.Normal distribution underestimates the risk because it is a thin tailed distribution.Generalized hyperbolic distributions have semi-heavy tails thus they are good candidates for risk management.A GARCH model is used to filter the negative return series to get i.i.d filtered negative return series and forecast the volatility.The generalized hyperbolic distributions are calibrated and the α quantile calculated after getting i.i.d filtered negative return series.The VaRα for negative return series can be restored using the forecasted volatility and α quantile for filtered negative return series.Backrest VaR on generalized hyperbolic distributions and normal distribution, find that all generalized hyperbolic distributions pass the VaR test while normal distribution fails at 99% VaR for both Dow and S&P 500 index even at 97.5% level for S&P 500.Use of skewed t distributions is not often known.It has the least number of parameters among all generalized hyperbolic distributions and a fast calibration algorithm.It has the biggest log likelihood among all generalized hyperbolic, Student t, and Gaussian distributions.Copulas are used to describe and construct dependence structure of multivariate distributions.There are two kinds of copulas.First, bivariate Archimedean copulas to model bivariate distributions successfully.The best-fit Archimedean copula can be determined.The Archimedean copula can be extended to n dimensions loosing almost all dependence information because all pairs of variables have the same Kendall's tau correlation coefficient.Second, they are elliptical copulas.Gaussian and Student t distributions are elliptical distributions.Gaussian and Student t copulas are derived from Gaussian and Student t distributions respectively.Tail dependence is to model co-occurrence of extreme events related to heavy tail property in univariate distributions.Upper tail dependence is to model probability that two uniform random variables tend to go to largest value 1 together whereas low tail dependence is modelled similarly for smallest value 0.The availability of fast algorithm to calculate cumulative distribution functions (CDF) and quantile of corresponding one dimensional Gaussian and Student t distributions will determine the success of copulas.Student t copula is tail dependent whereas Gaussian copula is tail independent.It was shown that all generalized hyperbolic copulas derived from symmetric generalized hyperbolic distributions are tail independent excluding Student t copula.Calibration of Student t copula is very fast if the degree of freedom ν is optimized by maximizing log likelihood.Calibration of Student t copula is separate from calibration of marginal distributions.Empirical distributions are generally suggested to fit the margins although they have poor performance in the tails.A hybrid of parametric and non-parametric method looks at the use of empirical distribution in the body and generalized Pareto distribution (GPD) in the tails with some using Gaussian distribution in the body. Student t copula and Gaussian distribution in the center and left tail and GPD in the right tail for margins to model multivariate losses is used by some.These issues can be avoided if we can effectively calibrate the full distribution directly using skewed t or Student t distributions.

Opinion regarding the quality and importance are default intensity is calculated based on underlying equity prices is an interesting concept.Higher equity prices can reduce the probability of default is useful to know.

The topic should be further studied because whether higher equity price can lead to higher bond price of a corporation is worth studying.Underlying bond price can be used to determine default intensities of the corporations to be studied.

Aas and Haff (2006)’s research problem shows that daily or weekly return from financial markets e.g. forex rates, interest rates and stock prices are skewed with polynomial and exponential tails. They determine that Generalized Hyperbolic (GH) Skew Student’s t-distribution is the best fit. The risk measures they use, VaR and expected shortfall, show how polynomial and exponential tails is important.MLE estimators is computed with expectation-maximization (EM) algorithm and GH Skew Student's t-distribution.

The key findings of their research show that daily or weekly return from financial markets e.g. forex rates, interest rates and stock prices are skewed with polynomial and exponential tails. They show that Generalized Hyperbolic (GH) Skew Student’s t-distribution is the best fit.

The important gap in the data is there is no exact time horizon of the data.

The trends and themes of the research is related to gaussian versus non-gaussian distribution because daily or weekly return from financial markets e.g. forex rates, interest rates and stock prices are skewed. This means it is non-gaussian distribution.

The relationship between key concepts are the GH skew Student’s t-distribution provides the best fit for Norwegian stocks, international bonds, EUR/NOK exchange rate and European 5-year interest rate.At 5% significance level for the kopek test, the null hypothesis is twice rejected for Azzalini’s skew Student’s t-distribution, once for the normal inverse gaussian (NIG) distribution but never for the GH skew Student’s t-distribution.GH skew Student’s t-distribution has the lowest Dα value for each distribution and level.This is used to backtest the predicted expected shortfall value for confidence level α.It is the best compared to other distributions in predicting expected shortfall for the given test data.

Possible contradictions are GH distribution is seldom used because it is not particularly analytically tractable; for very big sample sizes, it can be hard to differentiate between different.It is because of the flatness of the GH likelihood function in those values.The MLE utilizing the EM algorithm can lead to local maximum. Picking the right initial values to avoid that is important.

Research designs or methods seemedinadequate because the GH skew Student’s t-distribution skewness and kurtosis are not defined when ν ≤ 6 and ν ≤ 8.For risk measurement, VaR only measures a quantile of the distribution ignoring the tails beyond them. Expected shortfall measures the tail risks better.

The opinion regarding the quality and importance is it is important to apply to data from emerging markets and extend the quality of the data to include stocks, bonds, commodities and currencies during crisis.

Further studies should be conducted to apply the data during crisis to understand the impact on financial market returns.

Schmidt, Hrycej, and St¨utzle (2006)’s research problem is about multivariate generalized hyperbolic distributions is an appealing family of distributions with exponentially decreasing tails for multivariate data modelling.Robust and fast estimation procedures are scarce in a finite data environment.An alternate type of multivariate distributions with exponentially decreasing tails is proposed comprising affine-linearly transformed random vectors with stochastically independent and generalized hyperbolic marginals.The latter distributions have good estimation properties and attractive dependence structures explored deeply.Dependencies of extreme events like tail dependence can be modelled with this class of multivariate distributions.The pertinent estimation and random-number generation procedures are provided too.Different pros and cons of both kinds of distributions are discussed and elucidated through simulation.

The key findings of their research show the analysis is as follows:

1) Elucidation of statistical-mathematical properties of multivariate generalized hyperbolic (MGH) and multivariate affine generalized hyperbolic (MAGH) distributions,

2) Computational procedures for parameter estimation of MGH and MAGH distributions,

3) Random number generation for MGH and MAGH distributions,

4) Simulation study and real data analysis.

The main contributions from 1) to 4) are as follows.

For 1), the focus is on the dependence structure of both distributions by using copulae theory.It is shown that the dependence structure of MAGH distribution is very attractive for data modelling.The correlation matrix, an important dependence measure, is more intuitive and easier to handle for MAGH than MGH distribution.Certain parameter constellations imply independent margins of MAGH distribution in contrast to margins of MGH distribution which do not have this property.MAGH distributions can model dependencies of extreme events (tail dependence), an important property for financial risk analysis, unlike MGH distribution.For 2), the parameters of MAGH distribution can be estimated in a simple two-stage method are shown.The procedure reduces to covariance matrix estimation and parameters related to univariate marginal distributions.The estimation is simplified to a one-dimensional estimation problem.However, this kind of estimation for MGH distributions can only work in the elliptically contoured distributions subclass.For 3), a fast and simple random-number generator based on famous rejection algorithm is established. The explicit algorithm outperforms known algorithms in the literature for MGH distributions under particular parameter constellations avoiding difficult minimization routines.For 4), a detailed simulation study to illustrate suitability of MAGH distributions for data modelling is presented.

The important gaps in knowledge and data are multivariate generalized hyperbolic (MGH) distributions were introduced and investigated.They possess appealing analytical and statistical properties compare to robust and fast parameter estimation which are difficult in higher dimensions.In addition, MGH distribution functions have no parameter constellation which they are product of their marginal distribution functions.Many applications need multivariate distribution function to model marginal dependence and independence.Multivariate affine generalized hyperbolic (MAGH) distributions, a new class of multivariate distributions, is introduced and explored because of these and other shortcomings.Unlike the MGH distributions, they have an attractive stochastic representation making the estimation and simulation algorithms become easier.The simulation study shows the goodness-of-fit of MAGH distribution comparable to that of MGH distribution. The one-dimensional marginals of MAGH distribution are more flexible because of more flexibility in the parameter option.A subclass of MGH distributions, hyperbolic distributions, has been introduced through variance-mean mixtures of inverse Gaussian distributions.They are handicapped from not having hyperbolic distributed marginals.They are not closed to passing to marginal distributions.This class is extended to family of MGH distributions because of other theoretical aspects.Different parametric representations of MGH density functions are provided.MGH density function is invariant under affine-linear transformations and is an important property of the parameterization of

The multivariate hyperbolic density is obtained when λ = (n + 1)/2.The multivariate normal inverse Gaussian density is obtained when λ = -1/2.Hyperbolically distributed one-dimensional margins is obtained when λ = 1.MGH distributions with λ = 1 are closed with respect to passing to marginal distributions and under affine-linear transformations are shown.One disadvantage of multivariate generalized hyperbolic distributions and other families of multivariate distributions is margins Xi of X = (X1,…,Xn)' are not mutually independent for some option of scaling matrix Σ.The modelling of phenomena where random variables result from sum of independent random variables is not plausible.This is critical because independence can be an undisputable property of problem based on stochastic model.The covariance matrix has a complex relationship with matrix Σ in case of asymmetry where β ≠ 0.An alternative model is proposed.A distribution is considered made up of n independent margins with univariate generalized hyperbolic distributions with zero location and unit scaling.This canonical random vector is then subject to affine-linear transformation. Therefore, the transformation matrix can be modelled proportionally to square root of covariance-matrix inverse even for asymmetric case.This property holds for multivariate normal distributions. The MGH and MAGH distributions are fitted to various asset-return data: Dad/CA and Nikkei/CA returns both with 3989 samples and Dad/Dow with 1254 samples.The following distributions are utilized:

1) MGH/MH,

2) MAGH/MAH with minimum parameterization represented by (min) with all margins equally parameterized,

3) MAGH/MAH with maximum parameterization represented by (max) with each margin individually parameterized.

The symmetric pendant represented by (sym) for some of these distributions are estimated i.e. β = 0.Estimations observing the affine-linear transformation method already discussed represented by (PC) is explained.Lower cross entropy indicates a better fit.Cross entropy is approximately equal to another popular divergence measure, χ^2 divergence, for small divergences between distributions i.e. good fits.Substantially different results cannot be expected if alternative divergence measures are used.Using additional parameters or degrees of freedom like:

1) λ ≠ 1 (MAGH or MGH) not λ = 1 (MAH or MH),

2) β ≠ 0 (MAGH or MGH) not β = 0 (symmetric MAGH or MGH),

3) multiple λ, α and β parameters (maximum parametrizations) not single ones (minimum parametrizations).

A single additional parameter like λ or β is justified on a significance level of 0.05 with Hj - Hi > 3.84146/3989 ≈ 0.001 for Dad/CA and Nikkei/CA data and with Hj - Hi > 3.84146/1254 ≈ 0.003 for Dad/Dow data.This is not the case for minimally parameterized variants, especially generalized asymmetric variants are not justified on this significance level.Three additional parameters, λ, α, and β in maximally parameterized variants, are justified on a significance level 0.05 with Hj - Hi > 7.81473/3989 ≈ 0.002 for Dad/CA and Nikkei/CA data and with Hj - Hi >7.81473/1254 ≈ 0.006 for Dad/Dow data.The critical differences are 0.003 for Dad/CA and Nikkei/CA data and 0.009 for Dad/Dow data on significance level 0.01.Such significance thresholds are greater for Dad/CA data with the flexibility of maximally parameterized variants obviously valuable for modelling.The χ2 test is performed to evaluate the goodness-of-fit of respective distribution models to financial data. The option is difficult for multivariate distributions whereas there are diverse recommendations for option of class intervals in univariate case.However, the results of the test is based on this option.Simple choices used are: 62, 82 and 102 intervals of width 0.005 or 0.01. The hypotheses that Dad/Dow data do not come from individual MGH and MAGH distributions cannot be rejected on 1% significance level for MGH distributions and 82 intervals the hypothesis cannot be rejected on 5% significance level for 62 and 82 intervals.The hypothesis could be rejected on 1% significance level for other data sets.The hypothesis must be rejected for any other common parametric multidimensional distribution different from hyperbolic family.The dependence parameters can be approximately interpreted as correlation coefficients for MGH and MAGHmin distributions because of total or approximate symmetry of some distribution models.They are also close to corresponding sample correlation coefficient, column Corr. This is impossible for MAGHmax model because of different parameterization of one-dimensional margins.

The trends and themes of the research related to the topic are generalized hyperbolic distributions used for data analysis is becoming very popular in different areas of theoretical and applied statistics.It was originally used to model grain size distributions of wind-blown sand.The latter family of distribution has been used for multidimensional asset-return modelling in econometrical finance.The generalized hyperbolic distribution supplants the Gaussian distribution unable to explain fat tails and distributional skewness of financial asset-return data.

The relationships between key concepts are MAGH distribution has independent margins if scaling matrix Σ equals identity matrix I.However, no MAGH distribution is from the class of elliptically contoured distributions for dimension n ≥ 2 from density contour-plots.Under general affine transformations, the consideration of lower triangular matrix A' in stochastic representations for MGH and MAGH distributions is pertinent because any other decomposition of scaling matrix Σ would lead to a different class of distributions.Only elliptically contoured sublcass of MGH distributions is invariant with respect to different decompositions A'A = Σ.All decompositions of scaling matrix Σ result in same distribution because they enter characteristic function through the form Σ = A'A.This equivalence does not hold in asymmetric or general affine case.Distributional flexibility to fit real data is discussed. One excellent property of MAGH distributionsis, after affine-linear transformation, all-one-dimensional margins can be fitted separately through different generalized hyperbolic distributions.In contrast, the one-dimensional margins of MGH distributions are not so flexible because the parameters α and λ relate to the whole multivariate distribution and a strong structural behavior is determined.On the contrary, this structure leads a large subclass of MGH distributions to belong to elliptical contoured distributions inheriting many useful statistical and analytical properties from multivariate normal distributions.The family of elliptically contoured distribution is closed under linear regression and passing to marginal distributions.MACH distributions may have independent margins for some parameter constellation about the dependence structure. Models based on linear combination of independent factors are supported.On the contrary, MGH distributions are incapable of modelling independent margins.Extreme dependencies for bivariate distributions with zero correlation are yielded.Correlation matrix of MAGH distributions is proportional to scaling matrix Σ in a large subclass of asymmetric MAGH distributions, in contrast, Σ is hard to interpret for skewed MGH distributions.In addition, copula of MAGH distributions, dependence structure of an affine-linearly transformed random vector with independent components,is quite illustrative and has many attractive modelling properties.Extreme dependencies for bivariate distributions with zero correlation are yielded.The MAGH distributions can model tail dependence in contrast MGH distributions are always tail independent looking at dependence of extreme events.Thus, MAGH distributions are appropriate especially in risk management. Parameter estimation for MAGH distributions is simpler and more robust in contrast to MGH distributions.The parameters of MAGH distributions can be identified in a two-stage procedure with a considerable computational advantage in higher dimensions even in an asymmetric environment. This same procedure can be applied for elliptically contoured MGH distributions with β = 0.The random vector generation algorithms for MGH and MAGH distributions are equally efficient and fast regardless of the dimension.The simulations show that both distributions fit simulated and real data well.Therefore, the MAGH distributions have much to recommend them about their parameter estimation, dependence structure and random vector generation.It depends on the kind of application and user's taste which model to prefer.One-dimensional marginals of both kinds of distributions are quite flexible. The copula technique combines these marginals with respective dependence structure resulting in a multidimensional MGH or MAGH distribution.The dependence structure of affine-linearly transformed distributions like MAGH distributions in terms of copulae has not attracted much attention in the literature yet.The usefulness of copulae has been depicted in many applications especially finance.

A detailed analysis of dependence structure of MGH and MAGH distributions is carried out.

The respective copulae and various dependence measures like covariance and correlation coefficients, Kendall's tau and tail dependence is considered.Dependence structure of MAGH distributions is quite different to respective MGH counterpart although the contour plots does not show this.The behavior of common extreme events is different and that for any parameter constellation the margins of MGH distribution cannot be independent.The copula encodes all information on dependence structure unencumbered by information on marginal distributions and couples the marginal distributions to give joint distribution.The copula of an MGH or MAGH distribution does not depend on location vector like μ is not a copula parameter.Such results can be extended to n-dimensional MGH and MAGH distributions.The lower Frechet bound is not a copula function anymore for n ≥ 3.The covariance and correlation matrix are still the most popular in most practical applications among the large number of dependence measures for multivariate random vectors.These dependence measures should be considered with care for non-elliptically contoured distributions like MAGH distributions.Kendall's tau, the correlation coefficient is a measure of linear dependence between two random variables.It is not invariant under monotone increasing transformations.Scale-invariance presents an undisputable requirement for a proper dependence measure in general and play an increasing role in dependence modelling.Kendall's tau is the most well-known and will be determined for MGH and MAGH distributions.The tail dependence structure of MGH and MAGH distributions is emphasized.The result about dependence structure of extreme events (tail or extremal dependence) related to latter kinds of distributions is established.The importance of tail dependence in financial risk management is addressed.Bivariate standardized MGH distributions show more evidence of dependence in upper-right and lower-left quadrant of its distribution function than MAGH distributions.MGH distributions are always tail independent but non-standardized MAGH distributions can model tail dependence.The symmetric case β = 0 is considered for simplicity.

The inconsistencyis to use MAGH distribution to fit probability distributions of portfolio assets during crisis because no specific time horizon is stated.Risk measures are unknown.

Expected shortfall or value-at-risk may be a better risk measure not standard deviation during crisis.

The research designs or methods seemedinadequate are the parameters of MAGH distribution can be estimated with simple two-stage method after setting up the estimation procedure.It refers to estimation of covariance matrix and parameters corresponding to univariate marginal distributions.The estimation reduces to one-dimensional estimation problem.This type of estimation can be performed only for MGH distributions in the subclass of elliptically contoured distributions.The first stage starts with minimizing cross entropy.The Kullback divergence, Hk, is calculated to measure the similarity between true density f\* and its approximation f.Minimizing the Kullback divergence by changing f is a popular method to obtain a good approximation or fit of true density f\*.This is often used if common goodness-of-fit tests like χ2-square test is too complex.The latter is for high-dimensional distributions.Hk is a constant and can be removed.Resulting expression is cross entropy.f\* is unknown and approximated by its empirical counterpart.Minimizing cross entropy or Kullback divergence matches with concept of maximum likelihood.Two methods for optimization are: constrained nonlinear optimization methods, and unconstrained nonlinear optimization methods after suitable parameter transformations.The unconstrained approach is preferred because of robustness reasons.The optimization algorithm belongs to probabilistic ones with non-convex objective function and has two characteristic phases:

1. The objective function is determined at random points as part of the search space in a global phase.

2. the samples are changed to become candidates for local optimization routines in a local phase.

Results pertaining to convergence of probabilistic algorithms to global minimum are covered.A multi-level single-linkage procedure is used.The global optimization method produces random samples from search space by finding the samples belonging to same objective function attractor and eliminating multiple ones.A conjugate gradient method is started for remaining samples in local phase.A Bayesian stopping rule is applied to assess the probability that all attractors have been explored.The estimation of MAGH distributions is carried out.A big advantage of MACH distribution is its parameters can be easily identified in a two-stage procedure.The two-stage estimation may change the asymptotic efficiency of the estimation.The empirical results show that the two-stage estimation is quite strong with respect to finite-sample volatility of corresponding estimators.It is crucial for applications that the univariate densities are not necessarily identically parameterized.There is much freedom to choose the parameters λ,α, and β.

Two further extreme alternatives are possible:

1) Minimum parameterization: Equal parameters λ,α, and β for all one-dimensional margins.

2) Maximum parameterization: Individual parameters λi,αi and βi for all one-dimensional margins.

The suitable parameterization depends on the application type and available data volume. The optimization procedure presented is a special case of an identification-algorithm for conditional distributions explored.An efficient and self-contained generator of random vectors for families of MGH and MAGH distributions is provided.The generator based on a rejection algorithm is made up of several features not published yet.An explicit algorithm avoiding difficult minimization routines outperforms known algorithms in MGH distributions with general parameter constellations.The generation of random vectors reduces to generation of one-dimensional random numbers.The algorithm outperforms sampling algorithm efficiency proposed suites to a larger class of distributions.The algorithm avoids tedious minimization routine and time-consuming evaluations of Bessel function.The generation uses a rejection method with a three part rejection-envelop.The rejection method needs random numbers generation with density s.d(x) where scaling factor s has to be computed to obtain a density function s.d(x), x > 0.The random numbers are generated from an univariate inverse Gaussian distribution.The multivariate generalized hyperbolic random vectors is generated.This is done by exploiting the above introduced mixture representation.MAGH vector is generated.The empirical efficiency of MGH random vector generator for different parameter constellations is presented.A series of computational experiments with simulated data is carried out.The results from the experiments show that:

1) MGH density functions with parameter λϵ R appear to have very close counterparts in MGH-subclass with parameter λ = 1.

2) MAGH distribution can closely approximate MGH distribution with similar parameter values.

The following conclusions are drawn:

1) The fit of both distributions, MGH and MH, measured by cross entropy and visual closeness of density plots, is satisfying.

The presence of multiple parameter constellations for MGH distributions leading to quite similar density functions is implied.Similar results are observed for corresponding tail functions.

2) Generalized hyperbolic densities appear to have very close counterparts in MH distributions class even for large λ.

Thus, MH distributions class will be sufficiently rich for our considerations.Only MH distributions and corresponding MAH distributions (multivariate affine generalized hyperbolic distributions with λ = 1) will be considered next.Parameter estimates for the following bivariate distributions are compared:

1) MH distributions.

2) MAH distributions with minimal parameter configuration using same value of α and β for each margin.

3) MAH distributions with maximal parameter configuration using different values of αi and βi for each margin.

The following conclusions are drawn:

1) Most parameters show an appropriate fit about the sample bias and standard deviation.

The relative variability of estimates increases with decreasing α in fatter tailed distributions.

Such distributions appear to be more ill-posed pertaining to estimation of individual parameters.

2) It is negligible for the differences between parameter estimates either for MH, MAHmin, or MAXmas distributions although the data are from MH distribution.

3) The fit in terms of cross entropy is not too different between the various models.

The MAHmax estimates are nearer to MH reference distribution than MAHmin estimates in terms of cross entropy.They are even better than MH estimates in one case.The fitting capability of MAHmax model is at the expense of a larger variability and sometimes larger bias known as overlearning effect.

The opinion regarding the quality and importance is multivariate affine generalized hyperbolic distributions, a new class of multidimensional distributions with exponentially decreasing tails to analyze high-dimensional data is investigated.They are appealing because of unknown parameters estimation and random vector generation.They possess an attractive dependence structure.Several dependence properties are derived.An extensive simulation study displayed the flexibility and robustness of the model.Multivariate affine generalized hyperbolic distributions is recommended for multidimensional data modelling in statistical and financial applications.

The topic should be further studied because it would be crucial to know whether MAGH is a better distribution compared to MGH to fit probability distribution of asset classes during crisis with the right risk measures like expected shortfall or value-at-risk.Major flaw with probability space defined by triplet: sample space, set of events and assignment of probabilities to events.Assumption is once probability space is set, nature makes its move and selects a single outcome from the sample space.Questions: how does nature select the outcome?what if multiple outcomes are selected?Elliptical distributions are critical in portfolio theory because if returns on all assets available for portfolio formation are jointly elliptically distributed, then all portfolios can be characterized wholly by their location and scale.Any two portfolios with identical location and scale of portfolio return have identical distributions of portfolio return.Location and scale correspond to mean and standard deviation for multi-normal distributions.How will elliptical distributions for fitting asset classes with the right risk measures like expected shortfall or value-at-risk perform during crisis ?

## III.3 STANDARD DEVIATION, VaR OR EXPECTED SHORTFALL

Artzner et al. 1996’s research problem is an axiomatic structure for coherent measures of risk is given.It is shown that the coherent risk measures are precisely those given by the scenarios.Relations with existing and prescribed margin requirements are pointed out.An alternate representation and several examples are given.A connection to the theory of convex games is explored.

The key findings of their research are the ability to quantify risk is a basic requirement for risk management. This has been contemplated in different contexts like credit risk, insurance risk and market risk.Risk measures like capital requirements, value at risk, margin requirements, risk capital etc are different quantifications.Several properties for a coherent risk measure is proposed and discussed.The subadditivity property diverts away from the value at risk measurement.Scenario's based risk measures are initially shown to be coherent measures.The proof of the reverse statement goes through a detour.A measurement of certain prespecified risks can, under specific conditions, be extended to a coherent risk measure of all risks under contemplation. Two equivalent representations of coherent risk measures are allowed.Coherent risk measures which happen to be additive on comonotone risks are presented.

The important gaps in knowledge and data are only risks from the beginning to end of a specified period is considered to simplify the problem.This can be the period between rehedging, a fixed interval like two weeks, or period needed to liquidate a position which can vary for different positions.It is supposed that the length of the period is known and fixed.It is assumed that the position remains fixed over this period. It is supposed that the set of all possible states of the world at end of period is known whereas probabilities of the various states happening are not known. The state of the world might be described by a list of prices of all securities in the example of market risk.It is assumed that the set of all possible such lists are known not the probabilities.

The trends and themes of the research related to the topic is having the appropriate risk measure with the right probability distribution is crucial during portfolio optimization during crisis.

The relationships between key concepts are a risk measure is a mapping ρ from Ԍ to non-negative real numbers.Risk measure has a value no higher than the maximum possible net loss.The subadditivity condition is a natural requirement.For example, an individual would be motivated to open two accounts, X1 and X2, incurring the smaller risk measure of ρ(X1) + ρ(X2).A firm would be motivated to create two subsidiaries if a firm was required to meet a capital requirement which did not satisfy this property.Risk measure of the form ρ(X) = z with z as α-quantile of loss X for some fixed α is proposed.This risk measure is incoherent because it fails to satisfy the subadditivity condition.It is unreasonable to impose the subadditivity condition because position size directly influencesrisk. Monotonicity is another requirement to be satisfied.A formal construction of coherent risk measures along lines of actual risk measures using scenarios is given.The question of recoverability of scenarios out of the measure, a key instrument to understand a dual presentation of risk measures where a coherent risk measure is created by some kind of extension of a prespecified measurement defined on a subset of risks.

The inconsistencies or contradictions is it is uncertain how coherency of risk measures can be applied to probability distribution for portfolio optimization during crisis.

The research designs or methods seemed inadequate because the construction of coherent risk measures through generalized scenarios is done.The deterministic scenarios traditionally considered are simply probability measures with simple-point support.They are currently used at MATIF 1993 and at different U.S. exchanges which use SPAN 1995 to calculate risk measures.Closed convex hull can be written in terms of the coherent risk measures defined given a set of probability measures.Construction of coherent risk measures out of measures prespecified on certain risks is done.Considering at some exchange based risk measurement in 1977 CBOE lead the investor to calculate risk measure needed for his or her portfolio finding an optimal decomposition of it as a linear combination with positive coefficients of pairings. They were certain portfolios of risk where a good measurement was prespecified.The universality of the cover and/or scenarios constructions is discussed.Any coherent risk measure appears as given by a worst case method in a framework of generalized scenarios.An example of risk measure used in Life Insurance, the mittlere Risico, is constructed out of one scenario, related to the life table used by a company.A connection with convex games is established.The assumption of comonotonicity appears to be acceptable.The risk measure of the sum should be equal to the sum of the risk measure if the risks are such that they work in the same direction, that is, there is no cancellation by taking a joint position.Comonotonicity describes the loose statement "working in the same direction".

The opinion regarding the quality and importance is coherency of risk measures builds the foundation of portfolio optimization from asset diversification.The subadditivity criteria allows successful asset diversification to reduce risk during portfolio optimization.

The topic should be further studied because this will allow the determination of the appropriate risk measures to complement the right probability distribution for portfolio optimization during crisis.

Artzner et al. 1999’s research problem is studying both market and nonmarket risks without complete market assumption and discussing methods of these risk measurements. They present and justify a set of four desirable properties for risk measures and call them satisfying these properties coherent. They examine the risk measures provided and related actions needed by SPAN, SEC/NASD rules and quantile-based methods. They demonstrate the universality of scenario-based methods for providing coherent measures. They provide suggestions about the SEC method. They suggest a method to repair the failure of subadditivity of quantile-based methods.

The key findings of their research are providing a definition of risks both market and nonmarket risks. They present and justify a unified framework for analysis, construction and implementation of risk measures.Completeness of markets is not assumed.Such risk measures can be used as extra capital requirements to control risk assumed by market participants, traders and insurance underwriters and to allocate existing capital. They define acceptable future random net worth’s and provide a set of axioms about set of acceptable future net worths. They define risk measure of an unacceptable position, once a reference, prudent investment instrument has been specified, as minimum extra capital which invested in reference instrument, makes the future value of changed position become acceptable. They state axioms on risk measures and relate them to axioms on acceptance sets. They argue that they should hold for any risk measure used to effectively control or manage risks.The risk measures that satisfy the four axioms are coherent. They present a simplified description of three existing methods for measuring market risk: variance-quantile method of value-at-risk (VaR), margin system SPAN (Standard Portfolio Analysis of Risk) developed by Chicago Mercantile Exchange, and margin rules of Securities and Exchanges Commission (SEC) used by the National Association of Securities Dealers (NASD). They analyze existing methods in terms of axioms and show that the last two methods are actually the same that when slightly changed they are mathematical duals of each other. They make a specific recommendation for improvement of NASD-SEC margin system. They examine the results of using value at risk for risk management. They provide a general representation for all coherent risk measures in terms of generalized scenarios by applying a result of the separation theorem for convex sets already in mathematics literature. They give conditions for extending into a coherent risk measure a measurement already agreed on for a confined class of risks. They use representation results to suggest a specific coherent measure known as tail conditional expectation and give an example of construction of a coherent measure out of measures on separate classes of risks e.g. credit and market risks. The axioms are not sufficiently restrictive to specify a unique risk measure.They characterize a large class of risk measures.The selection of precisely which measure to utilize from this class should probably be made on basis of additional economic considerations.Tail conditional expectation is the least expensive among those coherent and accepted by regulators being more conservative than value at risk measurement under some assumptions.

The important gaps in knowledge and research are the assumption of position being held during the whole period can be relaxed substantially.Positions may change because of the agent's actions or those of counterparties.The risk of following a strategy specifying the portfolio held at each date as function of the market events and counterparties' actions over an arbitrary period of time. The first step begins with the current results in a simplified way starts the first step.Model-free measures can be used where only risks of positions are considered compared to model-dependent measure of risk when an explicit probability on space is used to construct it. Describing risk by a single number can lead to a great loss of information.However, the actual decision to take a risk or allow one to take it is fundamentally binary, yes or no, claimed to be the actual origin of risk measurement.Any coherent risk measure is in a framework of generalized scenarios for the given worst case method.Scenarios should be announced to all traders within the firm by the manager or to all firms by the regulator. Decentralization of risk management within the firm is possible only after such announcements.There is a problem preventing decentralized risk management in quantile-based methods after announcements of individual limits.Two operators ignorant of each other's action may each comply with their individual quantile limits with no automatic procedure provides an interesting upper bound for the measure of joint risk because of their actions.It is a formidable task of building and announcing a suitable set of scenarios for the regulation case.The fewer initial standard risks are considered, the more conservative is the coherent risk measure obtained compare to scenarios-based measures of risks.Too many scenarios and dually too few standard risks were considered with SEC rules.

The trends and themes of the research related to the topic are determining the right risk measures to use for the appropriate probability distribution is crucial.This is most critical during crisis with lots of jumps and shocks.Coherent risk measures like VaR and Expected Shortfall with generalized hyperbolic distribution are the appropriate options during crisis.

The relationships between key concepts are risk is defined in terms of changes in values between two dates because it is related to variability of future value of position because of market changes or more generally to uncertain events, thus it is better to consider future values only.There is no need for initial costs of components of position to be determined from universally defined market prices.The basic objects of the study shall be the random variables on the set of states of nature at a future date as possible future values of positions or portfolios currently held.A first crude and crucial measurement of position risk will be whether its future value belongs or does not belong to subset of acceptable risks decided by a supervisor like:

a) a regulator who takes into account unfavorable states allowing a risky position that may draw on resources of government e.g. as a guarantor of last resort;

b) an exchange's clearing firm has to make good on promises to all parties of transactions securely completed;

c) an investment manager who knows that his firm has given to its traders an exit option in which the strike price consists in being fired in event of big trading losses on one's position.

A trade-off between severity of risk measurement and level of activities in supervised domain in the cases mentioned is present.The axioms and characterizations to be provided do not single out a specific risk measure and additional economic considerations have to play a role in final choice of measure.The position should be altered for an unacceptable risk resulting from a position with an unacceptable future net worth.Another option is to search for some commonly accepted instrument when added to the current position making its future value acceptable to the regulator or supervisor.The current cost of getting sufficient of this or these instrument(s) is a good candidate for measure of risk of initially unacceptable position.It is supposed that set of all possible states of the world at end of the period is known and probabilities of various states occurring may be unknown or not subject to common agreement.The state of the world might be described by a list of prices of all securities and all exchange rates when dealing with market risk.It is assumed that the set of all possible such lists are known.This assumes that markets at date T are liquid, otherwise more complicated models are needed to distinguish the risks of a position and of a future net worth because with illiquid markets mapping from former to latter may not be linear.A final net worth that is always nonnegative does not need extra capital whereas a net worth that is always strictly negative certainly does.Risk aversion on the part of the regulator, exchange director, or trading room supervisor is required.A less natural requirement on the set of acceptable final net worths is given.Sets of acceptable future net worths are primitive objects to be considered to describe acceptance or rejection of a risk.There is a natural way to define a risk measure by describing how close or how far from acceptance a position is given some reference instrument.Risk is definedG be the set of all risks, that is the set of all real-valued functions on finite set of states of nature and as a final net worth because:

1) the necessity to extend accounting procedures dealing with future certain bad events like loss in inventories or degradation (wear and tear) of physical plant into measurement procedures for future uncertain bad events.

2) actual measurements in practice appear to be defined only for risks in both states with positive and negative final net worths exist.

3) multiperiod models may naturally introduce at some intermediate date prospect of such final net worths.

Cash in a publicly traded company is an increase in equity.Risk measure associated with an acceptance set is defined.Acceptance sets allows the handling of a question of importance to an international regulator and to risk manager of a multinational firm, specifically the invariance of acceptability of a position with respect to a variation of currencies.The scenario is more complicated for unacceptable positions.Acceptance set is associated with a risk measure.Risk measure is stated in the same units as final net worth except for the use of the reference instrument.The following four axioms should be adhered to for the risk measure to be coherent:

1) Translation invariance

2) Subadditivity

3) Positive homogeneity

The consequences of lack of liquidity when computing future net worth of a position should be considered if position size directly influences risk.Reverse inequality is imposed to model what a government or exchange might impose in a scenario where no netting or diversification occurs especially because the government does not prevent a lot of firms from all taking the same position.

4) Monotonicity

Relevance is adhered to but insufficient to prevent concentration of risks remains undetected.The acceptance set is claimed to be the central object and axioms in terms of associated risk measure are discussed.Propositions are provided to support that.

The contradiction is using non-coherent risk measures like VaR is not appropriate during crisis.Risk is present throughout the probability distribution not only at the tails during crisis.

The research designs or methods seemedinadequate because the three currently used methods of market risk measurement are:

1) SPAN created by Chicago Mercantile Exchange;

2) The Securities and Exchange Commission (SEC) rules observed by National Association of Securities Dealers , similar to rules used by Pacific Exchange and Chicago Board of Options Exchange;

3) The quantile-based Value-at-Risk (Var) method.

The relationship of these three methods with the abstract approach provided will be examined.Slightly more general forms for some of the methods are suggested.The distinction made above between model-free and model-dependent risk measures appears. The SEC rules on final net worth consider portfolios to be formal lists of securities and impose margin requirements on them whereas the SPAN approach takes the random variables, that is, gains and losses of the portfolios of securities, to be basic objects to measure.Value-at-risk (VaR) is commonly defined in terms of net wins or P/L thus ignores the difference between money at one date and money at a different date for small time periods and a single currency may be acceptable.Using quantile needs paying attention to discontinuities and intervals of quantile numbers.The quantile do satisfy subadditivity only if probabilities of exceedances are smaller than 0.5 if they are computed under a distribution for which all prices are jointly normally distributed.Any value outside the quantile will be ignored.Some works on quantile-based measures consider usually the computational and statistical problems they raise without first considering the implications of such method of measuring risks.VaR is rejected as measure of risk because:

1) it does not behave well with respect to addition of risks, even independent ones, causing severe aggregation problems.

2) its use does not encourage and sometimes prohibits diversification because it does not account for economic consequences of events, the probabilities of which it controls.

Any coherent risk measure starts as the supremum of expected negative of final net worth for some collection of generalized scenarios or probability measures on states of the world. The more scenarios considered, the more conservative is the risk measure obtained.A point mass scenario is chosen by the investor or supervisor whereas the simulation trial is chosen randomly according to a distribution they have prescribed beforehand.Coherent risk measures proposed are tail conditional expectation or TailVaR and worst conditional expectation.

The opinion regarding the quality and importance is theoretical proofs have been given on coherent risk measures and how coherent risk measures can be created from different classes of risks.Empirical proofs with actual data set should be given to support the theoretical proofs.

The topic should be further studied because coherent risk measures should be further investigated with the appropriate probability distribution to assist in portfolio optimization and risk management.This is crucial to maximizing return and minimizing loss during crisis.

Acerbi and Tasche (2002)’s researchproblem is the expected Shortfall (ES) in different variations is proposed as a solution for the deficiencies of Value-at-Risk (VaR) which is not a coherent risk measure.Most definitions of ES come to the same results when applied to continuous loss distributions.Differences may occur when underlying loss distributions have discontinuities.As such, the coherence property of ES can be lost unless care is taken of the details for its definition. Some of the definitions of ES is compared showing there is one which is robust in yielding a coherent risk measure despite of the underlying distributions.Thus, this ES can be estimated effectively even where the usual estimators for VaR fail.

The key findings of their research are making transparent relations between the developed notions.Four characterizations of expected shortfall are presented:

1) Integral of all quantile below corresponding level.

2) Limit in a tail strong law of large numbers.

3) Minimum of a certain functional introduced.

4) Maximum of WCE's when underlying probability space changes.

This shows that ES definition is complementary and in some aspects superior to other ideas. Any law invariant coherent risk measure has a representation with ES as main building block.WCE, TCE, CVaR (conditional value-at-risk), ES and α-tail mean (TM) will be mathematically defined precisely.Useful properties of α-tail mean and ES like integral representation, continuity and monotonicity in level α and coherence for ES. It is shown that α-tail mean arises naturally as limit of average of 100α% worst cases in a sample.ES and CVaR are two different names for same object.Inequalities and examples clarifying relations between ES, TCE, and WCE are discussed.A general representation of ES in terms of WCE is discussed.

The important gaps in knowledge and researchare Value-at-risk(VaR)isseriously criticized for not being sub-additive as a risk measure.The portfolio risk can be bigger than the sum of stand-alone risks of its components when measured by VaR.Risk management by VaR may fail to stimulate diversification. VaR does not consider the severity of an incurred damage event.Coherent risk measures was introduced to address the handicap. Worst conditional expectation (WCE) closely related to tail conditional expectation (TCE) is introduced.WCE is coherent and very useful only in a theoretical setting because it needs the knowledge of entire underlying probability space whereas TCE is useful in practical applications but not coherent. The goal is to create a risk measure both coherent and easy to compute and estimate.Expected shortfall (ES) at a specified level α is literal mathematically average loss in worst 100α% cases.It is a growing need to handle random variables with discontinuous distributions in the financial industry.The examples are portfolios with not-traded loans which are purely discrete distributions or portfolios with derivatives which are mixtures of continous and discrete distributions.Tail risk measures like VaR, TCE, and WCE applied to discontinuous distributions has a problem with their sensitivity to small changes in confidence level α.They are not continuous with respect to confidence level α. On the contrary, Esα is continuous with respect to α.The risk measured by Esα will not vary dramatically when there is a switch in confidence level by some basis points regardless of underlying distributions. The insensitivity property is derived as a result of a different representation of tail mean.The integral representation for continuous distributions is interesting.For Esα about its monotonicity in α, the smaller the level α, the greater is the risk.α-tail mean does not have a general representation of a conditional expectation of X given some event A ϵ σ(X).None of the quantities -qα, VARα, TCEα, or TCEα defines a sub-additive risk measure generally.

The trends and themes of the research related to the topic are VaR is usually used as risk measure during normal times.It is not a suitable risk measure during crisis because of its incoherence.ES is a more suitable risk measure during crisis.It is coherent.

The relationships between key concepts are basic definitions is given.Only losses of some asset or portfolio is focused on in this paper.The quantile, both lower and upper α-quantile of X, are defined.VaRα (Value-at-risk) is the smallest value with probability of absolute loss being at most this value is at least 1 - α.Tail conditional expectations is defined.Worst conditional expectations is defined.Condtional value-at-risk is defined.Tail mean and expected shortfall are defined.α-tail mean and ESα depends only on X distribution and level α not a particular quantile definition.Useful properties of tail mean and expected shortfall are explained.The most important property of ES might be its coherence.ES is coherent because it is monotonous, sub-additive, positively homogeneous and translation invariant.Motivation for tail mean and expected shortfall is discussed.The α-tail mean is the mean of the worst 100α% cases.This is very natural from insurance or risk management view appeared in literature by different types of conditional expectation beyond VaR a different concept for discrete distributions.TCE, WCE, CVaR are conditional expected values of random variable X.WCE and ES is different. This phenomenon can occur only when underlying probability space is too small not allowing a suitable representation of random variable under consideration as function of a continuous random variable.It is always possible to switch to a larger probability space to make WCE and ES coincide if only finitely many random variables are under consideration.A general representation of ES in terms of related WCEs is stated.

The contradictions are inequalities and counter-examples are given.Expected shortfall is compared with risk measures TCE and WCE defined.An example is presented showing that VaR and TCE are not sub-additive.No clear relationship between WCE and lower TCE is shown.Result in the spirit of Neyman-Pearson lemma is started.

The research designs or methods seemed insufficient is nothing is mentioned about this.

The opinion regarding the quality and importance is taking a conditional expectation of losses beyond VaR can fail to yield a coherent risk measure when discontinuities in loss distributions are present.Existing definitions for some type of expected shortfall redress the drawback did not give representations appropriate for efficient computation and estimation generally.Clarified the relations between these definitions and explicit highlighting which is most suitable for practical purposes.

The topic should be further studied because risk measures and probability distribution during crisis are different compared to normal times.Risk measures used during crisis can be VaR, ES and standard deviation. Probability distribution used during crisis can be generalized hyperbolic distributions etc.Portfolio optimization during crisis will require the appropriate probability distribution and risk measure.

**III.4 TYPES OF OPTIMIZATION**

Markowitz (1952)’s research problem is selecting a portfolio involves two stages.First, we begin with observation and experience and end with beliefs about future performances of available securities.Second, we begin with relevant beliefs about future performances and end with the portfolio choice.Only the second stage is considered in this paper.

The key findings of his research are relevant beliefs about future performances resulting in portfolio choice is examined.The investor should maximize discounted expected or anticipated returns.This is rejected both as hypothesis to explain and as a maxim to guide investment behavior.The investor should consider expected return a desirable thing and variance of return an undesirable thing.It has many sound points as a maxim for and hypothesis about investment behavior.Geometrically relations between beliefs and portfolio choice according to expected returns-variance of returns rule is explained.The investor should maximize discounted or capitalized value of future returns.Expected or anticipated returns must be discounted because the future is not known with certainty. Anticipated returns could include an allowance for risk. The rate at which we capitalize returns from particular securities could change with risk.The hypothesis or maxim that the investor should maximize discounted return must be rejected.It is never implied that a diversified portfolio is preferable to all non-diversified portfolios if market imperfections are ignored.Diversification is both observed and sensible, thus this does not imply its superiority must be rejected both as a hypothesis and as a maxim.

The important gaps in knowledge and research are it is not implied in diversification no matter how the anticipated returns are formed; whether the same or different discount rates are used for different securities; no matter howthose discount rates are decided on or how they change over time.The results depend on assumption that the anticipated returns and discount rates are independent of particular investor's portfolio.The hypothesis implies that the investor places all his funds in the security with the greatest discounted value.Any of two or more securities or any combination of them is as good as any other if they have the same value.A diversified portfolio is in no case preferred to all non-diversified portfolios.An investor would place all his funds in the security with maximum anticipated returns if he wished to maximize anticipated return from the portfolio.The investor should diversify and maximize expected return.The investor should diversify his funds among all securities giving maximum expected return.The law of large numbers will insure that actual yield of the portfolio will be almost the same as the expected yield.It is a special case of the expected returns-variance of returns rule.It assumes that there is a portfolio which gives both maximum expected return and minimum variance and commended such portfolio to the investor.The law of large numbers applied to a portfolio of securities cannot be accepted.Securities returns are too intercorrelated.Diversification cannot remove all variance.Maximum expected return portfolio is not necessarily the one with minimum variance.There is a rate at which the investor can get expected return by taking on variance or reduce variance by giving up expected return.Expected returns or anticipated returns is insufficient.The expected returns-variance of returns rule is considered.The main limitations are:

1) the results are not derived analytically for n-security case whereas they are presented geometrically for 3 and 4 security cases;

2) static probability beliefs are assumed.

The probability distribution of yields of different securities is a function of time.The difficult question of how investors do form their probability beliefs is not considered.Two conditionsmust be satisfied before it would be practical to use efficient surfaces in the manner described.First, the investor must desire to act according to E-V maxim.Second, arriving at reasonable ci and σij is mandatory.

The trends and themes of the research related to the topic are portfolio selection depend on getting the expected return with the risk undertaken.Having the right risk measures and probability distribution during crisis is crucial for portfolio optimization.

The relationships between key concepts are an isomean curve is the set of all points (portfolios) with a given expected return.An isovariance line is defined to be the set of all points (portfolios) with a given variance of return.Examining the formulae for E and V reveals the shapes of the isomean and isovariance curves.Usually, the isomean curves are a system of parallel straight lines whereas the isovariance curves are a system of concentric ellipses.Different reasons recommend the use of expected return-variance of return rule both as a hypothesis to explain well-established investment behavior and as a maxim to guide one's own action.It serves better as an explanation of and guide to investment which is different from speculation.Expected return is rejected previously because it never implied the superiority of diversification.However, the expected return-variance of return implies diversification for a wide range of μi, σij.This does not mean that E-V never implies the superiority of an undiversified portfolio.One security might have an extremely higher yield and lower variance than all other securities such that one particular undiversified portfolio would give maximum E and minimum V.E-V results in efficient portfolios almost all of which are diversified for a large, presumably representative range of μi, σij.E-V hypothesis imply diversification for the right kind for the right reason.The sufficiency of diversification is not thought by investors to rely only on number of different securities held.For example, a portfolio with sixty different railway securities would not be as well diversified as the same size portfolio with some railroad, public utility, mining, different kinds of manufacturing etc.This is because it is generally more probable for firms within the same industry to do poorly at the same time than for firms in dissimilar industries.

It is insufficient to invest in many securities in trying to make the variance small.It is essential to avoid investing in securities with high covariances among themselves.Diversification across industries shoul is done because firms in various industries especially industries with different economic characteristic have lower covariances than firms within an industry.Variance is a famous measure of dispersion about the expected.The investor's choice would still be in the set of efficient portfolios if the concern is with standard error, σ = √V, or with coefficient of dispersion, σ/E.An investor diversifies between two portfolios by buying of shares from two different investment companies.The variance of the resulting portfolio will be less than the variance of either original portfolio if two original portfolios have equal variance.The variance will not be increased.The variance will not be decreased if the returns from both portfolios are perfectly correlated.E, V efficiency is reasonable as a working hypothesis and maxim for a great variation of investing institutions considering yield to be a good thing and risk a bad thing with gambling avoided.E-V is used in theoretical analyses or in actual selection of portfolios.The various effects of a change in the beliefs usually held about a firm, or a general change in preference as to expected return versus variance of return, or variation in the supply of a security in theoretical analyses may be enquired.

The contradictionis whether having the relevant beliefs about future performances to determine the portfolio choice will work during crisis is unknown.

The research designs or methods seemed inadequate because observation and experience to determine beliefs about future performances of available securities is not available.

The opinion regarding the quality and importance is thereis no mentioning of the appropriate probability distribution and risk measures to use during crisis.No empirical testing is done to support the theoretical methods mentioned.

The topic should be further studied because only the second stage in the process of selecting a portfolio is considered.It begins with relevant beliefs about securities involved and finishes with the selection of a portfolio.The formation of the relevant beliefs based on observation is not considered in the first stage.

Lintner (1965)’s research problem is about risk and uncertainty on asset prices on rational decision rules for individuals and institutions to use in choosing security portfolios, and on proper selection of projects to include corporate capital budgets, have attracted the attention of professional economists and students of capital markets and of business finance.The present paper is to push back the frontiers of the knowledge of logical structure of these related matters under idealized conditions.

The key findings of the research are the problem of selecting optimal security portfolios by risk-averse investors with alternative of investing in risk-free securities with a positive return or borrowing at same rate of interest and can sell short if they wish.Derive a set of stable equilibrium market prices which minimally fully and explicitly show the presence of uncertainty different from diverse expectations effects.Derive more implications of such uncertainty.Aggregate market value of any company's equity is equal to capitalization at risk-free interest rate of a uniquely defined certainty-equivalent of probability distribution of aggregate dollar returns to all stock holders.This certainty equivalent is expected value of these uncertain returns less adjustment term proportional to their aggregate risk for all company.The factor of proportionality is same for all companies in equilibrium may be regarded as market price of dollar risk.The relevant risk of each company's stock is measured by the sum of variance of its own aggregate dollar returns and their total covariance with those of all other stocks.Some implications of such results for normative aspects of capital budgeting decisions of a company with stock traded in the market is considered. Further assumptions needed to make capital budgeting decisions independent of decisions on how budget is financed are imposed.It is assumed that common stock portfolios are not inferior goods that value of all other common stocks is invariant with any effect of capital budgets changes on covariances between values of different companies' stocks will not be considered.The capital budgeting problem becomes a quadratic programming problem. This capital budgeting-portfolio problem is formulated with its solution given and its more important properties scrutinized.The minimum expected return in expected present value dollars needed to justify the funds allocation to a given risky project is shown to be an increasing function of the following factors:

(i) risk-free rate of return;

(ii) market price of dollar risk;

(iii) variance in project's own present value return;

(iv) project's aggregate present value return-covariance with assets already held by other company, and

(v) total covariance with other projects simultaneously included in capital budget.

The five factors are involved explicitly in corresponding derived formula for minimum acceptable expected rate of return on investment project.All means and covariances of present values must be determined at riskless rate r\*.It is shown that there can be no risk-discount rate used in calculating present values to accept or reject individual projects.The cost of capital defined for uncertainty is an inappropriate rate to use in such decisions even if all new projects have same risk as existing assets.Briefly examines the complications introduced by institutional limits on amounts which either individuals or corporations may borrow atgiven rates by rising costs of borrowed funds.

The important gaps in knowledge and research are according to Tobin's separation theorem, proportionate composition of non-cash assets is independent of their aggregate share of investment balance, therefore optimal holding of cash for risk averters exist in purely competitive markets with quadratic utility functions or multivariate normal rates of return.He assumed that funds are to be allocated only over risk-free cash and default-free bonds of uncertain resale price and no short sales or borrowing allowed.The best portfolio-mix of risk assets can be determined by a single simple set of simultaneous equations without recourse to programming methods and when covariances are zero a still simpler ratio scheme gives the optimum whether short sales are permitted or not.A single quadratic programming solution is needed and sufficient when covariances are not all zero and short sales excluded.Focus on set of risk assets in risk averters' portfolios continuing extensions of Tobin's classic work.Develop different significant equilibrium properties in the risk asset portfolio.Establish conditions under which stocks will be held long or short in optimal portfolios when risk premiums are negative or positive.Develop expressions for different combinations of expected rate of return on a given security, and its standard deviation, variance, and/or covariances result in same relative holding of a stock, ceteris paribus. The direct evidence of appropriate functional relationships between required rates of return and relevant risk parameter(s) and how risk classes of securities may best be delineated.Modigliani and Miller assumed corporations were divided into homogeneous classes with property that all shares of all corporations in any given class differed by a scale factor and (a) perfectly correlated with each other and (b) were perfect substitutes for each other in perfect markets.No comment on measure of risk or uncertainty or other attributes relevant to identification of different equivalent return class.Market value of firm independent of capital structure and linear relation between expected return on equity shares and debt-equity ratio for firms within a given class are derived from the above assumptions and further assumption that corporate bonds are riskless securities.It involves no interclass comparisons nor any assertion to what is a sufficient compensation to investors for taking a given degree of risk.It is a general presumption among economists that relative risks are best measured by standard deviation or coefficient of variation of rate of return.In simplest cases considered, when all covariances are invariant or zero, the indifference functions are linear between expected rates of return and their variance, not standard deviation.The constant term will be larger and slope lower, the higher the fixed level of covariances of given stocks with other stocks.The indifference function between the ith expected rate of return and its pooled covariance with other stocks is hyperbolic with variances fixed.No simple relation between expected rate of return needed to maintain an investor's relative holding of a stock and its standard deviation exists.The indifference functions are complex and non-linear if it is assumed that correlations between rates of return on different securities are invariant when covariances are non-zero and variable.It is assumed that given current security prices each investor acts on his own probability distribution over rates of return given these market prices.It is assumed that investors' joint probability distributions is about dollar returns rather than rates of return.It is assumed that all investors assign identical sets of means, variances, and covariances to distribution of such dollar returns.The results of this paper are not being presented as directly applicable to practical decisions because many of the factors which matter very significantly in practice had to be ignored or assumed away.Assumptions simplification is to permit a rigorous development of theoretical relationships and theorems which reorient much current theory especially on capital budgeting providing a basis for further work.The quadratic utility of income or wealth function has several undesirable restrictive and implausible properties despite its mathematical convenience,multivariate normality is doubtless especially in considering common stocks.It implies negative marginal utilities of income or wealth too soon in empirical work unless risk-aversion parameter is very small.It cannot account for degree of risk-aversion empirically found.Offering more return at same risk would sate investors that would reduce their risk-investments because they were more attractive.Denying negatively sloped demand curves for riskless assets in liquidity preference theory is a conclusion which cannot be avoided by limiting arguments on quadratic utilities once borrowing and leveraged are admitted.Insurance premiums which people pay to hedge given risks rise progressively with wealth or income.

The trends and themes of the researchare using Bienayme-Tchebycheff inequality, it is shown that investors operate on safety first principle i.e. make risky investments to minimize upper bound of probability that realized outcome will fall below a pre-assigned disaster level should maximize ratio of excess expected portfolio return over disaster level to standard deviation of portfolio return which is the criterion of max θ when his disaster level is equated to risk-free rate r\*.This result does not depend on multivariate normality and uses a different argument and form of utility function.It is essential to express return on any arbitrary mix in terms of returns on individual stocks in the portfolio.

Short sales are excluded by assumption in most writings on portfolio optimization is arbitrary for some purposes.The short seller must pay to person who lends him stock any dividends which accrue while the stock is sold short and borrowed and his capital gain or loss is negative of any price appreciation during this period while computing a short sale return.

The short seller will receive interest at riskless rate r\* on sale price placed in escrow and may or may not also receive interest at same rate on cash remittance to lender of the stock.

It is assumed that both interest components are always received by short seller and margin requirements are 100%.The solution of a single bilinear or quadratic programming problem is needed to determine the optimal portfolio in more generally realistic cases when covariances are nonzero and short sales are not admitted.

The relationships between key concepts are Separation Theorem states that:

Given assumptions about borrowing, lending, and investor preferences discussed earlier in this section, optimal proportionate composition of stock i.e. risk-asset portfolio is independent of ratio of gross investment in stocks to total net investment.The seperation theorem is proven by Tobin through deriving the detailed solution for optimal mix of risk assets conditional on a given gross investment in the portfolio and then formally prove the critical invariance property in the theorem.More restrictive assumptions are used by Tobin pertaining to available investment opportunities and no borrowing is permitted.The problem fits well into a traditional Fisher framework with varying available combinations of expected values and standard deviations of return on alternative stock portfolios taking place of original production opportunity set and with alternative investment options being concurrent rather than between time periods.Alternative and more transparent proofs of separation theorem are available not involving actual calculation of best stocks allocation over individual stock issues.Presented a simple algebraic proof, set out logic of argument leading to the theorem, and portray the necessary geometry of the problem.Established the relation between the investor's total investment in any arbitrary mixture or portfolio of individual stocks, total net return from all investments including riskless assets and any borrowing, and risk parameters of investment position.The investor's net expected rate of return on his total net investment is linear to risk of return on total net investment determined by standard deviation of return in any arbitrarily chosen stock portfolio.As the investment w in the mix is increased, the standard deviation thus variance of return on total investment is proportionately bigger.An investor will minimize variance of his over-all return related with any expected return will choose by limiting all the stock investment to largest θ (slope of market opportunity line) value.The portfolio minimizes the variance related with any mean of net return per dollar of total net investment independent of mean and ratio of stocks to total net investment.The separation theorem is established assuming available portfolios insure a maximum θ.

It has four corollaries:

(i) Any investor whose options maximize expectation of any specific utility function consistent with these conditions will have identical decisions about the proportionate composition of his stock (risk-asset) portfolio given the assumptions regarding borrowing and lending already stated.

This is true regardless of specific utility function on expectation maximized.

(ii) Only a single point on Markowitz Efficient Frontier is related to investor's decision about the investments in risk assets under these assumptions.

With the same assumptions,

(iii) the parameters of investor's particular utility in the related set determine only ratio of total gross investment in stocks to total net investment including riskless assets and borrowing;

(iv) the investor's wealth is related to determining the absolute size of investment in individual stocks but not to relative distribution of gross investment in stocks among individual issues.

The difference between what is proposed in this paper and standard Fisher two-period production-opportunity case is in the concurrent nature of the comparisons not inter-period and rotation of market opportunity lines around common pivot of riskless return not parallel shifts in present value lines.Investor risk-aversion in preference for expected return and preference against return-variance, ceteris paribus, exists.Tobin proved that either concave-quadratic utility functions or multivariate normality of probability measurements and any concave utility were sufficient to validate their premise but were not alleged to be necessary conditions.

Theorem:

Under Idealized Uncertainty, in purely competitive markets of risk-averse investors,

a) total market value of any stock in equilibrium is equal to capitalization at risk-free interest rate r\* of certainty equivalent (Ri - W) of its uncertain aggregate dollar return ~Ri;

b) the difference Wi between expected value ¯Ri of these returns and their certainty equivalent is proportional for each company to its aggregate risk depicted by sum (∑jvRij) of variance of these returns and their total covariance with those of all other stocks; and

c) factor of proportionality (γ = λ/T) is same for all companies in the market.

It will be a contradiction to apply such valuation techniques during crisis because the underlying probability distribution is usually non-gaussian which the valuation techniques is based on.

The research designs or methods seemed improper, insufficient, or inadequate because of market assumptions:

(1) Each individual investor can invest any part of his capital in certain risk-free assets, for example, deposits in insured savings accounts, government bonds of appropriate maturity giving certain yield, all of which pay interest at a common positive rate exogenously determined;

(2) he can invest any fraction of his capital in any or all of a given finite set of risky securities

(3) traded in a single purely competitive market, free of transaction costs and taxes at given market prices which do not depend on his investments or transactions.

(4) any investor may borrow funds to invest in risk assets.

It is assumed that interest rate paid on such loans is same as what is received from investing in risk-free savings accounts, no limit on amount can be borrowed at this rate.

(5) he makes all purchases and sales of securities and all deposits and loans at discrete points in time in selecting the portfolio at any transaction point each investor will consider only

a) the cash throw-off i.e. interest payments and dividends received within the period to next transaction point and

b) changes in market prices of stocks during this same period.

Return of any common stock is the sum of cash dividends received and its market price change.The return on any portfolio is quantified including interest received or paid.

Investors assumptions:

(1) Each investor has decided part of his total capital will be in cash and non-interest bearing deposits for liquidity or transactions.

Risk-free assets with positive returns influence those with no return after liquidity and transactions requirements are satisfied at margin.An investor's capital is the fund available for profitable investment after optimal cash holdings are deducted.

(2) Each investor will be assigned a joint probability distribution involving his best judgments about the returns on all individual stocks or at minimum specified expected value and variance to every return and a covariance or correlation to every pair of returns.

All expected values of returns are finite. All variances are non-zero and finite.

All correlations of returns are less than one in absolute value i.e. the covariance matrix is positive-definite.The investor calculates the expected value and variance of total return on any possible portfolio, or mix of any specified amounts of any or all of individual stocks forming the suitably weighted average or sum of these components expected returns, variances and covariances.

(3) The investor will prefer the mixture having smaller variance of return if any two mixtures of assets have same expected return.

The mixture of assets with greater expected value is preferred if any two mixtures of assets have same variance of returns.Such preferences are implied by maximization of expected value of von Neumann-Morgenstern utility function if either (a) investor's utility function is concave and quadratic or (b) investor's utility function is concave and assigned probability distributions like returns on all possible portfolios vary at most by location and scale parameter if joint distribution of all individual stocks is multivariate normal.Borrowing is negative lending because interest rates on riskless savings bank deposits and on borrowed funds are assumed to be the same.Any portfolio can be described in terms of (i) gross amount invested in stocks, (ii) fraction of the amount invested in each individual stock, and (iii) net amount invested in loans with negative value showing that investor has borrowed not lent.

The total net investment consists of algebraic sum of stocks plus loans is a given amount.

This involves finding jointly optimal values for (1) ratio of gross investment in stocks to total net investment, and (2) ratio of gross investment in each individual stock to total gross investment in stocks.

The opinion regarding the quality and importance is there is no mention about the appropriate probability distribution and risk measures to utilize during crisis.

The topic should be further studied because it is important to know the appropriate probability distribution and risk measure to use during crisis.

Fama (1970)’s research problem is the theoretical and empirical literature on efficient markets model is reviewed.Empirical work related with adjustment of security prices to three relevant information subsets is considered after a discussion of the theory.First, weak form tests with information set is just historical prices are discussed.Second, semi-strong form tests is whether the prices efficiently adjust to other information that is obviously publicly available e.g. announcements of annual earnings, stock splits, etc. are considered.

Third, strong form tests is whether given investors or groups have monopolistic access to any information related for price information are reviewed.It is concluded with a few exceptions the efficient markets model stands up well.Theory to empirical work is proceeded to keep proper historical perspective a large extent the empirical work in this area preceded the development of the theory.The theory is presented first to more easily decide which of the empirical results are most related from the theory's viewpoint. The empirical work will then be reviewed in more or less historical sequence.

The key findings of previous research are there are three tests to determine an efficient capital market:

1) Weak form tests with information set are just historical prices.

2) Semi-strong form tests is whether the prices efficiently adjust to other information that is obviously publicly available e.g. announcements of annual earnings, stock splits, etc.

3) Strong form tests are whether given investors or groups have monopolistic access to any information related for price information.

Large daily price variations tend to be followed by large daily variations.The signs of the successor variations appear random but show that the phenomenon indicates a denial of the random walk model not of the market efficiency hypothesis.Two departures from complete randomness in common stock price variations from transaction to transaction are noted. First, reversals (pairs of consecutive price variations of opposite signs) are from two to three times as likely as continuations (pairs of consecutive price variations of the same sign).Second, a continuation is slightly more frequent after a preceding continuation than after a reversal.Niederhoffer and Osborne present convincing evidence of statistically significant fluctuations from independence in price variations from transaction to transaction with their analysis of their findings presents interesting insights into market making process on major exchanges, the kinds of dependence uncovered do not imply market inefficiency.The best documented source of dependence with tendency toward excessive reversals in pairs of non-zero price variations appears to be a direct result of ability of investors to place limit orders and orders at market and this negative dependence in itself does not imply existence of profitable trading rules.The Niederhoffer-Osborne analysis of market making points clearly to the existence of market inefficiency with respect to strong form tests of efficient markets model.The biggest contribution of Mandelbrot's work has been to excite research on stable distributions and estimation procedures to be applied to stable variables.Roll's work is novel because it is the first weak form empirical work that is consciously in the fair game not random walk tradition.

The important gaps in knowledge and research are an efficient market prices fully reflect available information is so general that it has no empirically testable implications.The process of price formation must be specified in more detail so that the model is testable.We must define more exactly what is meant by fully reflect.One option would be to posit that equilibrium prices or expected returns on securities are produced as in the two parameter world.Generally, however, theoretical models especially empirical tests of capital market efficiency have not been this specific.Most of the available work is on the assumption that conditions of market equilibrium can be stated in terms of expected returns.The equilibrium expected return on a security is a function of its risk generally like the two parameter model such theories would posit that conditional on some relevant information set.Different theories would differ primarily in how risk is defined.It is assumed that conditions of market equilibrium can be stated as expected returns raises the purely mathematical concept of expected value to a status not necessarily implied by general belief of market efficiency.The expected value is just one of many possible summary measures of a distribution of returns and market efficiency per se does not permeate it with any special importance. The results of test based on this assumption rely to some extent on its validity and market efficiency.It assumes the unavoidable price one must pay to give the theory of efficient markets empirical content.The assumptions that conditions of market equilibrium can be made known in terms of expected returns and equilibrium expected returns are created on the basis of and therefore fully reflect the information set have a major empirical implication by ruling out the possibility of trading systems relying only on information in it that have expected profits or returns in excess of equilibrium expected profits or returns.The expected return or fair game efficient market models have other important testable implications.The random walk model is best regarded as an extension of the general expected return or fair game efficient markets model giving a mode detailed statement about economic environment.The fair game model only states that conditions of market equilibrium can be given in terms of expected returns with little details of stochastic process generating returns.A random walk forms within context of such a model when the environment is fortuitously such that evolution of investor tastes and process generating new information combine to give equilibria where return distributions repeat themselves through time.A frictionless market with all information is freely available and investors agree on its implications is not descriptive of markets in practice.Such conditions are adequate for market efficiency but unnecessary.Large transactions costs that inhibit flow of transactions do not imply that when transactions take place prices will not fully reflect available information as long as transactors take account of all available information.The market may be efficient if sufficient numbers of investors have ready access to available information.Disagreement among investors about implications of given information does not imply market inefficiency unless there are investors who can consistently make better evaluations of available information than are implicit in market prices.Transaction costs, information not freely available to all investors and disagreement among investors about implications of given information are potential sources of market inefficiency.They exist to some extent in real world markets.Measuring their effects on price formation process is the major goal of empirical work in this area.

The trends and themes of the research related to the topic are the main role of the capital market is allocating ownership of the economy's capital stock.The perfect market is in which prices give accurate signals for resource allocation: a market in which firms can make production-investment decisions and investors can select among the securities that represent ownership of firms' activities under assumption that security prices at any time fully reflect all available information.An efficient market is a market in which prices always fully reflect available information.All empirical research on efficient markets theory has been concerned with whether prices fully reflect particular subcategory of available information.

Initial works were concerned with weak form tests in which information subset of interest is just past price or return histories.Attention was turned to semi-strong form tests in which speed of price adjustment to other obviously publicly available information, for example, announcements of stock splits, annual reports, new security issues etc when extensive tests appeared to support the efficiency hypothesis at this level.Strong form tests is whether any investor or groups, for example, managements of mutual funds, have monopolistic access to any information relevant for prices formation have recently appeared.In early literature, discussions on efficient markets model were based on the even more special random walk model although it is argued that most of the early authors were concerned with more general versions of the fair game model.Research on security prices did not start with theory development of price information which was then subjected to empirical tests.On the contrary, the force for theory development came from accumulation of evidence in the middle 1950's and early 1960's that behavior of common stock and other speculative prices could be well approximated by a random walk.This resulted in the theory of efficient markets based on random walks but usually implying some more general fair game model.However, it was not until Samuelson's and Mandelbrot's works in 1965 and 1966 respectively that the role of fair game expected return models in efficient markets theory and relationships between them and random walks theory were thoroughly studied.The first statement and test of random walk model was by Bachelier in 1900.His fundamental principle for prices behavior was that speculation should be a fair game where the expected profits to the speculator should be zero.The process implied by his fundamental principle is a martingale from modern theory of stochastic processes.

The relationships between key concepts are a submartingale in prices has an important empirical implication.It is the assumption that expected returns conditional on information with fully reflected price are non-negative directly implies that such trading rules only on the information with fully reflected price cannot have bigger expected profits than a policy of always buying-and-holding security during the future period in question.Tests using such rules will be an important part of the empirical evidence on efficient markets model.

Current price of a security fully reflects available information was assumed to imply that successive price changes or one-period returns are independent.It was usually assumed that successive variations or returns are identically distributed.These two hypotheses form the random walk model.Market conditions that might assist or hinder efficient adjustment of prices to information are in order.First, it is easy to determine sufficient conditions for capital market efficiency.It is possible to consider a market in which

(i) there are no transactions costs in trading securities,

(ii) All available information iscostless available to all market participants, and

(iii) all agree on implications of current information for current price and distributions of future prices of each security.

The current price of a security fully reflects all available information in such a market.

The inconsistencies and contradictionsare evidence in contradiction of fair game efficient markets model for price variations or returns covering periods longer thana single day is more difficult to find.Preponderantly negative and small serial correlations in weekly common stock returns, and this results appearin the four day returns analyzed.It does not appear in runs test where there is some slight indication of positive dependence but not much evidence of any dependence at all.There is no indication that whatever dependence exists in weekly returns can be used as the basis of profitable trading rules.Looking hard one can locate evidence in the filter tests of both Alexander and Fama-Blume that is inconsistent with submartingale efficient markets model if it is interpreted in a strict sense.

The results for very small filters (1 percent in Alexander's tests and 0.5, 1.0, and 1.5 percent in the tests of Fama-Blume) show that it is possible to create trading schemes on very short-term (preferably intra-day and at most daily) price swings that will outperform buy-and-hold on average.The average profits on individual transactions from such schemes are tiny but they produce transactions so frequently that over longer periods and ignoring commissions they outperform buy-and-hold by a substantial margin.Such outcomes are evidence of persistence or positive dependence in very short-term price fluctuations.This is consistent with evidence for slight positive linear dependence in successive daily price variations generated by serial correlations.Minimum trading costs are enough to wipe out their advantage over buy-and-hold because small filters produce a lot of frequent trades.Filter tests, like serial correlations, produce empirically noticeable variations from strict implications of efficient markets model.The departures are so small that it seems hardly justified to use them to declare the market inefficient despite any statistical signifance they might have from an economic perspective.Strong form tests of the efficient markets model is about whether all available information is fully reflected in prices that no individual has higher expected trading profits than others because he has monopolistic access to some information.This model is not expected to be an exact description of reality and preceding discussions have already showed the existence of contradictory evidence.When studying the performance of mutual funds, the major goals are to determine:

(a) whether in general fund managers seems to have access to special information allowing them to generate abnormal expected returns;

(b) whether some funds are better at uncovering such special information than others.

The major theoretical and practical problem in using mutual fund industry to test the efficient markets model is producing a norm against which performance can be judged.The norm must show the results of an investment policy based on assumption that prices fully reflect all available information.One has the problem of finding appropriate definitions of risk and evaluating each fund relative to a norm with its chosen level of risk if one believes that investors are generally risk averse and must be compensated for any risks undertaken.Jensen uses the Sharpe-Lintner model of equilibrium expected returns to derive a norm consistent with these goals.The market line represents the results of a naïve investment strategy which the investor thinks prices reflect all available information might follow.The performance of a mutual fund is measured relative to this naïve strategy.Jensen argues that although the results certainly do not imply that the strong form of martingale hypothesis holds for all investors and for all time, they provide strong evidence in support of that hypothesis.These analysts are extremely well endowed.They operate in securities markets daily and have wide-ranging contacts and associations in both business and financial communities.They are apparently unable to forecast returns accurately enough to recover their research and transactions costs is a striking piece of evidence in favor of the strong form of martingale hypothesis as far as the extensive subset of information available to these analysts is concerned.

The research designs or methods seemedinadequate because empirical tests of random walk model are tests of fair game properties are more strongly in support of model than tests of additional and from viewpoint of expected return market efficiency, superfluouspure independence assumption.It is equally surprising that evidence against independence of returns over time is as weak as it is.First, the efficient markets model is the hypothesis that security prices at any point in time fully reflect all available information.It is an extreme null hypothesis although it is argued that the model stands up quite well to the data .

It is not expected to be literally true.Dividing the tests into weak, semi-strong and strong form allows one to pinpoint the information level at which the hypothesis breaks down. There is no important evidence opposing the hypothesis in the weak and semi-strong form tests and only limited evidence against the hypothesis in strong form tests.Kendall, Working, and Roberts research that series of speculative prices may be well described by random walks was based on observation.None of them provided much economic rationale for the hypothesis and Kendall felt that economists would usually reject it.Osborne's work suggested market conditions similar to those assumed by Bachelier would lead to a random walk.However, in his model, the independence of successive price variations comesfrom the assumption that investor’s decisions in an individual security are independent from each transaction farfetched from an economic model.There is an awareness that the fair game assumption is insufficient to lead to a random walk.Fair game models imply impossibility of different kinds of trading systems.Tests of serial covariances of returns are critical.It is shown that the serial covariances of a fair game are zero like a random walk.Thus, these tests are also relevant for the expected return models.The fair game model does not necessarily imply that the serial covariances of one-period returns are zero.It is assumed that the expected return and the entire distribution of returns are stationary through time in random walk.This implies estimating serial covariances by taking cross products of deviations of observed returns from overall sample mean return.This procedure which represents a rather gross approximation from viewpoint of general expected return efficient markets model does not seem to greatly affect the results of the covariance tests for common stocks.There is no evidence of significant linear dependence between lagged price variations or returns.The measured serial correlations are always close to zero in absolute terms.It is unlikely that the small absolute levels of serial correlation always observed can be used as the basis of substantially profitable trading systems.Zero serial covariances are consistent with a fair game model but there are kinds of nonlinear dependence that imply existence of profitable trading systems and yet do not imply nonzero serial covariances.Alexander's work concludes that there is some evidence in his results against independence assumption of random walk model.On the contrary, market efficiency does not need a random walk and from the perspective of the submartingale model, the conclusion that filters cannot beat buy-and-hold is support for the efficient markets hypothesis. The weight of the empirical evidence is such that economists would usually agree that whatever dependence exists in historical returns series cannot be utilized to make profitable predictions of the future.There is little in evidence that would cause rejection of stronger random walk model as a good first approximation for returns that cover periods of a day or longer.The nature of the distribution of price variations is a critical issue for efficient markets hypothesis because it affects both the kinds of statistical tools relevant for testing hypothesis and interpretation of any results obtained.Bachelier who assumed that price variations from transaction to transaction are independent, identically distributed random variables with finite variances first proposed the model implying normally distributed price variations.Central Limit Theorem leads us to expect that price variations will have normal or Gaussian distributions if the transactions are fairly uniformly spread across time and the number of transactions per day, week, or month is very large.Instead, high tails are observed in data distributions versus what would be expected in normal distributions.It is suggested that these departures from normality could be explained by a more general form of the Bachelier model.The limiting distributions for price variations over longer differencing intervals could be any member of the stable class including the normal as a special case if it is not assumed that distributions of price variations from transaction to transaction have finite variances.Non-normal stable distributions have higher tails than normal and can account for this empirically observed price variations distributions.It is concluded that non-normal stable distributions describe common stocks daily returns distributions better than the normal after extensive testing.Economists have been reluctant to accept the results because of the wealth of statistical techniques available for dealing with normal variables and relative paucity of such techniques for non-normal stable variables.Tests of fair game and random walk properties appear to go well when conditional expected return is estimated as average return for data sample on common stocks.Variation in common stock returns about their expected values is so big relative to any variations in expected values that the latter can be ignored safely.Roll demonstrates this result does not apply for Treasury Bills. Testing the fair game model on Treasury Bills need explicit economic theory for evolution of expected returns through time.Roll uses three existing theories of the term structure and two market segmentation hypotheses for this purpose.After extensive testing, Roll concludes that:

(i) The two market segmentation hypotheses fit the data better that the pure expectation hypothesis with a slight advantage for the liquidity preference hypotheses;

(ii) the market for Treasury Bills is efficient.

His results would not be so strongly in support of the efficient markets model if he simply assumed that his data distributions were normal. Thus, accounting of observed high tails of data distributions significantly affected the interpretation of the results.Single security tests are carried out for weak form tests supporting the fair game efficient markets model.Only the price or return histories of individual securities are examined for evidence of dependence that might be used as the basis of a trading system for that security.Tests of whether securities are suitably priced against each other is not discussed.An economic theory of equilibrium expected returns is needed to determine whether variations between average returns are suitable.The goal of the Sharpe-Lintner model is to determine the extent to which returns on a given security are related to the returns on other securities.Variation through time in one-period riskless interest rates is probably frivolous relative to variation in other factors affecting monthly common stock returns, thus more powerful statistical methods would be essential to study the effects of variations in the riskless rate on a monthly basis.The only way to produce the strong empirical conclusions about the Sharpe-Lintner model is to test it directly.Any alternative model of equilibrium expected returns must be kind of consistent with the market model given the evidence to support it. Semi-strong form tests of efficient markets models are about whether current prices fully reflect all obviously publicly available information.Each individual test is concerned with adjustment of security prices to one kind of information generating event like stock splits, announcements of financial reports by firms, new security issues etc.Each tests only brings supporting evidence for the model that by accumulating such evidence the validity of the model will be established.Splits and adjustment of stock prices to new information is examined.Security returns around split dates is examined to check first if there is any unusual behavior and, if so, to what extent it can be accounted for by relationships between splits and other more fundamental variables.It appears that firms tend to split their shares during periods when prices of their shares have increased more than would be implied by their normal relationships with general market prices which probably reflecta sharp improvement relative to the market in the earnings prospects of these firms sometime during the years immediately preceding a split.Subsequent to the split there is no net movement up or down in the cumulative average residuals although the behavior of post-split returns will be very different depending on whether or not dividend increases occur despite of the fact that a large majority of split securities do experience dividend increases when all splits are scrutinized together.Thus, the market makes unbiased forecasts of implications of a split for future dividends and these forecasts are fully reflected in the security prices by end of the split month.The results support the conclusion that the stock market is efficient with respect to its ability to adjust to information implicit in a split.Other studies of public announcements are done.This is carried out using variants of method of residual analysis to study the effects of different types of public announcements and all of them also support the efficient markets hypothesis.The available semi-strong form evidence on effect of various kinds of public announcements on common stock returns is all consistent with the efficiency markets model.The strong point of the evidence is its consistency rather than its quantity.

Few different kinds of public information have been examined although those treated are among the obviously most important.The amount of semi-strong form evidence is big compared to strong form tests available.

The opinion regarding the quality and importance is the theory of efficient markets is about whether prices at any time fully reflect available information.The theory only has empirical content within the context of a more specific model of market equilibrium: a model that specifies market equilibrium nature when prices fully reflect available information.All of the available empirical literature is implicitly or explicitly based on assumption that market equilibrium conditions can be stated in terms of expected returns.This assumption is basis of expected return or fair game efficient markets models.The empirical work can be separated into three categories depending on the nature of information subset of interest. Strong-form tests are concerned with whether individual investors or groups have monopolistic access to any information related to price information.Semi-strong-form tests include all obviously publicly available information.Weak form tests is just historical price or return sequences.Weak form tests of efficient market model are most voluminous with the results strongly in support.Some of the statistically significant evidence for dependence in successive price changes or returns has been found consistent with fair game model and the rest does not appear to be sufficient to declare the market inefficient.There isn't much evidence against the fair game model's more ambitious random walk at least for price variations or returns covering a day or longer.There is consistent evidence of positive dependence in daily price variations and returns on common stocks and dependence can be used as the basis of marginally profitable trading rules.The dependence appears as serial correlations that are consistently positive and also consistently close to zero and as a slight tendency for observed numbers of runs of positive and negative price variations to be less than numbers that would be expected from a purely random process.The dependence also appears in the filter tests and as a tendency for very small filters to produce profits in excess of buy-and-hold.Any systems like the filters that attempt to turn short-term dependence into trading profits of necessity produce many transactions that their expected profits would be absorbed by the minimum commissions (security handling fees) that floor traders on major exchanges must pay.This positive dependence does not appear of sufficient importance to warrant rejection of the efficient markets model using a less than completely strict interpretation of market efficiency.Other existing evidence of returns dependence gives interesting insights into process of price formation in the stock market, whereas it is irrelevant for testing the efficient markets model.Large daily price changes tend to be followed by large changes with unpredictable sign.Important information cannot be completely evaluated immediately and that the initial first day's adjustment of prices to the information is unbiased which is enough for the martingale model.A tendency is found toward excessive reversals in common stock price varies from transaction to transaction.This is because of the mechanism with orders to buy and sell at market are matched with existing limit orders on the specialist's books.There appears to be no way it can be utilized as the basis of a profitable trading rule with the way this tendency toward excessive reversals arises.The results are a strong refutation of theory of random walks at least applied to price variation from transaction to transaction whereas they do not form refutation of the economically more relevant fair game efficient markets model.Semi-strong form tests in which prices are assumed to fully reflect all obviously publicly available information have also supported the efficient markets hypothesis.The information in stock splits involves the firm's future dividend payments is on average fully reflected in price of a split share at time of the split.Similar conclusions are drawn with respect to information contained in:

(i) Annual earnings announcements by firms and

(ii) new issues and large block secondary rises of common stock.

Strong-form efficient markets model is one in which prices are assumed to fully reflect all available information is probably best known as a benchmark against which deviations from market efficiency can be judged.Two such deviations have been observed.First, specialists on major security exchanges have monopolistic access to information on unexecuted limit orders and use such information to generate trading profits.It is questionable whether the market making function of the specialist could not as effectively be carried out by some other mechanism that did not imply monopolistic access to information.Second, corporate insiders often have monopolistic access to information about their firms.Currently, corporate insiders and specialists are the only two groups whose monopolistic access to information has been documented.There is no evidence that deviations from the strong form of the efficient markets model pervade down any further through investment community.The efficient markets model appears to be a good first and second approximation to reality for the purposes of most investors.

The topic should be further studiedbecause the evidence in support of efficient markets model is extensive, and contradictory evidence is sparse.In successful scientific research, knowing having been in the past allows one to pose and hopefully answer an even more interesting set of questions for the future.The most pressing field of future attempt is the development and testing of models of market equilibrium under uncertainty.A more substantial framework for more sophisticated intersociety tests of market efficiency is possible when the process generating equilibrium expected returns is better understood assuming some expected return model is relevant.Using the right probability distribution and risk measure during crisis is crucial for portfolio optimization.

Markowitz (1991)’s research problem is the work on portfolio theory considers how an optimizing investor would behave whereas the work of Sharpe and Lintner on Capital Asset Pricing Model (CAPM) is about economic equilibrium assuming all investors optimize in a particular manner the author proposed.The author's work and Sharpe and Lintner provide parts one and two of a microeconomicsof capital markets respectively.Sharpe will talk about CAPM whereas the author will be limited to portfolio theory.

The key findings of previous research are there are three major ways in which portfolio theory differs from theory of the firm and theory of consumer.First, it is about investors not manufacturing firms or consumers.Second, it is about economic agents acting under uncertainty.Third, it is a theory able to be used to direct practice, at least by large e.g. institutional investors with enough computer and database resources.

The important gaps in knowledge and researchare the theory of the producer assumed that the competitive firm knows the price it will be selling the goods it produces.There is a delay between decision to produce, time of production and time of sale in the real world. The price of the product at time of sale may be different from that which was expected when the production decision was made.This uncertainty of final sales price is important in actual production planning but was ignored in classical economic models.Levy and Markowitz measure the efficacy of f(E, V) by correlation between it and EU.Y. Simaan defines optimization premium to be the percent investor would be just willing to pay from the portfolio for privilege of choosing the true EU maximizing the portfolio not confined to the mean-variance.The reason to perform a mean-variance analysis not a theoretically correct expected utility analysis is convenience, cost or feasibility.It is much more expensive to find a utility maximizing portfolio than to trace out an entire mean-variance frontier.Data requirements for an expected utility analysis can be much more than those of a mean-variance analysis because estimates of first and second moments generally are insufficient for the the former.Determining the investor's utility function can be a problem.

Simaan's criteria measures the worth paid out of the portfolio of added expenses of finding an EU maximizing portfolio as a percent of the portfolio.He solves for the optimization premium analytically give specific assumptions.L. Ederington evaluates EU approximations with thousands of synthetic time series produced by randomly selecting from actual time series.He evaluates approximations except the first three of four moments and uses the first two.Theoretically, more moments are better than fewer but how much ?

Ederington finds that for some utility functions the mean-variance approximation is so good that there is virtually no room for improvement.Ederington finds that three moments provides little improvement to approximation whereas four moments improves the approximation considerably.

The trends and themes of the research related to the topic are microeconomics studied forty years ago involved how optimizing firms and consumers would behave and the nature of the economic equilibrium resulting from such behavior.The critical line algorithm for tracing out the efficient frontier given estimates of expected returns, variances and covariances for any number of securities subject to various types of constraints was published in 1956 by the author.The relationship between the mean-variance analysis and the fundamental theories of action under risk and uncertainty of Von Neumann and Morgenstern and L. J. Savage was explored by the author's 1959 book.Sharpe, Blume, King, and Rosenberg greatly clarified the problem of estimating covariances starting in the 1960s.Several analysts reported success in using publicly available accounting figures, combined with security analysts' earnings estimates, to estimate expected returns.Their estimates do not eliminate uncertainty but on average, securities with higher estimates outperform those with lower estimates.

The relationships between key concepts are uncertainty cannot be disregarded so easily in analyzing optimizing investor behavior.An investor who knew future returns with certainty would invest in only one security with the highest future return.The investor would be indifferent between any of these, or any combination of these if several securities had the same, highest, future return.The investor would not prefer a diversified portfolio.However, diversification is a common and reasonable investment practice to reduce uncertainty.The existence of uncertainty is necessary to rational investment behavior analysis.A rational agent acting under uncertainty would act according to probability beliefs where no objective probabilities are known and these probability beliefs or subjective probabilities combine exactly as do objective probabilities.It is unclear and irrelevant whether the probabilities, expected values, etc. discussed are for subjective or objective distributions assuming this.John Burr Williams, The Theory of Investment Value, proposed that value of a stock should equal present value of its future dividend stream. Dividends are uncertain thus a stock is valued as the expected value of its discounted future dividend stream.The investor must also be only interested in the expected value of the portfolio if the investor is concerned only with expected value of securities.One just needs to invest in one security with maximum expected return or one such if several tie for maximumto maximize the expected value of a portfolio.Action based on expected return only like action based on certainty of the future must be rejected as descriptive of actual or rational investment behavior.Investors are concerned with risk and return and these should be measured for the portfolio as a whole.Variance or equivalently standard deviation is a measure of risk of the portfolio.Variance of the portfolio, that is the variance of a weighted sum, involved all covariance terms led to its plausibility.There were two criteria, expected return and risk, where the investor would select a point from the set of Pareto optimal expected return, variance of return combinations known as the efficient frontier.The question on whether mean and variance are proper and sufficient criteria for portfolio choice needs to be answered.The theory of rational choice under uncertainty is considered.The third way in which portfolio theory is different from classical microeconomic theory of the firm or consumer should be recalled.A set of rules which investors can be followed will be sought.An approximate method which is computationally feasible to a precise one which cannot be computed is preferred.This is the difference between Kenneth Arrow's work on economics of uncertainty and the author's.Kenneth Arrow sought a precise and general solution.The author sought a good approximation that could be implemented.Both lines of inquiry are valuable.The discussion of principles of rational behavior under uncertainty in Part IV of the author's book begins with a variant of L. J. Savage's axioms.One should select a strategy which maximizes expected utility for a many-period game from such axioms.This implies that the investor should act each period to maximize expected value of a single period utility function.This single period utility function may depend on portfolio return and other state variables.It is assumed that it depends only on portfolio return.The question to be answered is if you know the expected value and variance of a probability distribution of return on a portfolio can you guess fairly closely its expected utility?

The inconsistency is it is uncertain whether the mean-variance efficient frontier with appropriate risk measure like expected shortfall can be complemented with the appropriate probability distribution function to be used consistently during crisis.

The research designs or methods seemed improper, insufficient, or inadequate because not all expected maximizers are equally catered by mean-variance approximations.Levy and Markowitz have two observations about an expected utility maximizer with U = -e-10(1 + R).First, an investor who had -e-10(1 + R) as his or her utility function would have some really strange preferences among probabilities of return.He or she would not insist on certainty of return.Second, even if some unusual investor did have the questioned utility function such an investor could determine in advance that f(E,V) was not a good approximation for this EU.Levy and Markowitz present other empirical results too.They explain the difference between assuming that an investor has a quadratic utility function against using a quadratic approximation to a given utility function to develop an f(E,V) approximation.They show that f(E,V) is not subject to Arrow, Pratt objection to a quadratic utility function that it has increasing risk aversion.Levy and Markowitz show that a large class of f(E,V) approximations have the same risk aversion in the small and original EU maximizer.

The opinion regarding the quality and importance is thetheory of rational behavior under uncertainty can continue to provide insights to which practicable procedures give near optimum results.It can further assist in evaluating the adequacy of mean and variance or alternate practical measures as criteria.

The topic should be further studied because first, all the experimentation and analysis to now is rather spotty of where mean-variance works and where it does not.Developing a more systematic characterization of the utility functions and distributions for which the mean-variance approximation is good, bad and marginal.Second, if the investor has a utility function for which mean-variance gives a close approximation and the investor does not know precisely his choice.The investor need not determine his or her utility function to obtain a near optimum portfolio.He or she need only pick carefully from one-dimensional curve of efficient EV combinations in the two dimensional EV space.The investor must select carefully from a three-dimensional surface in a four-dimensional space to pursue a similar way when four moments are needed.This causes serious operational problems itself although we overcome computational problems because of nonconvexity of portfolios sets with given third moment or better.Maybe some other measure of portfolio risk will be a two parameter analysis for some of the utility functions which are a problem to variance.Until now, no one has determined whether there is a substantial class of utility functions for which mean-semi-variance succeeds while mean-variance does not provide a sufficient approximation to EU.Third, the derived, single period utility functions can contain state-variables and return or end of period wealth.Expected utility can be estimated from return and state-variable means, variances and covariances given that utility is approximately quadratic in the relevant area.No one has investigated such quadratic approximation in which state variables other than portfolio value are needed in practice.

Bouchaud et al. 1998’s research problem is a method of optimization of asset allocation where stock price fluctuations have fat tails represented by power laws.Three distinct elements of risk by three different components of the distributions of price changes are distinguished generalizing over previous works with stable Levy distributions.They are: unexpected gains to be kept, harmless noise intrinsic to financial activity, and unpleasant losses, the only element to be minimized.The independent treatment of distributions tails for positive and negative changes and generalization to large events of covariance of two random variables give explicit formulae for optimal portfolio.Probability of loss or equivalently Value-at-Risk as key quantity to study and minimize gives a simple option to optimization of asset allocations in general where the characteristic exponents are different for each asset.

The key findings of their research are the strength of the efficient market hypothesis is its conceptual and mathematical simplicity.It is on the elementary ideas of mean and variance, the algebraic manipulations are eased by the hypothesis of Gaussian statistics with a score of mathematical results available.The power-law hypothesis may force one to dump the idea of mean and variance to find different, less elementary objects as building-blocks of the theory.The mathematical tool-box is also much more involved and somewhat poorer. Previous theoretical approaches of asset allocation developed for stable Levy laws is extended to more general situations where power laws hold only on the distribution tail, that is, on the most noteworthy events which have an immediate meaning for all operators on the financial markets, or are characterized by different exponents possibly larger than 2, that is are not stable in the Levy sense.Replacing the standard return/risk ratio by a return/probability of large losses ratio is proposed.Risk should be decomposed into three distinct components: unexpected gains to be kept, harmless noise intrinsic to financial activity and unpleasant losses which is the only part to be minimized.Notations are introduced and class of probability distribution or probability density that are adopted for the description of real data are specified.A generalization of the concept of covariance of two random variables adapted to large events is proposed.Important results on addition of power-law distributed random variables will be recalled with heuristic proofs given and used to establish results on optimal portfolio in extended sense mentioned.Applicability of these ideas is discussed and concrete example of portfolio optimization is presented. Diversification of extreme risks is possible when large events are to some extent uncorrelated.A critical step for obtaining a useful portfolio theory is to generalize the usual idea of covariance to tail of the distributions.The toolbox available for determination of statistical properties of sums and products of μ-variables must be used because any definition of covariance will involve products and linear combinations of price variations of different assets.

The important gaps in knowledge and data are efficient market hypothesis suggests that market prices are unpredictable, prices vary randomly and no consistent positive profit can be derived from speculation.The example of stock price fluctuations is the multivariate normal distribution.The standard efficient portfolio theory results in several difficulties in practice.The weights of the different assets must be revised all the time and the optimal portfolio usually retain a small fraction of all assets, an aspect unreasonable and impractical by many operators.

The trends and themes of the research related to the topic are many natural phenomena must be described by power law statistics has only been fully accepted in the past ten years despite early insightful studies.Intense activity has developed to understand both physical content of mathematical tools devised by Levy and others and source of these ubiquitous power law tails.This has resulted in interesting ideas like the seminal concept of self-organized criticality.It is in economy and finance that these power law distributions were first noted by the works of Pareto and in the early sixties by the works of Mandelbrot and Fama.Their works were initially discarded and did not have a large practical impact until the standard model of efficient markets was rapidly developing with significant successes like the Capital Asset Pricing Model (CAPM) or Black-Scholes option pricing theory.The power-law or paretian hypothesis remained asleep until recently in spite of the continuing confirmation that price variations distributions and other commodities present a strong leptokurtosis abnormally large ratio, sometimes much bigger than 3 characterizing Gaussian statistics of the fourth moment over second moment squared indicating abnormally large variations.The recent revival of such ideas in finance is partly because of the 1987 crash and more recent smaller ones spotlighting the crucial importance of large, catastrophic events and limitation of Gaussian theories, dramatically underestimating their probability of occurrence.The idea of value-at-risk (VaR) has become central quantity to assess risk.A correct calculation of this VaR and its necessary control needs a new theoretical tools.The possibility that stable Levy distributions could present price variations more accurately than normal distribution from a series of authors have been explored.Such analysis of stock market variations consistently shows that a Levy distribution or more precisely a truncated Levy distribution of returns is a better representation of the data.

The relationships between key concepts are the change in the value of a given asset is denoted by ν (daily, weekly, monthly, etc).Any progress on asset allocation relies on determination of mathematical form for probability density P(ν) of ν's because it controls the existence or not of mean return and variance.The repeated observations of a strong anomalous leptokurtosis leads to the proposal that Gaussian paradigm should be replaced by Levy paradigm.Thinking of Levy laws is natural because they are attributed by power-law tails and are stable as is the Gaussian distribution with respect to the addition of variables.

The first property gives hope in considering the large observed fluctuations and relating leptokurtosis.The second property ensures that the mathematical description is invariant with respect to the chosen time step whether daily, weekly, monthly and so forth leading to the property of stationarity.The probability density P(ν) is defined by a power law with scale factor for positive or negative price variations by C+/C-, exponent of the tail of probability density for positive or negative price variations by μ+ or μ-.It can be identified with index α in particular where P(ν) is a Levy distribution Lα when μ+ or μ-.Recent empirical studies show that an appropriate description of price changes involve Levy distributions.This can be too restrictive and a more flexible description might be needed requiring power tails with exponents bigger than 2 which is unstable or rather cross-overs from a power law to an exponential behavior resulting in a truncated Levy distribution.It is interesting that the limit μ → ∞ corresponds to exponential tails.Rank ordering technique is used to retrieve the values of exponents and scale parameters for implementation of the proposed asset allocation strategy.The scale coefficient C+/- reflects the scale of fluctuations of price variations ν.The power law structure of probability density P(ν) is a self-similar process for which no intrinsic characteristic scale exists.There is no precise model proposed to explain this in finance.It appears to be a quite reasonable assumption at least in a restricted range of variations by analogy with other collective systems.This assumption may break down at large variations showing the possible existence of characteristic scales.The important practical implication is that if N is not very large these basic parameters for determination of an optimal portfolio within the usual Gaussian framework are varying with time leading to very strong instabilities in the optimal weights. A particular model of correlation between all assets which is a generalization of the known one factor β-model is considered.Next, the generalized multi-factor model is considered.

It composes the natural generalization of the calculation of covariance in power-law distributions of asset price variations.Now, the extreme risk optimized portfolio can be determined.

Thecontradictions are the central limit theorem is used as a strong argument for the standard approach to portfolio optimization.However, the Gaussian limit is reached when V\* grows as σ √log(M) when M tends to infinity leaving the Gaussian as the limit distribution very slowly.For μ < 2, the weight increases with M signaling the breakdown of Gaussian convergence and attraction to the μ-stable Levy distribution.

The research designs or methods seemedinadequate because the optimal portfolio theory for strongly fluctuating assets is determined.The existence of stable Levy distributions and exponential tails can lead to mean return and variance determination which could be ill-conditioned.The asymmetry of distributions is automatically done by considering separately tails for large positive and large negative variations ν.The rather delicate problem of determination of asymmetry parameter of stable Levy distributions best fitting the asset time series is avoided.The optimal portfolio is characterized by set of portfolio asset weights such that amplitude of large loss of the distribution of the portfolio returns minimized.Another option is to minimize the amplitude of loss from portfolio return distribution while maximizing the probability of large gains proportional to amplitude of gain from portfolio return distribution.This gives the generalization of the efficient frontier in usual portfolio theory known as tail chiseling technique where the frequency of very large, unpleasant losses is minimized for certain level of return.There is a fundamental difference although it looks quite similar to previous works.There is no minimization of global coefficient Cp of stable Levy distribution as best representation of whole distribution of price variations.It follows the idea that it is better to have less fluctuations in gains and losses than to let large losses coexist with large gains.It focuses on the scale parameter weighting the large losses.This is similar to minimizing the Value-at-Risk (VaR), λ, of the portfolio.This is justified by common observation that losses or gains from behavior of portfolio is over a small fraction of total investment period.Defining a portfolio strategy on importance of large gains and losses should focus correctly at part of the market moves significant for behavior of the portfolio.An alternative definition of average return Ri of the ith asset is determined by tails of the distribution.Another interesting criterion to determine the optimal portfolio could be to minimize amplitude of loss for a given value of amplitude of gain in the portfolio return distribution.Generalization of Sharpe's ratio is the quality factor, Q.Different loss tolerance levels λ will correspond to different asset allocations.The efficient frontier for the portfolio with 18 different assets has common roots with safety first criterion consisting of minimizing the probability of loss larger than some predetermined value or equivalently of minimizing admissible loss threshold, safety level or Value at Risk, with a given low probability of tolerance.The general multi-factor case is considered.The Markowitz formulae and exponential tail distributions are considered.

The opinion regarding the quality and importance is many studies indicate the existence of power-law tails in return probability distributions with rather low value of index usually cut-off above a certain characteristic value.The usual notion of variance, although formally well defined, may be irrelevant to the situation because it is dramatically sensitive to large events.The expected return is not very well defined.It is proposed to substitute these quantities by their natural dialogues in case of power-law distributions.Rank ordering to extract tail covariance have been suggested.Explicit formulae for optimal portfolio in an extended sense based on minimization of the probability of large losses is proposed. Treating seperately the tails for positive and negative variations allowing an independent minimization of the probability of large losses keeping the potential for large gains is proposed.Similar ideas were proposed for option pricing in the power-law world.

Risk should be decomposed into three parts with probability of loss as key quantity to study and minimize, provides a simple option to optimization of asset allocations where exponents are different for each asset, or for more general distributions.Optimal portfolio with Value at Risk is different for the Gaussian (Markowitz) optimal portfolio.

The topic should be further studied because how to remove harmless noise intrinsic to financial activity should be studied.The appropriate probability distribution with the right risk measures should be determined for portfolio optimization during crisis.

Cont (2001)’s research problem is a set of stylized empirical facts from statistical analysis of price changes in different kinds of financial markets is presented.Some general issues popular to all statistical studies of financial time series are first discussed.Different statistical properties of asset returns are described:distributional properties, tail properties and extreme fluctuations, pathwise regularity, linear and nonlinear dependence of returns in time and across stocks.Properties popular to a wide variety of markets and instruments are emphasized.It is shown how these statistical properties invalidate many of the common statistical ways to study financial data sets and scrutinize some of the statistical problems in each case.

The key findings of their research are a pedagogical overview of the stylized facts is presented.The aim is to focus more on properties of empirical data than on those of statistical models and give new insights provided by methods on statistical techniques applied in empirical finance recently.The goal is to let the data speak for themselves as much as possible.This is done by using non-parametric methods making only qualitative assumptions about properties of stochastic process generating data with statistical methods.

There is no assumption that they belong to any prespecified parametric family.The shape of the singularity spectrum is not dependent on asset considered.All series show the same, inverted parabola shape also observed on the USD/DEM high-frequency exchange rate data using structure function method.The non-trivial spectrum is very different from what is expected from diffusion processes, Levy processes or jump-diffusion processes, used in continuous-time finance with the singularity spectrum theoretically known.There is no discontinuous jump component in the signal because the Hoelder exponent does not extend down to zero.The rare examples of stochastic processes for which singularity spectrum is like the one observed in empirical data are stochastic cascades or their causal versions, multifractal random walks.Applying same techniques to Monte Carlo simulations of different stochastic models in finance to check whether peculiar shapes of spectra obtained are not artefacts because either to small sample size or discretization can be done.Preliminary results seem to rule out such possibility.Three different multifractal formalism give similar estimators for singularity spectrum.

The important gaps in knowledge and research are study of these new data sets has led to conclusion of some old contentions about the nature of data and has generated new challenges.The ability to capture in a synthetic and meaningful fashion the information and properties in this big amount of data is critical.Stylized facts is a set of properties, common across many instruments, markets and time periods is observed by independent studies. Non-parametric methods have the great theoretical advantage of being model free but can only provide qualitative information on financial time series and to get a more precise description semi-parametric methods without fully specifying the form of the price process imply existence of a parameter describing a property of the process.One disadvantage of the singularity spectrum is finite sample properties are not well known.This can cause one to believe that one will generically get non-trivial multifractal spectra even if data set studied is not multifractal in which the results indicated should be interpreted with great caution.The convergence of the empirical spectrum to its true value can be quite slow even in genuine multifractal process.

The trends and themes of the research related to the topic are statistical properties of prices of stocks and commodities and market indexes have been studied with data from different markets and instruments for more than half a century.The availability of large data sets of high-frequency price series and application of computer-intensive methods to analyze their properties have opened new horizons to researchers in empirical finance in the last ten years and have contributed to consolidation of data-based approach in financial modelling.The risk attribute of a financial asset is related to the irregularity of variations of its market price.

The risk is directly related to the (UN) smoothness of the trajectory and is crucial of empirical data what one would like a mathematical model to reproduce.Each stochastic models class produces sample paths with certain local regularity properties. The local regularity of the sample paths should try to reproduce empirically observed price trajectories for a model to show sufficiently the intermittent character of price fluctuations.

The concept of a singularity spectrum of a signal was introduced to give a less detailed and more stable characterization of local smoothness structure of a function in a statistical sense.

It has been shown that the singularity spectrum is the same for almost all sample paths for large classes of stochastic processes.Multifractal formalism enables singularity spectrum to be calculated from sample moments of increments.

The relationships between key concepts are many market analysts have used event-based method to explain or rationalize a given market movement by relating it to an economic or political event or announcement revealed by a causal examination of most financial newspapers and journals.Different assets are not necessarily influenced by same events or information sets.Price series from different assets and a fortiori from different markets will show different properties.Result of more than half a century of empirical studies on financial time series shows that if one scrutinizes their properties from a statistical point of view, the random changes of asset prices share some quite non-trivial statistical properties.They are common across a big range of instruments, markets and time periods known as stylized empirical facts.Stylized facts are obtained by taking a common denominator amidst the properties observed in studies of different markets and instruments.One gains in generality and loses in precision of statements about asset returns.Stylized facts are commonly formulated in terms of qualitative properties of asset returns andmay not be precise enough to distinguish among different parametric models.These qualitative stylized facts are so constraining that it is not easy to show even an ad hoc stochastic process possessing the same set of properties and one has to go to great lengths to reproduce them with a model.The following are a set of stylized statistical facts common to wide set of financial assets:

1.Absence of autocorrelations: linear autocorrelations of asset returns are usually insignificant excluding very small intraday time scales ≈ 20 minutes for microstructure effects in play.

2.Heavy tails: unconditional distribution of returns display a power-law or Pareto-like tail with finite tail index, higher than two and less than five for most data sets studied.This excludes stable laws with infinite variance and normal distribution.The precise form of the tails is hard to determine.

3.Gain/loss asymmetry: large drawdowns in stock pricesand stock index values, not equally large upward movements.This does not apply to exchange rates with higher symmetry in up/down moves.

4. Aggregation Gaussianity: The distribution appears like normal distribution when the time scale ∆t is increased over which returns are calculated.Distribution shape is not the same at different time scales.

5. Intermittency: a high degree of variability is shown by returns at any time scale. This is quantified by irregular bursts in time series of a wide choice of volatility estimators.

6.Volatility clustering: different measures of volatility show a positive autocorrelation over several days quantifies that high-volatility events tend to cluster in time.

7. Conditional heavy tails: residual time series still show heavy tails after correction of returns for volatility clustering like via GARCH-type models.

8.Slow decay of autocorrelationin absolute returns: the autocorrelation function of absolute returns decays slowly as function of time lag approximately as power law with exponent β ϵ [0.2, 0.4] sometimes interpreted as long-range dependence.

9. Leverage effect: most measures of volatility of an asset are negatively correlated with returns of that asset.

10.Volume/volatility correlation: trading volume has correlation with all measures of volatility.

11.Asymmetry in time scales: coarse-grained measures of volatility can predict fine-scale volatility better than vice versa.

The regularity of a function may be characterized by its local Hoelder exponents.

The inconsistency is it is uncertain whether the stylized facts highlighted can be applied during crisis to obtain the correct portfolio optimization with the right probability distribution and risk measure.

The research designs or methods seemedinadequate because most basic condition of any stastical analysis of market data is existence of some statistical properties of data under investigation remaining stable over time.Invariance of statistical properties of return process in time is related to stationarity hypothesis.It is unclear whether returns in calendar time verifies it: seasonality effects like intraday, variability, weekend effects etc.There is a need to ensure that empirical averages do converge to quantities they are estimating though stationarity is essential to ensure one can mix data from different timings to estimate moments of the returns.Ergodicity is essential to ensure time average of quantity converges to its expectation.It is usually satisfied by IID observations not conspicuous.A statistical estimator defined as a sample average does not need to be equal to quantity it estimates defined as a moment of theoretical distribution of observations.Hypothesis testing in financial econometrics where one calculates the likelihood of a model to hold given value of statistic and rejects or accepts the model by comparing test statistic to threshold value.Normal distribution is insufficient for modelling marginal distribution of asset returns and their heavy-tailed character.Non-Gaussian attributes of price changes distribution has been repeatedly observed in different market data.Kurtosis and skewness are insufficient for identifying returns distribution leaving considerable margin for distribution option.Fitting different functional forms to distribution of stock returns and stock price variations is common. Parametric models like normal distribution, stable distributions,Student distribution,hyperbolic distributions,normalinverseGaussiandistributionsand exponentially truncated stable distributions are some.A parametric model must have minimum four parameters: location parameter, scale (volatility) parameter, parameter describing decay of tails and asymmetry parameter permitting left and right tails to have different behaviours for it to successfully reproduce all above properties of marginal distributions.Normal inverse Gaussian distributions, generalized hyperbolic distributions and exponentially truncated stable distributions meet such requisites.Non-Gaussian distribution makes it essential to use other measures of dispersion besides standard deviation to capture variability of returns.Higher-order moments or cumulants can be measures of dispersion and variability.Such moments need to be well defined given heavy-tailed nature of distribution.The tail index k of a distribution may be defined as order of finite highest absolute moment.Representing the sample moments or cumulants as function of sample size n is suggested.The sample moment will finally reach the region defined around its theoretical limit and fluctuate around that value if the theoretical moment is finite.The sample moment will either diverge as function of sample size or exhibit erratic behaviour and large fluctuations where the true value is infinite.This is applied to time series of cotton prices and conjectured that theoretical variance may be infinite because the sample variance did not converge to a particular value as the sample size increased and continued to change non-stop.The sample variance reaches to a limit value after a transitory phase of wild oscillations.The fourth moment behaves in a usually more erratic manner.The standard deviations of sample kurtosis involves a polynomial containing theoretical moments up to order eight!The eighth moment of distribution have a very large numerical value with the fourth moment change wildly.One of the important characteristics of financial time series is their high variability revealed by heavy-tailed distributions of their increments and non-negligible probability of occurrence of violent market movements.Theselarge market movements focus attention of market participants because their magnitude such that they are part of an important fraction of the return aggregated over a long period.The intermittent nature of financial time series and to model sufficiently the tails of the returns distribution is important. They are essential for calculating the Value-at-Risk needed to determine regulatory capital requirements.Extreme value theorem for IID sequence is discussed.Price movements in liquid markets do not show any significant autocorrelation.The absence of significant linear correlations in price increments and asset returns has been widely documented.Statistical arbitrage will reduce correlations excluding very short time scales representing the time the market reacts to new information if price changes show significant correlation.Traditional tools of signal processing based on second-order properties in time domain - autocovariance analysis, ARMA modelling or in spectral domain - Fourier analysis, linear filtering - cannot differentiate between asset returns and white noise.The need for nonlinear measures of dependence to characterize dependence properties of asset returns is crucial.Absence of autocorrelations gave some empirical support for random walk models of prices in which returns are independent random variables.Independence implies that any nonlinear function of returns will have no autocorrelation.Simple nonlinear functions for returns like absolute or squared returns show significant positive autocorrelation or persistence. Volatility clustering is about large price variations are more likely to be followed by large price variations.Log prices are not random walk.Empirical studies with returns from different indices and stocks show that this autocorrelation function is positive and decays gradually staying significantly positive over several days, sometimes weeks.This is the ARCH effect because it is a future of (G)ARCH models.It is model-free property of returns without any reliance on the GARCH hypothesis.Such persistence implies some degree of predictability for amplitude for motivation.The existence of such nonlinear dependence opposed to absence of autocorrelation in returns is commonly interpreted by stating there is correlation in volatility of returns but no returns themselves.Definition of volatility is model independent and volatility correlations are not observable whereas correlations of absolute returns can be calculated.Autocorrelation functions (ACF) were initially developed for analyzing dependence for Gaussian time series and linear models which they sufficiently capture the dependence structure of series under study.It is less conspicuous when considering nonlinear, non-Gaussian time series.Heavy-tailed feature of such time series can make interpretation of a sample ACF problematic.Sample ACF of heavy-tailed nonlinear time series can possess non-standard statistical properties which invalidate many econometric testing procedures for detecting or measuring dependence.There is a great deal of variability in sample autocorrelations of squared returns raising doubts about statistical significance of quantitative estimates derived from them.Estimators of autocorrelation function of returns and their squares can be highly unreliable and even where they are consistent they may have large confidence intervals associated with them for heavy-tailed time series like GARCH.One must be very careful when drawing quantitative conclusions from autocorrelation function of powers of returns.Statistical analysis of risk of management of portfolios with a large number of assets usually more than a hundred need information on joint distribution of returns of different assets.The key tool to analyses interdependence of asset returns is covariance matrix of returns.Heteroskedastic attribute of individual asset provide results in instability in time of covariances.Using sample covariance matrix as input for portfolio optimization by classical methods like mean-variance optimization and capital asset pricing model (CAPM) and arbitrage pricing theory (APT) where correlations between large number of assets are represented by a small number of factors is questionable.Conditional correlations can be defined by conditioning on an aggregate variable like market return before computing correlations to examine the residual correlations with common factors already accounted for.Covariance matrix, independently of the significance of its information content, is insufficient as a tool for measuring dependence because it is based on an averaging procedure emphasizing the centre of the distribution returns compare to correlations used for portfolio diversification are quite useful when stock prices have large fluctuations.A more relevant measure is the conditional probability of a large (negative) return in one stock with a large negative movement in a different stock.The two assets may have extremal correlations while their covariance is zero because covariance does not measure the correlation of extremes. The local Hoelder exponent is not a robust statistical too for characterizing signal roughness because it may change from sample path to a sample path in stochastic process.

The opinion regarding the quality and importance is the authors have tried to present a set of statistical facts which come from empirical study of asset returns and common to a big set of assets and markets.The properties mentioned are model free in that they do not result from a parametric hypothesis on return process but from general hypotheses of qualitative nature.

They should be constraints a stochastic process needs to verify to reproduce the statistical properties of returns accurately.Most currently existing models fail to reproduce all these statistical features at once showing that they are very constraining.

The topic should be further studied because several issues have not been discussed.One crucial question is whether a stylized empirical fact is necessary from an economic view point.Can these empirical facts be used to confirm or eliminate certain modelling methods used in economic theory?Another question is whether these empirical facts are relevant from a practitioner's standpoint.Does the presence of volatility clustering imply anything interesting from a practical standpoint for volatility forecasting? If so, can it be put to use to implement a more effective risk measurement or management approach?Can one exploit such correlations to implement a volatility trading strategy?How can one include a measure of irregularity like singularity spectrum or extremal index of returns in a measure of portfolio market risk?Further study is needed to get a suitable characterization of finite sample behaviour of the estimators proposed for different quantities of interest.

Surya and Kurniawan (2013)’s research problem is it talks about optimal portfolio selection problems using Expected Shortfall as the risk measure.The multivariate Generalized Hyperbolic distribution is the joint distribution for risk factors of underlying portfolio assets like stocks, currencies and bonds.The optimal portfolio strategy is found using the multivariate Generalized Hyperbolic distribution.

The key findings of previous research are the RORC optimization model is crucial because it is a tool to compare the performance of portfolios used with a benchmark portfolio.JKSE portfolio is used as the benchmark.The portfolio produced using Generalized Hyperbolic distribution model performs better than JKSE portfolio with performance measured by RORC with Expected Shortfall as risk measure.This shows that the optimization method used is superior to one used by JKSE.

The important gaps in knowledge and researchare financial data are often not normally distributed.They show properties that normally distributed data do not have.Empirical return distributions almost always show excess kurtosis and heavy tail.The logarithm of relative price variations on financial and commodity markets show a heavy-tailed distribution.A Levy process with Variance Gamma distributed increments to model log price processes is proposed.Variance Gamma is a special case of Generalized Hyperbolic (GH) distribution.Other subclasses of GH distribution were proven to give an excellent fit to empirically observed increments of financial log price processes especially log return distributions like Hyperbolic distribution, the Normal Inverse Gaussian Distribution and the Generalized Hyperbolic skew Student's t distribution.The student's t and normal distributions are limit distributions of GH.These are reasons for the popularity of Generalized Hyperbolic family distributions because they give a good fit to financial return data and are also extensions to the well-known student's t and normal distributions.It has a critical problem.It can lead to a centralized portfolio when applied to nonelliptical distributions though it is coherent for elliptical distributions against the diversity principle.It is also a generally nonconvex function of portfolio weights causing portfolio optimization to be an expensive computational problem.Expected shortfall (ES) is the more recent and commonly used risk measure.It was made popular because of VaR's downside.It always results in a diversified portfolio as a coherent risk measure compare to VaR.It takes into account the behaviour of return distributions at and beyond a selected point.It displays the behaviour of the distributions' tails with a much wider scope like VaR.These attributes make it more favourable than its classic counterpart and only this measure is focused.

The trends and themes of the research related to the topic are alternative risk measures must be considered because return data are nonnormality with heavy tails and volatility is not designed to capture extreme large losses.Value-at-Risk (Var) as a risk measure can satisfy this.It determines the point of relative loss level that is exceeded at a specified degree not measuring return deviations fromits mean.It can measure the behavior of negative return distributions at a point far away from the expected return when appropriately adjusted.It can take into account the extreme movements of assets return.It gives an easy representation of potential losses because it is none other than the quantile of loss distribution for a continuous distribution.

The relationships between key concepts are the Generalized Hyperbolic (GH) distribution is based on the Generalized Inverse Gaussain distribution(GIG) and Multivariate Normal Mean-Variance Mixture Distribution (MNMVM).The Generalized Hyperbolic distribution possess the linearity property.The general properties of Expected Shortfall relevant with uncertain loss in a portfolio because of the volatility of financial market is discussed.Risk measurement must account for the randomness of loss and be used to determine the capital reserve to account for future loss.The subadditivity and positive homogeneity attributes together imply the convexity of the Expected Shortfall which is very useful in dealing with portfolio optimization problems.The most critical aspect in portfolio optimization is modelling portfolio risk.The portfolio loss function is defined because the risk comes from portfolio loss value over the holding period.The risk factors are chosen to be logarithmic price of financial assets, yields or logarithmic exchange rates because the distribution models of their time increments have been empirically known.First-order approximation gives a convenient computation of loss because it shows loss as the linear combination of risk-factor changes.It is best used when risk-factor variations have small time horizon and when portfolio value is almost linear in risk factors.The loss of the stock portfolio is defined.The loss of the zero coupon bond portfolio is defined.The loss of the fixed coupon bond portfolio is defined.The loss of the currency portfolio is defined.The weight for assets valued in foreign currency varies slightly from weight of other domestic assets. This weight can be considered as usual weight multiplied by exchange rate of currency of which it is denominated.Such interpretation is consistent with conversion process of its foreign value to its base value.The distribution of the portfolio loss using the linearity property of Generalized Hyperbolic distribution is determined because the risk mapping for each of the portfolio's assets have been done.Only the linearized portfolio loss is considered because of this linearity property to ensure future calculations are tractable.One of the advantages of modelling risk-factor increments with Generalized Hyperbolic distribution as linearized portfolio is also Generalized Hyperbolic distributed. Portfolio loss and profit function are approximated by its linearized counterpart argued by linearity property of Generalized Hyperbolic distribution.Portfolio optimization for both symmetric and asymmetric cases are defined.

The inconsistency is that it is uncertain whether expected shortfall on generalized hyperbolic distribution will work well during crisis.

The research designs or methods seemedinadequate because numerical results for 4 stocks, 3 foreign currencies, 2 Indonesian government-issued zero coupon bonds and 1 Indonesian government-issued international fixed coupon bond are discussed.The base currency is IDR and the stocks are chosen from international blue-chip companies.First-order approximation of portfolio loss is accurate and robust, and is justified.Only the asymmetric Generalized Hyperbolic model is focused because portfolio optimization in symmetric framework can be solved analytically.An advantage of modelling with asymmetric model is that it gives an additional degree of freedom without fixing the parameter gamma to 0.From observing the calibration result whether value of gamma and its skewness are close to 0 or not will determine whether the better model is asymmetric or symmetric.Calibration results are percentage risk-factor increments of assets presented for both univariate and multivariate asymmetric Generalized Hyperbolic distributions.The calibration is done using the EM algorithm.Goodness of fit of calibrated parameters of Generalized Hyperbolic on data isanalyzed.Some univariate data examples will be first analyzed to gain some confidence that Generalized Hyperbolic gives a good fit.Comparison between histogram of empirical distribution and pdf of theoretical distributions are done to look at how the kurtosis and skewness of theoretical distributions match those from the actual distributions.Numerical results of markowitz optimization problem are obtained.Numerical results of RORC optimization problem are obtained.Backtesting is done by comparison with JKSE index.This is to check whether the portfolio's performance in a frictionless world can surpass the stock market's performance.The in-sample period is from 6 February 2008 until end of 2010 and the out-of-sample period is from start of 2011 until 4 march 2011.The JKSE composite index data is first fit and multivariate risk increment data from the portfolio within the in-sample period with the Generalized Hyperbolic distribution.

Next, the expected return of the JKSE index is extracted from the calibrated distribution and value input to optimization engine for portfolio to find the minimum expected shortfall that can be obtained and the optimal portfolio weights too.The Expected Shortfall of daily losses between that of obtained optimal portfolio is compared with the one from JKSE composite index in the out-of-sample period.

The opinion regarding the quality and importance is a way to solve portfolio optimization problems when expected shortfall is used as risk measure and when asset return distribution is modelled by Generalized Hyperbolic distribution is developed.Analytical solutions to Markowitz and RORC portfolio optimization problems are obtained in the framework of symmetric Generalized Hyperbolic.Such solutions can be obtained with the linearity property of Generalized Hyperbolic.The optimal weights from Markowitz optimization model with Expected Shortfall are equal to those from using volatility as risk measure by linearity of Generalized Hyperbolic distribution.The problem is reduced to classical Markowitz optimization problem.Optimization problems using Expected Shortfall are solved numerically in asymmetric framework.Initially, evaluating Expected Shortfall as function of portfolio weights is evaluating high dimensional integral.This almost intractable problem can be greatly simplified into a one dimensional integral problem with the linearity property of the Generalized Hyperbolic distribution.The Markowitz-optimal composition from using Expected Shortfall have also been found to be not necessarily equal to those from using volatility.The compositions are very similar for high expected returns when no shortings are allowed.The nonconvexity of the RORC problem can be avoided by changing it into a number of convex Markowitz optimization problems.That depends on the desired accuracy.Optimal weights cannot be achieved for RORC optimization version when asset shortings are allowed because the existence condition for maximum RORC is not satisfied although it tends to asymptotic value.There is no existence condition of the maximum RORC when Expected Shortfall is used in asymmetric framework.The trend of the plot is forever increasing to an asymptotic value.The plot behaves similarly with plot when volatility is used as risk measure is produced where optimal weights cannot be achieved.Such findings give the confidence that maximum RORC initially is unachievable and RORC tends to some asymptotic value.

The topic should be further studied because it is crucial to apply expected shortfall with generalized hyperbolic distribution during crisis to check whether it still works or not.

**III.5 BACKTESTING**

Blomvall and Lindberg (2003)’s research problem is an investment model on stochastic programming is built.Buying is done at the ask price and selling at the bid price in the model.The model is applied to investing in a Swedish stock index, call options on the index and risk-free asset.It is shown that options can be utilized to make a portfolio that outperforms the index by reoptimizing the portfolio on a daily basis over a ten-year period.It is shown that a portfolio that dominates the index in terms of mean and variance can be created with ex post analysis at given level of risk with the possibility of achieving a higher return using options.

The key findings ofresearch are the model is made as realistic as possible to make it plausible that returns actually could have been achieved.Three different asset types in the model is used.First is the OMX-index, a composite index in Sweden similar to S&P 500.It is made up of 30 most traded companies in Sweden.The weight in index is proportional to company's capitalization.The second asset type is call options on OMX-index.Only the options with the shortest time to maturity are used.There are 10-40 such options available during 1990-1999 for each day.

The important gaps in knowledge and researchare the Markowitz Mean-Variance model has been of large use to portfolio managers over the years.Its simplicity is the reason for its weakness and advantages.The probability distributions of asset returns stay the same and no explicit evolution of asset prices over time are produced in this model.It is difficult to study the effects of market imperfections like transaction costs and taxes.It is assumed in the model that the investor wants to optimize only the mean and variance.The model does not consider aspects like fat tails and skewness es in returns.Stochastic programming model is used to handle the problem.Several good reviews of this multistage framework are given.The multistage models can be solved using Stochastic Programming algorithms.Some of the first applications of Stochastic Programming in finance was on a rolling two-stage model used to solve an investment problem in stocks and bonds.Other studies on portfolios made up of fixed-income securities are done.Portfolios made up of mortgage-backed securities have been comprehensively treated.A mortgage-backed security consists of a collection of different mortgages.Asset-liability management for pension funds use models like Towers-Perrin, Pacific Financial Asset Management Company and the Russel-Yasuda Kasai.Pension-fund models are for over long horizons but short-term investments models exist too.A linear Stochastic Programming model is utilized to solve an investment problem with derivative instruments.Linear Stochastic Programming has been used to hedge a derivative portfolio optimally with stochastic volatility and transaction costs.Only limited numerical results on the model without performance testing on extensive time series is done.

The trends and themes of the research related to the topicis the Markowitz Mean-Variance model is based on the Gaussian distribution with standard deviation as risk measure.

The relationships between key concepts are a Stochastic Programming model is traditionally represented by a time-staged tree with each node representing a fixed outcome of random variables up to that stage.Each node in the tree shows a state where one outcome has been realized for values of random variables.The stages representing depth from the root node shows the time evolution.Another state can be associated with each node.Objective function only depends on the states in the terminal nodes.The formation of Multistage Stochastic Programming problems is different from conventional formulations because it uses predecessors and successors to define the tree structure.Utility theory has been well-discussed in economic theory.The preferences of a rational investor is to maximize his expected utility under some reasonable assumptions.One special utility function which outperforms any other utility function exists in the long run in growth of wealth when some natural limiting assumptions are made.The investor will have more money with probability 1 than if any other utility function had been used thecatch with the utility function investment is over an infinitely long time period to secure more money.Concomitant risk is high.Kelly strategy is a relevant approach has been proposed with fraction of the capital invested in log-optimal portfolio.The logarithmic utility function is used.A multistage asset investment problem with discrete outcomes of prices can be modeled.A mathematic programming problem with nonlinear objective and linear constraints is stated.Interior point solver is developed.The advantage of interior point method is the sub problem, the second-order approximation, can be determined efficiently.Power utility functions can be solved in polynomial time.Both linear and logarithmic utility functions can be solved in polynomial time.A parallel implementation of algorithm one instance with 16,384 scenarios, 98,300 variables and 180,214 constraints was solved in 2.9s on 32-node 900 MHz Athlon PC cluster with 20 Mb of RAM.

The inconsistenciesare scenario generation technique is used.8 year of historical quarterly returns with each return assigned equal probability.It assumes that probability distribution form is of same form as historically.Extreme price movements occur from time to time are automatically included.The current stock volatility is crucial in predicting the probability distribution because the current volatility to a large extent affects the returns for the next few days during the short time horizon.The probability distribution of the returns is less connected to the current volatility in the longer run.The stock price model is modified to capture the volatility effect.It is assumed that stock prices follow a lognormal process modeled with Brownian motion discrete version.Getting a small error will need a large set of historical data.Approximately 8 years of daily historical returns to estimate the mean is used.Such techniques may not work during crisis situation.

The research designs or methods seemed improper, insufficient, or inadequate are evaluation on how well the investment performed from February 14, 1990 to June 1,1999 is carried out.The model is initialized with data from 1986 - 1989.The evaluation is done by solving a stochastic two-stage optimization problem for the first date, buy assets at current market prices, then step model forward one time period and evaluate the portfolio.The procedure is repeated when this is done.It is necessary to make it possible to buy as many times as possible because we want to take advantage of incorrectly priced options.The model is stepped forward one day at a time.2328 optimization problems is solved to evaluate the investment during the 9.5 years.Only a two-stage model is used because of the difficulty of determining future option prices.The dates for the second stage is selected as the same date as options mature because at this date the value of the option is given by index price.The time horizon will never be longer than one month.The optimization problems are solved using 80 scenarios to model the future outcomes of OMX-index.The 2328 optimization problems are solved as a sequence in chronological order.No borrowing of money and short selling of assets are allowed in the option portfolio.It is impossible to determine how many assets could be traded at a given bid or ask price because back-testing of returns.The arithmetic mean of daily returns is used with the measure not predicting the actual performance of portfolio when the money is reinvested to do an ex post analysis.Kelly strategy is to invest only a share in the log-optimal portfolio.

The opinions regarding the quality and importance are it can be concluded that a portfolio that dominates the OMX-index is found by studying the four first moments, the tails of the returns and Sharpe-ratio measure.It is impossible if the market is efficient.Options are derivatives of underlying asset.The portfolio should not yield a higher return for a given risk level if options are included in the portfolio.It can be concluded that the Swedish option market has been inefficient during the examined period.It would be desirable to account for the liquidity in a more realistic manner to give a better indication of how serious the market inefficiency is.

The topic should be further studied because having the appropriate probability distribution and risk measures during crisis is critical to the efficiency and success of portfolio optimization.

Christoffersen and Pelletier (2004)’s research problem is financial risk model evaluation or backtesting is crucial as an internal model's way to market risk management given by the Basel Committee on Banking Supervision. However, current backtesting methods possess relatively low power in realistic small sample settings.The contribution is exploration of new tools for backtesting based on duration of days between violations of Value-at-Risk (VaR).Monte Carlo results show that in realistic scenarios new duration-based tests possess better power properties than previously suggested tests.

The key findings of the research are available methods for backtesting are still few while alternative methods for calculating portfolio measures like VaR already investigated by others.The contribution by the authors is exploration of a new tool for backtesting on duration of days between violations of risk metric.The key insight is if one-day-ahead VaR is correctly specified for coverage rate p, then the daily conditional expected duration until next violation should be a constant 1/p days.Various ways of testing this null hypothesis is suggested.Monte Carlo analysis comparing new tests to those currently available is conducted.The results show that in many realistic scenarios the duration-based tests have better power properties than the previously suggested tests.The size of the tests is easily controlled using Monte Carlo testing approach.The kind of omnibus backtesting procedures suggested are complements to not substitutes for statistical diagnostic tests done on different perspectives of risk model in model estimation stage.The tests suggested can be viewed either as a final diagnostic for an internal model builder or alternatively as a suitable diagnostic for an external model evaluator for whom only limited, aggregate portfolio information is available.

The important gaps in knowledge and research are financial risk model evaluation or backtesting is crucial as an internal model's way to market risk management given by the Basel Committee on Banking Supervision.However, current backtesting methods possess relatively low power in realistic small sample settings.Better methods proposed do better but depend on information like shape of left tail of portfolio return distribution usually not available.The most popular risk measure is Value-at-Risk (VaR) defined to be the conditional quantile of return distribution but that does not describe the shape of tail to the left of the quantile.Violation is an event where ex post portfolio loss exceeds the ex-ante VaR measure.It is critical to determine clustering of violations from backtesting.An institution's internal risk management team and external supervisors need to find clustering in violations.Large losses happening in quick succession are more plausible to cause disastrous events like bankruptcy.Evaluation of VaR techniques were mostly on artificial portfolios because of lack of real portfolio data in previous literature.Recently, performance of actual VaR forecasts from six large and anonymous U.S. commercial banks are reported.Banks are usually conservative with fewer than expected violations; their exceedances are big and seem to be clustered in time and across banks.Majority of the violations occur during the August 1998 Russia default and Long-Term Capital Management (LTCM) debacle.Rejecting a particular bank's risk model because of clustering of violations is very important if such violations happen to be correlated across banks from a regulator's perspective concerned about systemic risk.

The trends and themes of the research related to the topic are bounded support of uniform variable renders standard inference difficult.One is forced to depend on nonparametric tests with notoriously poor small sample properties.Confining attention to the left tail of the distribution has merit in backtesting of risk models if the left tail has the largest losses most likely to impose bankruptcy risk.

The relationships between key concepts are specific implementation of hit sequence tests suggested are discussed.The observations from a realistic portfolio return process is simulated and risk measures from the popular HS risk model is calculated providing the hit sequences for testing.Large-sample distributions of likelihood ratio tests are well-known but may not lead to reliable inference in realistic risk management settings.Nominal sample sizes can be reasonably large like two to four years of daily data but scarcity of violations like 1% VaR renders the effective sample size small.There are two additional advantages of using a simulation procedure.First, possible systematic biases from use of continuous distributions to study discrete processes are accounted for.Second, Monte Carlo testing procedures are consistent even if parameter value is on boundary of parameter space. Bootstrap procedures could be inconsistent.The power of the proposed duration tests under Monte Carlo study is assessed.Sample size is usually determined by practical considerations like amount of effort involved in valuing the current portfolio holdings with past prices on underlying securities.A sense of appropriateness of duration-dependence alternative is given before calculating actual finite sample power in suggested tests.Data and other resource constraints force risk managers to backtest their models on relatively short backtesting samples.The power experiment is conducted with samples sizes from 250 to 1500 days in increments of 250 days.The backtesting samples correspond to roughly one through six years of daily returns.The focus is only on the independence tests because HS risk models under study are correctly specified unconditionally.Practitioners usually work very hard to expand their databases allowing them to increase their rolling estimation sample period.The results suggest that such efforts may be unwarranted because lengthening the size of the rolling sample does not eliminate the distributional problems with HS.

The inconsistencies are VaR as portfolio risk measure can be criticized on several fronts. The quantile nature of VaR implies the shape of the return distribution to the left of the VaR is ignored.This shortcoming can be crucial for portfolios with highly nonlinear distributions including options.VaR measure is criticized both from a utility-theoretic perspective and from a dynamic trading perspective.Risk managers are interested in the entire distribution of returns particularly the left tail.Backtesting distributions rather than VaR havebecome important.Having the correct risk and return measures during crisis for portfolio optimization is crucial.Generalized hyperbolic distribution with expected shortfall is a better option.

The research designs or methods seemed improper, insufficient, or inadequateis detection of violation clustering is very critical because of widespread dependence on VaR’sdetermined from historical simulation (HS) technique.A sample of historical portfolio returns using current portfolio weights is first made in the HS methodology.The VaR is calculated as unconditional quantile from historical sample. The HS method largely ignores the last 20 years of academic research on conditional asset return models.Time variability is only captured from the rolling historical sample. The model-free nature of HS technique is seen as a great benefit by many practitioners despite forceful warnings.The widespread use of HS technique encourages the focus on backtesting VaR calculated using this method.The VaR forecast cannot promise violations of a certain magnitude but only their conditional frequency, p.This is a major disadvantage of VaR measure.Markov alternative is about clustered violations signalingrisk model misspecification. Violation clustering is critical because it implies repeated severe capital losses to institution together could result in bankruptcy.Markov first-order alternative may have limited power against general forms of clustering.More general tests for clustering uses only information in the hit sequence is to be established.It is assumed that the VaR is for a one-day horizon.No overlapping observations have to be used to apply this backtesting framework to a horizon of more than one day.The tests stated so far are capable of detecting misspecified risk models when temporal dependence in hit sequence is a simple first-order Markov structure.Developing tests with power against more general forms of dependence still rely on estimating only a few parameters is needed.The intuition behind this duration-based tests is clustering of violations will cause an excessive number of relatively short and long no-hit durations relating to market turbulence and market calm respectively.Duration tests can capture higher-order dependence in hit sequence by testing the unconditional distribution of durations.Dependence in hit sequence may show up as an excess of relatively long no-hit durations (quiet periods) and an excess of relatively short no-hit durations relating to violation clustering.Any information in the ordering of the durations is completely lost.The information in temporal ordering of no-hit durations could be obtained using framework of exponential autoregressive conditional duration (EACD) model.The information set only contains past durations but could be extended to include all conditioning information used to compute VaR for example in the test specifications.

The opinion regarding the quality and importance are a new set of procedures for backtesting risk models are presented.The key insight is that if the one-day VaR model is appropriately specified for coverage rate, p, then the daily conditional expected duration until next violation should be constant 1/p days.Different ways of testing this null hypothesis are suggested.Monte Carlo analysis comparing the new tests to those currently available is conducted.The results show that in many of the scenarios considered the duration-based tests have much better power properties than previously suggested tests.The size of the tests is easily controlled through finite-sample p-values calculated using Monte Carlo simulation.Majority of financial institutions use VaR as risk measure.Many calculate VaR using the HS approach.The main focus of this article is backtesting VaRs from HS.

The topic should be further studied because extensions to density and density tail backtesting are suggested.There are several immediate potential extensions to the Monte Carlo results.First, it may be interesting to calculate the power of the tests with different GARCH specifications.Second, consider structural breaks in underlying return models. Lastly, a class of dynamic hazard models is recently introduced.Exploring them for backtesting can be interesting.More complicated portfolios including options and other derivatives could be considered.Scrutinizing duration patterns from misspecified risk models could suggest other alternative hypotheses than those suggested here.Such extensions are for future work.The current regulator practice of backtesting on samples of only 250 daily observations is plausibly futile as the power to reject misspecified risk models is very low.

Skoglund, Erdman, and Chen (2010)’s research problem shows that backtesting the performance of value-at-risk (VaR) risk model in predicting future losses of a portfolio of stocks, futures and options.Stock data including crisis period from 6 April 2001 to 17 June 2009 is used.Univariate generalized autoregressive conditional heteroskedasticity (GARCH) models for stock returns with different assumptions and distribution of univariate model residuals like normal distribution, t-distribution, and empirical distribution with t-distributed tails are modeled.Model dependence using normal copula, t copula and discrete normal mixture copula to show co-dependence between financial return series.GARCH models for estimating and forecasting volatility contributes significantly to the accuracy of VaR models after backtesting evaluating different models.Making VaR models match the expected number of VaR exceedances of losses and VaR model exceedances having expected durations between them are the contributions.Stochastic volatility, an important is not the only component, of VaR model specifications.The options of univariate model for the GARCH residuals and copula affect VaR model performance.Standard practice models used in VaR estimation e.g. delta-normal and covariance simulation models with or without RiskMetrics covariance matrix and historical simulation model are shown to underestimate risk quite badly.However, discrete normal variance mixture copula with univariate GARCH model performs well.

The key findings of their research show that performing VaR risk model validation is done by backtesting.Backtesting performance of several VaR models is compared especially during a period including the recent crisis from 6 April 2001 to 17 June 2009.Compare the ability of range of different VaR models to predict future losses of a portfolio of stocks, futures and options.ABB, Astra Zeneca, Ford, AIG, Microsoft, Ericsson and Volvo stock returns are modeled using univariate generalized autoregressive conditional heteroskedasticity (GARCH) models with different assumptions on univariate model residuals distribution like normal distribution, t-distribution and empirical distribution with t-distributed tails.Copula is used to model the co-dependence.Normal, t and discrete normal variance mixture copulas are considered.These models are compared to standard market models like delta-normal model, historical simulation and normal distribution covariance matrix simulation.It is impossible to discuss much about sign of financial returns.There is no compelling reason to go beyond simple parametric conditional mean specifications like the growth or mean-reverting model.Maximum likelihood is applied to compute the parameter estimates for normal and t-distributed residuals.Normal maximum likelihood can be applied whether the conditional distribution is assumed normal or not.The resulting estimator is a quasi-maximum likelihood estimator and is still generally consistent.Normal likelihood application where it is invalid may result in efficiency loss relative to the true and unknown maximum likelihood estimator.Semiparametric maximum likelihood estimator of GARCH models and efficient generalized method of moments estimator of GARCH models are introduced.Semiparametric estimator is a two-step estimator.Consistent estimates of parameters are obtained using quasi-maximum likelihood estimator and are used to estimate a non-parametric conditional density in the first step.Using this non-parametric density to adapt the initial estimator for the second step.The efficient generalized method of moments estimator is asymptotically more efficient than the quasi-maximum likelihood estimator with coefficient of skewness and excess kurtosis of conditional density in explaining the differences.A comparison of the performance of univariate GARCH models with different assumptions about density for the GARCH residuals, z\_t, is done.The co-dependence model was used as a parsimonious multivariate specification to model exchange rates.The constant conditional covariance matrix model can be represented with a diagonal volatility matrix with univariate GARCH specifications on the diagonal.

The important gap in data is the value-at-risk (VaR) method to measuring portfolio risk is well known in the financial industry.However, recently during the financial crisis, performance of VaR models has been questioned especially whether the VaR models were able to predict the magnitude of losses experienced.The ability of multivariate normal distribution to accurate capture risks is questioned.The distribution of model residuals, zt, does not seem to support the symmetry or exponentially decaying tail behavior shown by the normal distribution.The high quartiles may be severely underestimated using the normal distribution approximation for zt.A Student t distribution is considered as an alternative to the normal distribution.GARCH with Student t errors is applied.Student t distribution has heavier tails than normal showing polynomial decay in the tails.It may be able to capture fat tails maintaining symmetry which is troublesome because many financial returns are asymmetric having a much fatter left than right tail.However, purely non-parametric methods e.g. kernel methods or empirical distribution function, has no assumptions about nature of the empirical distribution function but consist of several drawbacks, especially their poor behavior in the tails.The empirical density is for describing the center of the distribution.The parametric models is for describing the tails.Co-dependence of the univariate GARCH filtered residual vector with zt where zjt for every j is approximately independent and identically distributed over time.It is assumed that the correlation between the random variables (z1t,…,znt) is time-independent.The present specification allows GARCH models for each of the univariate returns series y1t,…,ynt and the previous assumption does not imply time-independent correlation between the financial time series themselves.Empirical data from seven liquid stocks has been analyzed to compare the models' performances.ABB, Astra Zeneca, Ford, AIG (New York Stock Exchange), Microsoft, Ericsson (Nasdaq) and Volvo (over-the-counter) consisting of 2,063 samples each from April 6, 2001 to June 17, 2009 on business days.Liquidity risk is not a substantial component of profit and loss because the stocks are assumed to be liquidly traded under severe market stress.The recent financial crisis turbulence is in the data period for comparing the models' performance under significant stress.The logarithmic returns were computed then the models fitted using a normal or t maximum likelihood for GARCH models and Kendall's τ for the copula correlation parameters.Each day the different models were re-estimated with the last 250 days' returns rolling forward and 5,000 samples were generated from the first observation in their sample of returns.The simulated returns were from April 4 2002 to June 17 2009, a total of 1,812 days for comparison between the models.The simple risk models like delta-normal model (DNSTD, DNRM) and covariance simulation models (COVSTD, COVRM) underestimate risk very severely because their ability to capture the expected exceedances are rejected at low levels of significance.None of the models are sufficient measurements of VaR at 99% and 99.5% level even though the simple risk models using RiskMetrics covariance matrix have slightly better performance.The historical simulation model performs better than the simple models' performance because its expected number of exceedances is at the 99.5% VaR level but drastically underestimates risk at the 99% level.The t and normal mixture copula models with asymmetry overestimate risk at the 99% level after scrutinizing the copula models with normal, t-distributed and empirically distributed with t tails GARCH residuals.The risk is overestimated at the 99.5% level excluding normal GARCH residuals and an asymmetric normal mixture copula (N\_NM\_ASM8) and empirical GARCH residuals with t tails and a normal mixture copula (E\_T\_NM\_ASM8).The only model that performs well with a 5% rejection criterion for the p-value is the model with the symmetric normal mixture copula (NNM) using the normal model for GARCH residuals.The normal copula and the t5 and t10 copulas (TN, TT5 and TT10 respectively) are not rejected at the 5% level of significance at any 99% and 99.5% VaR levels using a univariate t-distribution for the GARCH residuals.The normal mixture copula model (TNM) overestimates risk but not as drastically as t and normal mixture copulas with asymmetry.The normal and t copula models underestimate risk and the copulas with asymmetry overestimate risk for models with empirical GARCH residual distribution and t tails.The symmetric normal mixture copula (E\_T\_NM) model is not rejected at 5% significance level for any of the VaR confidence levels.The p-values for the independence test for all the models are displayed.The p-values for the duration-based test calculated using the continuous Weinbull distribution with results at 99% VaR confidence levels are displayed because the results for the 99.5% level are similar.

The trends and themes of the research is financial return series are normal and independent over time is one of the most common assumptions in finance.Stylized facts of financial return series question these assumptions.Financial return series tend to display volatility clusters questioning the independence assumption.Observed fat-tailed nature of financial time series after correcting for volatility clusters questions the normal distribution assumption.The stylized facts are:

* + 1. They tend to be uncorrelated but dependent.
    2. Volatility is stochastic and autocorrelation function of absolute returns tends to decay very slowly.
    3. They tend to be heavy-tailed and asymmetric. Large returns occur more frequently than predicted by normal distribution.

Lack of autocorrelation in financial returns from stylized fact 1 shows linear association between consecutive observations is not big.The modeling and forecasting of volatility is a key issue when considering financial time series from stylized fact 2.An accurate modeling of volatility can improve risk management if it is stochastic and forecastable from stylized fact 2.The GARCH family of stochastic processes explain much of the dependence structure referred to by stylized fact 1.The GARCH filtered residuals show little or no dependence being approximately independent and identically distributed.GARCH(1,1) is the most commonly used model.The modeling of the constant conditional correlation matrix is from a more general copula point of view.The copulas are divided into elliptical copulas based on elliptical distributions, non-elliptical parametric copulas and copulas consistent with multivariate extreme value theory.Other constructions are based on copula transformations and copulas using Bernstein polynomials as an example.One of the most necessary and difficult issues of multivariate analysis is modeling dependence between random variables.The idea of copulas give a structured way of focusing on and analyzing the effect of different dependence structures.Modeling dependence between random variable is one of the most pertinent and difficult of multivariate analysis.Copulas provide a structured manner of focusing on and analyzing the impact of different dependence structures.Dependence between real-valued random variables X1,…,Xn is from the joint distribution function.Copula is to decouple multivariate distribution functions construction into marginal distributions specification and dependence structure.

The relationship between key concepts is the GARCH model is able to capture many stylized facts of univariate financial return series e.g. volatility clustering.It can be estimated with maximum likelihood for each desired marginal error distributions.It can be used as estimated or filtered GARCH residuals can be extracted and modeled with empirical density with t-distributed tails once it has been estimated.Model co-dependence of GARCH filtered residuals using copula with focus on elliptical copulas like normal, t and discrete normal variance mixture copulas.Measurement of dependence in copulas proceeds using Kendall's tau, Spearman's rho and coefficient of upper and lower tail dependence compare to Pearson linear correlation only depends on the copula.Elliptic copulas are derived from elliptic class of distributions like normal, Student t and logistic distribution.The Gaussian copula is copula of multivariate normal distribution.An explicit relation exists between Kendall's tau, τ, Spearman's rho, ρs, and linear correlation, ρ of random variables, X1, X2.The student t copula is the copula of the multivariate Student t distribution.The relationship between Spearman's rho, ρs, and linear correlation, ρ, for the above Gaussian copula is not in the t copula case.A normal mixture distribution can be the distribution of a random vector.A discrete normal mixture distribution uses discrete random variable values with different probabilities represented by ordinary and stress states.The normal mixture distribution can be expressed with asymmetry parameter γ where its negative values produce a greater level of tail dependence for joint negative returns.Joint negative returns show stronger dependence than joint positive returns in many financial time series.A normal mixture copula is the copula of a normal mixture distribution.The VaR model should be able to match closely the number of realized exceedances with the expected number with the preference of it distributing the exceedances randomly rather than it where exceedances are correlated over time.The unconditional coverage test assumes exceedances are independent over time but the independence test is a test of the independence of the violation sequence against the alternative of a first-order Markov chain.Duration test is another test for independence of the exceedances.It is based on the distribution of the periods between exceedances known as the durations.The durations should have meant of 1/α where α is the expected number of exceedances and exponentially distributed or memory less with a correctly specified VaR model.The exceedances follow a Weinbull distribution where a special case of the Weinbull distribution is the exponential under the alternative hypothesis.

Possible contradictionsarewanting to understand multivariate structures that can handle the scale of financial applications because the analysis is more towards the explicit dependence structures usage. In reality, this may not work if it is an explicit independence structures usage or there is no dependence structures usage. The focus is on elliptical copulas especially normal and t copulas.The seperation of modeling of univariate marginal distributions and dependence structure are in use of copulas for co-dependence modelling.However, financial data are usually not normal and better represented by non-elliptical distributions instead.

Research designs or methods seemedinadequate because performing backtesting comparison of VaR risk measure for several multivariate models may be biased toward the models and risk measures selected.They are considered based on univariate GARCH model specifications for each financial time series returns, univariate residuals model and copula model.The assumption about the univariate model density is it is either normal, t-distributed or empirically distributed with t-distributed tails.The copula model is either normal, t with 10 or 5 degrees of freedom or discrete normal variance mixture copulas.The model performance comparison is on empirical data from New York Stock Exchange (ABB, Astra Zeneca, Ford, AIG), Nasdaq (Microsoft, Ericsson) and over-the-counter (Volvo) each with 2,063 samples on business days from April 6 2001 to June 17 2009.This includes the recent financial crisis turbulence to compare the models' performance under a period of significant stress.The portfolio is made up of plain equity holding in each of the stock, forward agreements and European and American option holdings.Backtesting assumes that what happens in the past will happen again in the future.This may not always be true in reality.The model specification framework has two components.First, it is the specification and estimation of models for univariate marginal distributions including parametric GARCH specifications and fitting of non-parametric densities to the tails of the univariate residuals.Second, it is the specification of a co-dependence model between financial time series encoded in copula format for vector zt.Spearman's rho, Kendall's tau and coefficients of upper and lower tail dependence are best-known copula dependence measures.They are all bivariate ideas that can be extended to multivariate case by applying them to all pairs of components in the vector.Linear correlation is a linear measure of dependence.Kendall's tau is a copula property.Spearman's rho is the correlation of ranks.Spearman's rho is the usual linear correlation, ρ, of the probability transformed random variables.The coefficient of upper and lower tail dependence is another important copula quantity.The coefficients of upper and lower tail dependence measure probability of joint extremes of the copula.A simple method on Kendall's τ can estimate the parameters of the normal and t copulas because of the explicit relationships between Kendall's τ and linear correlation.It consists of constructing an empirical estimate of linear correlation for each bivariate margin then using the relationship between linear correlation and Kendall's τ stated above to infer an estimate.A different method is using the linear correlation of the probability transformed random variables i.e. Spearman's ρ or for the Gaussian case, the explicit relation between linear correlation and Spearman's ρ. The normal copula is asymptotically independent whereas the t copula shows asymptotic tail dependence.It may be more interesting to investigate the finite tail dependence for normal and t copulas although the asymptotic ideas are interesting from a practical perspective.A backtesting comparison of the VaR risk measure for different multivariate models is considered.The multivariate models are based on univariate GARCH model specifications for each financial time series returns and a copula.The univariate model density is assumed to be either normal, t-distributed or empirically distributed with t-distributed tails using a common threshold of 5%.The copula model is either normal, t with 10 or 5 degrees of freedom or normal mixture copula with parameters w1 = 1, w2 = 400, p1 = 0.9, p2 = 0.1.Each of the t and normal mixture copulas can have asymmetry with asymmetry parameter set to -0.8 or -0.3. The multivariate models using notation univariate model name and copula model name with univariate model abbreviations N for normal, T for t-distribution, E for empirical density using notation ET for empirical density with t-distributed tails.The abbreviation for copula model is N for normal copula, T for t copula and NM for normal mixture copula.The notation NN will represent the model with normal univariate density GARCH model and a normal copula.T\_NM\_ASM8 will represent the model with univariate t-distributed GARCH residuals and discrete normal variance mixture copula with asymmetry parameter -0.8.E\_T\_NM\_ASYM8 represents the model with the same copula as T\_NM\_ASM8 with empirically distributed univariate GARCH residuals with t-distributed tails.Standard VaR models like delta-normal model with an arithmetic or RiskMetrics covariance matrix are represented by DNSTD and DNRM respectively.The historical simulation model is represented by HISTSIM.The covariance matrix simulation model using either arithmetic or RiskMetrics covariance matrix is represented by COVSTD and COVRM respectively.The standard model performance measurement is a portfolio backtesting procedure to measure the quality of VaR predictions using simulated model returns and a portfolio.The standard model performance measurement is a portfolio backtesting procedure to measure the quality of VaR predictions using simulated model returns and a portfolio.The comparison portfolio is made up of equity holdings in each stock and forward agreements and European and American option holdings.Each of the models' one-day VaR forecasts were compared to the actual return for that day and the number of days when the actual loss was larger than the VaR forecast was determined known as the number of exceedances.The expected number of exceedances is stated for the 99% and 99.5% VaR confidence level.The unconditional coverage (UC) likelihood ratio test checks whether the VaR violations happen with expected probability.Exceedances are assumed to be independent and identically distributed Bernoulli random variables.A likelihood ratio statistic that is distributed as X12 under the null hypothesis is formulated.The p-values of the test for all the models and for the 99% and 99.5% VaR confidence levels are given.The naive models with no stochastic volatility component are unable to succeed in creating independent exceedances comparing the models' ability to spread exceedances over time.The independence and duration tests p-values show that the delta-normal model and the covariance based simulation model are rejected at any significance level because the models do not react to high volatility regimes and exceedances are expected to come in clusters.The delta-normal model with RiskMetrics covariance matrix and the covariance simulation model with RiskMetrics covariance are rejected with the covariance simulation model with RiskMetrics covariance matrix is rejected at slightly lower levels of confidence.This shows that RiskMetrics covariance matrix cannot capture volatility regimes sufficiently.The historical simulation model performed well at 99.5% level using one of the independence test.The other independence and duration tests show that the model is incapable of spreading exceedances independently and is not a realistic model of VaR.The models based on RiskMetrics covariance matrix and historical simulation, the copula models with GARCH univariate models are not rejected at 5% significance level for any of the models compared to unconditional models.The GARCH models can capture volatility clusters and react to volatility increases with higher risk estimates avoiding producing correlated exceedances.Models that use GARCH volatility prove to satisfy the production of independent VaR exceedances requirement but only a few of the copula models with univariate GARCH volatility could match the expected VaR exceedances.Normal GARCH residuals model with a symmetric normal mixture copula (NNM), models with univariate t-distribution for GARCH residual and normal copula and t5 and t10 copulas (TN, TT5 and TT10respectively), and empirical GARCH residual distribution and t tails with symmetric normal mixture copula (E\_T\_NM) are not rejected at 5% level for either unconditional coverage or independence of exceedances.

The opinion regarding the quality and importance is the unconditional coverage of and the ability of the model to produce independent VaR exceedances must be considered when judging a sufficient model for VaR.Backtested the performance of VaR models in predicting future losses of a portfolio of stocks, futures and options to validate the risk model.Historical stock data about ABB, Astra Zeneca, Ford, AIG, Microsoft, Ericsson and Volvo from April 6 2001 to June 17 2009 , the recent crisis period, to compare VaR models' ability to match the expected number of exceedances and their ability to spread exceedances randomly over time.Backtesting evaluations on the ability of VaR models to capture expected exceedances show that simple risk models like delta-normal and covariance simulation models underestimate risk drastically because their ability to capture expected exceedances is rejected at low levels of significance.RiskMetrics covariance matrix for those simple models improves the performance but they do not give sufficient measurements for 99% and 99.5% VaR level.Historical simulation model performs better with expected number of exceedances at 99.5% VaR level but underestimates risk drastically at 99% VaR level.More sophisticated copula dependence models with univariate GARCH models perform quite well in terms of expected exceedances.Models with normal variance mixture copula and GARCH normal residuals or empirical residuals with t tails are not rejected at normal significance levels for any of 99% and 99.5% VaR confidence levels.Models with t-distributed GARCH model residuals and normal, t10 or t5 copula are not rejected at normal significance levels.Models with GARCH volatility is successful in creating independent exceedances because of their ability to spread exceedances over time from the independence and duration test.Models using RiskMetrics or arithmetic average variance and covariance cannot create independent exceedances.It shows that GARCH models are a key model component in well-specified VaR models.Inclusion of GARCH models was necessary to create independence exceedances in the backtesting comparison but insufficient for creating both independent exceedances and matching expected VaR exceedances.Only a few copula models with GARCH variances including the symmetric normal mixture copula, univariate t-distribution for the GARCH residual, and normal copula and t5 and t10 copulas, empirical GARCH residual distribution and t-tails with symmetric normal mixture copula are not rejected at 5% level for unconditional coverage.It is important to know the right risk measure and distribution to use during crisis.

Further studies should be conducted because financial return commensurates with risk taken.During crisis, the financial risk escalates with diminishing returns and ballooning losses.Using the wrong distribution and risk measure can exacerbate the loss during crisis.It is even more important to determine the right distribution and risk measure during crisis.

Liu and Hung (2010)’s research problem is the objective of this paper is about applying alternative GARCH-type models to daily volatility forecasting and applying Value-at-Risk (VaR) to Taiwanese stock index futures markets suffered most from the global financial tsunami during 2008.This study uses three range-based proxies (PK, GK and RS), and one return-based proxy (realized volatility), for use in empirical exercise rather than using squared returns as proxy for true volatility.The forecast evaluation is carried out using different proxy measures on symmetric and asymmetric loss functions with back-testing and two utility-based loss functions are utilized for further VaR assessment with respect to risk management practice.Empirical results show that EGARCH model gives the most accurate daily volatility forecasts whereas the performance of the standard GARCH model and GARCH models with highly persistent and long-memory characteristics are relatively poor.The RV-VaR model usually underestimate VaR and has been rejected because of lack of correct unconditional coverage.On the contrary, GARCH genre of models can give satisfactory and reliable daily VaR forecasts.The unobservable volatility can be proxies with parsimonious daily price range with freely available prices when applied to Taiwanese futures markets.The GARCH-type models remain valid downside risk measures for both regulators and firms in a turbulent market.

The key findings of their research are alternative GARCH-type models (GARCH, GJR-GARCH, QGARCH, EGARCH, IGARCH and CGARCH) is applied to daily volatility forecasting and applying VaR to Taiwanese stock index futures markets that suffered most from global financial tsunami that occurred during 2008.Forecasting models that perform well over this turbulent period are probably useful in practical applications because they have performed well in the rigorous test with a data set covering extreme financial crisis.Three range-based proxies (PK, GK and RS) and a return-based proxy (RV) are adopted for use in the empirical exercise because true volatilities are not observed.Volatility forecast evaluation is conducted with various volatility proxies in terms of mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE) and mean mixed error statistics (MME) about forecasting accuracy.The first three loss functions are symmetrical whereas the last one is asymmetric that penalizes volatility forecasts asymmetrically during low and high volatility periods.The asymmetric criterion is practical and of interest to traders with long and short positions and option buyers and sellers.These models are also evaluated in terms of their ability to give adequate VaR estimates with inclusion of a realized-volatility-based VaR model (RV-VaR) besides directly evaluating these daily volatility forecasts.Whether it is better to use a RV-VaR model or GARCH-type model in getting daily VaR estimates is examined.Predictive performance of RV-based VaR model is compared with that of GARCH-based VaR models with Kupiec's backtesting with respect to risk management practice.The utility-based regulatory loss function (RLF) and firm loss function (FLF) are employed for further VaR assessment which can be closely shaped to economic agent utility in relation to specific issues for models meeting the prerequisite of correct unconditional coverage.

The important gaps in knowledge and researchare it has been long aware that return volatility changes over time and periods of high volatility usually cluster.Pattern of volatility clustering or autoregressive conditional Heteroskedastic (ARCH) effectsuggests that probability of large losses is bigger than that suggested by standard mean-variance analysis.Thus, Value-at-Risk (VaR) models should include ARCH effects when they exist for assets in a given portfolio.Failure to investigate ARCH effects results in sub-optimal portfolio management for housing market investors.The ARCH modeling developed with its subsequent generalization (GARCH) generated to respond to stylized facts.The volatility forecasting method is dominated by different adaptations of GARCH models including symmetrical model (standard GARCH), asymmetrical models (EGARCH, QGARCH, GJR-GARCH), highly persistent model (IGARCH) and long-memory model.Volatility models are comprehensively reviewed and assessed for their forecasting performances.A lot of research showed that distribution of financial returns is usually skewed to the left, has a high peak and fat tails.The skewed generalized T (SGT) distribution identified is closely shaped to the above empirical phenomena.Joint estimation of conditional mean and variance of returns are needed when implementing the parametric technique based on several popular GARCH-type models.Processes must be extended by assuming that conditional return innovations are SGT distributed to further enhance the robustness of GARCH estimation in non-normality.A large volume of literature uses squared returns as proxy for true volatility in forecasting exercise.The squared returns give a convenient measure of true underlying volatility of return process when the conditional mean of return is zero.Squared returns are an unbiased and very noisy measure of volatility.Daily realized volatility (RV) based on cumulative squared returns from intraday data can give a good alternative for daily return volatility.RV has at least one empirical limitation in that it needs intraday returns for 5-30 min and needs tedious procedures making it difficult to implement despite the high frequency of RV.Range-based volatility estimators are extensively developed to reach certain efficiency using free and available price data including daily high, low, opening and closing prices.Scaled high-low range values are used to develop PK daily volatility estimator based on assumption that intraday prices follow a Brownian motion process motivated by daily price range.GK estimator using opening and closing prices together with price range is created under the same assumption with PK.RS estimator is developed by addressing draft issue of security price process.Model ranking is highly dependent on proxy used for true volatility implying efforts is necessary to compare the volatility forecasting performance of different adaptations of GARCH model with a wide range of powerful proxy measures whereas most of the literature on model evaluation has considered just one proxy each time.This allows the possibility of recognizing the forecasting performance of models.It is critical to explore whether model forecasting performance is constant across various volatility proxies.Efficient forecasts can be made by parsimonious daily price range with freely available prices in absence of high-frequency price data for many assets if model forecasting performance is constant.

The trends and themes of the research related to the topic are volatility forecasting has grown in importance during the last twenty years because large movements in asset prices are becoming more prevalent in the modern world of finance.Various financial crises like collapse of Baring's Bank in 1995, near bankruptcy of Long Term Capital Management in 1998, recent bailouts of Bear Sterns, Lehman Brothers and American International Group are regarded to be Black Swan events further highlighted the importance of volatility forecasting especially its critical role in risk management.Such considerations have led both practitioners and researchers in empirical finance to ponder the problem of efficiently producing volatility forecasts.

The relationships between key concepts are the data examined are daily and intraday data of Taiwanese Stock Index Futures prices from Coney database.The sample period for daily data is April 24 2001 to December 31 2008 for a total of 1910 trading days with opening, high , low and closing prices.The sample is divided into two parts.The first 1660 observations (April 25 2001 to December 31 2007) are for in-sample estimation whereas the remaining 249 observations (January 1 2008 to December 2008) are for out-of-sample forecast evaluation.Other data set analyzed is made of 5-min intraday trading prices in 2008.The average daily return has a negative value closely approaching zero and very small compared with standard deviation of daily returns.The returns series show pronounced skewness and kurtosis.The returns series is skewed toward the left with more positive than negative outlying returns showing more positive than negative outlying returns are in Taiwanese futures markets whereas distribution of daily returns has a fatter tail and higher peak than normal distribution needing a sophisticated distribution that includes fat tails, leptokurtosis and skewness.Jarque-Bera (JB) statistic confirms that daily return is non-normally distributed.Q^2 statistic shows a linear dependence of squared returns and strong ARCH effects needing conditional models allowing time varying volatility.PP and KPSS unit root tests results show that no evidence of non-stationarity in Taiwanese Futures Exchange (TAIFEX) futures returns.

The contradictions are intraday data is used to calculate daily VaR.Using RV to calculate one-day-ahead VaR for two stock indexes and two exchange rate returns does not significantly improve on performance obtained with daily return-based model.Daily return-based model produces more accurate VaR forecasts than RV-based model at lower confidence levels.However, using realized range-based and RV models to calculate daily VaR for Chinese Shanghai and Shenzhen composite indexes and get empirical results showing that VaR forecasts based on high frequency data can improve the accuracy of VaR forecasts.The accuracy of RV-based VaR is compared with daily return-based GARCH-type models during 2008 considered to be a kind of stress testing under exceptionally turbulent market scenario.

The research designs or methods seemedinadequate becauseGARCH genre of volatility models need joint estimation of conditional mean variance equations.SGT distribution is utilized for empirical distribution of return series showing fat tails, leptokurtosis and skewness.Several popular GARCH-type models are used to model and forecast returns volatility of Taiwanese stock index futures.Two proxy categories for measuring volatility are used to compare the forecasting ability of various models used.The first category is range-based volatility with PK, GK and RS and second category is return-based volatility (RV) on squared intraday returns.The Mincer and Zarkonwiz regression test is carried out to test predictive power introduced.The symmetric loss functions is defined including MAE, MSE and RMSE.The likelihood ratio test developed by Kupiec to test whether the true failure rate is statistically consistent with theoretical failure rate of VaR model is employed to backtest VaR results.The EGARCH model gives the most accurate daily volatility forecasts whereas IGARCH model is less recommended based on symmetric loss functions defined.EGARCH model seems to provide the best performance in forecasting Taiwanese futures market volatility during financial tsunami when model selection is on both symmetric and asymmetric loss fucntions.Performances of standard GARCH model and GARCH models with highly persistent and long-memory properties are worse than expected.Asymmetry in volatility dynamics should be accounted for.Model forecasting performance is constant across various volatility proxies in most cases showing that unobservable volatility can be made by parsimonious daily price range with freely available prices.

The opinion regarding the quality and importance is volatility forecasting become more important in the last twenty years because large swings in price movements of assets have become more prevalent in the modern financial world.Alternative adaptations of GARCH model to daily volatility forecasting is applied and VaR to Taiwanese stock index futures markets that suffered most from global financial tsunami during 2008 is applied.It is about stress testing under a turbulent market scenario.Several solid points are suggested using a series of rigorous tests.First, EGARCH model provides the most accurate daily volatility forecasts when model selection is on both symmetric and asymmetric loss functions whereas performances of standard GARCH model and GARCH models with highly persistent and long-memory characteristics are worse than expected suggesting asymmetry in volatility dynamics should be taken into account when applied to Taiwanese futures markets.Second, model forecasting performance is constant across various volatility proxies in most cases showing the unobservable volatility can be made by parsimonious daily price range from freely available prices. Third, the RV-RaR model cannot be applied to current turbulent period because it underestimates the true VaR values and lacks requisite accuracy.GARCH-type models are capable of giving both satisfactory and reliable daily VaR forecasts although they tend to over-predict daily volatility.VaR-based risk management using GARCH-type models with SGT returns innovations is valid for Taiwanese stock index futures which suffered most from the financial tsunami in 2008.The IGARCH (EGARCH) model is the most efficient VaR model for a regulator (a firm) at low (high) confidence level during 2008.

The topic should be further studied becauseIGARCH model is the best performing model for a regulator based on ARLF statistic at both confidence levels followed by EGARCH, GJR-GARCH, GARCH, QGARCH and CGARCH models by identifying the best forecasting performance with minimum average economic loss.RV-VaR model is not applicable to current turbulent period because it underestimates the true VaR values and lacks the requisite accuracy.GARCH models are surprisingly capable of providing both satisfactory and reliable daily VaR forecasts although they tend to over-predict daily volatility.VaR-based risk management using GARCH-type models with SGT returns innovations is valid for Taiwanese stock index futures which suffered most from global financial tsunami.IGARCH (EGARCH) model is the most efficient VaR model for a regulator (a firm) at low (high) confidence level.Finding the appropriate probability distribution and risk measure during crisis is crucial for portfolio optimization during crisis.

Lechner and Ovaert (2010)’s research problem is the financial markets have shown great instability and high volatility in the last few years.Risk managers use Value-at-Risk (VaR) to capture amount of risk a financial firm takes on a single trading day.Many past methods have included a normality assumption capable of often produce misleading figures because most financial returns are characterized by skewness (asymmetry) and leptokurtosis (fat-tails). They provide an overview of VaR and describe some of the most current computational approaches.

The key findings of their research are recent research has used the third and fourth moments to estimate the shape index parameter of the tail.Other approaches like extreme value theory focus on extreme values to calculate the tail ends of a distribution.There is no one particular model that is best for computingValue-at-Risk (VaR) although all of the models have proven to capture the fat-tailed nature better than a normal distribution by emphasizing benefits and limitations of the Student-t, autoregressive conditional Heteroskedastic (ARCH) family of models and extreme value theory.An overview of VaR with some of the most recent computational approaches is given.Methods of computing VaR, detecting non-normality using quantile-quantile (Q-Q) plots, sample mean excess function (MEF), moment analysis, characterizing returns distribution with parametric models like Student-t distribution, autoregressive conditional Heteroskedastic (ARCH) method and extreme value theory (EVT) are introduced.Comparisons made between parametric models from various sources of recent research are summarized.Importance of risk management and regulatory requirements is discussed.Means of backtesting and measuring model accuracy are presented.Conclusions are made from the information presented.

The important gaps in knowledge and researchareVaR does not always imply reliability.Many ways is used to calculate VaR from simple historical simulation to complex semi-parametric approach.Many VaR estimates are evaluated from historically estimated probability density functions (PDFs) limiting the predictive power of future risk measures.The major error in predicting future VaRs from historical data is the actual shape of the PDF which can change significantly from one used in the past.Economists are now focusing their efforts on producing a robust parametric VaR model accounting for leptokurtosis and skewness in distribution returns and computed in a reasonable amount of time.Normal APARCH and Student-t APARCH model with estimates from RiskMetrics (GARCH model) which does not permit the power term to be estimated within the model is compared. VaR values produced by Normal APARCH model are ideal at lower confidence levels for asset returns with fatter tails and volatility clustering. VaR predictions produced by Student APARCH model are more accurate than those forecasted by both RiskMetrics and Normal APARCH model at high confidence levels.The optimal power term was between one and two supporting the use of a model that estimates the power term.The two APARCH models were more successful for determining heteroskedasticity and true volatility when compared to results of RiskMetrics.Normal APARCH model generates a more accurate VaR for lower confidence levels.Student APARCH model performs better in conditional and unconditional coverage producing the most accurate and efficient risk estimates for higher 99.5 and 99.9% confidence levels.RiskMetrics model was found to underestimate risk especially for higher confidence levels (99.9 percentile) and produced greatest risk for unexpected loss making it the least efficient model. VaR values produced by Normal APARCH model are preferred for lower confidence levels whereas Student APARCH model is more accurate and provide better out-of-sample results at higher confidence levels. APARCH skewed Student-t model accommodates both skewness and excess kurtosis of a financial return distribution and performs better because it provides a more accurate description of fat tails.Similar results when comparing a Student-t to a normal distribution are produced.The Student-t and normal distributions are approximately equivalent while at a higher level the Student-t model outperforms the normal model at a 95 percent confidence level.Non-Gaussian parametric formulae accurately captures leptokurtosis in financial data and are similar to a full historical evaluation when model parameters are optimized through empirical analysis.Extreme value approach is inappropriate for all heavy-tailed distributions.It works reasonably well for strongly heavy-tailed data involving large jumps whereas EVT index estimators are inappropriate for weakly heavy-tailed data.Extreme values come from rare events, thus size of observations of these rare events is very insignificant compared to the whole sample size.It is essential to have an ample size for events being critiqued because EVT is mainly concerned with tail part information and ignores the other returns data.EVT can be useful for measuring size of extreme events.This can be approached in different ways depending on data availability and frequency, desired time horizon, and level of complexity one is willing to implement.An extreme value approach with parametric method than any other approach because the theory accounts explicitly rare events in the distribution tails.EVT has three main advantages over other classical methods:

1) it is parametric allowing out-of-sample VaR measurements for high probability values whereas historical method cannot compute VaR for high probability values because it is based on a limited number of observations.

2) it does not assume a distribution considerably reducing model risk and producing better fitted tail models.

3) it focus on extreme events with event risk explicitly taken into account whereas large unexpected shocks are usually ignored with the GARCH method.

The trends and themes of the research related to the topic are billions of dollars can be lost in a short time because of failure in controlling financial risks as shown in history.The development of reliable methods to monitor financial risk has become increasingly important because of such extreme events.Risk measures are crucial for characterizing financial investment decisions with each institution measuring the amount of risk it takes on over a period of a day, week, month, or year.It is mandatory for a financial institution to meet capital requirements to cover potential losses because of sources of risk during normal operations: credit, operational and market risks imposed by the Basel Committee on Banking Supervision.A financial firm may want to know potential losses to is portfolio to allocate its funds and plan for payments to investors efficiently.Value-at-Risk (VaR) has become the most popular among risk managers as the best and simplest method to predict losses of an asset, portfolio, or an entire firm.The VaR of a portfolio measures the maximum loss during a specific time horizon within a given confidence level, that is 99 percent, conditioned such that the composition remains unchanged.The average loss of the portfolio will not be greater than USD 2M over the one-day horizon on 99 out of 100 trading days with a 99 percent confidence level if VaR is USD 2M for instance. VaR has become a standard for risk management because of its simplicity and accuracy of estimating risk at a reasonable computational cost.

The relationships between key concepts are there is no one particular method better than the other, rather each approach is geared toward different scenarios of risk.A risk manager might select a less-accurate model as long as it satisfies the confidence level requirement to achieve faster results.Efficient computing time is equally as important because calculations must be completed for every trading day although accuracy is the most important requirement on any financial model because it could jeopardize millions of dollars. VaR is useless to the trading managers the next day if the model takes too long to compute.Another choice is to use the faster method on a daily basis and utilize less computationally efficient models intermittently and regularly to track both correlation and significant deviations between the models.

The contradictionis VaR is not a coherent risk measure when applying to a crisis situation to measure risk given the right probability distribution is used.

The research designs or methods seemedinadequate because it involves comparing the Student-t, autoregressive conditional Heteroskedastic (ARCH) family of models, and extreme value theory (EVT) to capture the fat-tailed nature of a returns distribution.The use of VaR has been increasingly important over the past twenty years because:

(1) regulatory mandates for better control of financial risks;

(2) globalization of financial markets; and

(3) innovations in computational technologies.

VaR become so crucial because it can summarize downside risk in a single number.The cutoff return, R\*, associated with the lowest portfolio value for VaR has to be correctly estimated.The methods are divided into two groups: nonparametric and parametric. Nonparametric methods use a historical distribution to compute VaR directly whereas parametric methods define the historical data by a statistical distribution with characteristic parameters.The calculus of VaR require dealing with time horizon, confidence level and underlying conditional distribution of returns.One performance limitation on any VaR model is calculations must be carried out daily usually overnight and analyzed for the next trading day.The tradeoff for calculating VaR is between speed and accuracy.The fastest ways for calculating VaR rely on simplifications, making assumptions about returnsdistribution and variations in underlying risk factors.VaR has several weaknesses when measuring exposure to extreme market events:

1) probability of extreme market events is not captured well by normal distribution; and

2) high curvature of a derivative portfolio with respect to underlying extreme situation has yet to be addressed.

3) Does not quantify loss when the confidence threshold is exceeded; and

4) not sub-additive.

Two characteristics are specific to financial market data:

1) extreme outcomes are usually bigger and occur more common than for a normal distribution (fat tails); and

2) size of market movement varies over time (conditional volatility).

Recent methods do not assume normality with the data fitted based on tail distribution. Choosing among the countless number of methodologies which can utilize different assumptions and highlighting different parameters is one of the biggest challenges in calculating VaR.It is not the best method to calculate VaR even though it is computationally simpler to assume normality.The normal model obtains its properties from central limit theorem states that aggregate effect of individual elements will be distributed in a symmetric bell shape although the elements may be asymmetric.A nonparametric normal model can be useful near the mean but if measurement of risk near extremes is needed, then a more accurate description of the tail's shape is critical.Normal or Gaussian-based statistics usually produce significant errors when dealing with skewed data.Q-Q plot is used to determine whether a set of data correlates to a specific distribution that is normal.It reveals much information including whether the distribution is leptokurtic with fat tails or shows signs of asymmetry from skewness.A distribution with fat tails shows that the size and probability of the worst possible outcomes or expected losses is higher than a normal distribution as seen with most financial returns distributions.The distances between adjacent observations increase at a faster rate compared with normal distribution rate when moving away from the mean if the distribution has fatter tails.Determining the existence of fat tails is critical because assuming normality underestimates the probability of extreme events.Q-Q plots are usually plotted against Gumball distributions (thin-tailed) with the shape parameter of the function is zero.The data is correlated with Gumbel distribution if the points on the graph lie in a straight line.A fat-tail or short-tailed distribution is represented by concavity or convexity in the graph.Sample MEF can be used to estimate non-normality by taking the sum of threshold excesses divided by number of exceedances for MEF of Pareto distribution.It describes the overshoot of an excess with reference to the threshold.A positive shape parameter and a heavy tail is observed if the empirical MEF is a positively sloped straight line.A correlation with a Gumbel distribution with a medium-sized tail occurs if a horizontal MER is found.A negative slope represents a short-tailed distributions.Skewness and excess kurtosis measure deviation from normality.Any significant value of skewness indicates an asymmetrical distribution whereas excess kurtosis shows heavy tails.Third or fourth moments for capturing skewness and kurtosis respectively are considered when risk analysts want to find out if a series of data contains non-normality.Parametric method assumes some distribution function for calculating risk measures.Non-parametric models can give inaccurate results should the market environment move or change because it relies on historical returns to predict future returns.Parametric models use two kinds of PDFs: Gaussian (normal) distribution and non-Gaussian distribution like Student-t.Recent studies have focused on applying the EVT and ARCH models because financial returns do not follow Gaussian distribution.Leptokurtosis of financial returns and Student-t model create an unconditional distribution with thick tails.The tail behavior depends on shape parameter or tail index which has to be determined under the Student t-distribution.The strongest sensitivity and pronounced leptokurtosis was related to the smallest values of ν for Student-t distribution.An accurate estimate of tail index was compulsory to produce reliable VaR values when a distribution had particularly high levels of leptokurtosis.Generalized ARCH (GARCH) process, a member of the ARCH family of financial models, captures distribution fat-tails and takes into account for conditional volatility by allowing for a changing return distribution over time.GARCH fails to model asymmetric effect of volatility even though it has shown to generate fatter tails.Negative shocks to market caused by bad news have a greater impact in increasing future volatility than positive shocks caused by good news.The variance is portrayed as a linear function of squared prediction errors. Focusing on power terms to transform data is the recent studies on ARCH method.Higher moments like skewness, kurtosis etc is essential to describe data accurately if non-normal distribution is in the data.Power ARCH (PARCH) and asymmetric PARCH (APARCH) uses power term that is optimized in the model rather than being imposed by the researcher to transform the data.APARCH model is capable of capturing potential asymmetry in the distribution caused by return shocks on future volatility.Tail behavior of futures returns distribution are associated more with negative shocks than positive one.APARCH model has been found to produce far better in-sample fits.EVT estimates the tail index of a fat-tailed distribution and calculates VaR and extreme VaR based on the index.It is more accurate in figuring out the VaR of a portfolio because it focus on the shape of the tails not the whole returns distribution.Using it can be hard to estimate portfolio risk because each asset has its own peak threshold value.It focuses on the tails to give a robust framework and better risk estimate especially useful in extreme market movements.Frechet distribution can be obtained which has a polynomially decaying tail and fat-tail.Density functions for Freiceht, Gumbell and Weibull are available.Weibull distribution can be used to predict losses related to extreme events.Weibull distribution gives a good approximation for loss function.Frechet distribution is obtained for fat-tail distributions of returns like Student-t model.Distribution of maxima focuses on behavior of large observations that exceed a certain threshold is discussed.EVT models statistical behavior of extremesover a given time horizon through Frechet, Weibull, and Gumbel distributions.The result is consistent with many statistical models of returns in finance and generally more accurate when specifically modelling tail ends of a distribution.

The opinion regarding the quality and importance is the basic advantages, disadvantages and mathematics of current parametric methodologies used to assess Value-at-Risk (VaR) because accurate VaR measures reduce a firm's capital requirement and reassure creditors and investors of the firm's risk level.Accurately estimating the VaR of a portfolio or an entire firm is crucial because managers and investors have certain risk thresholds that they can bear.VaR methods can be used by firms to determine financial risk and necessary amount of capital to set aside.Regulators are worried about capital of a bank incurring maximum loss if risk is underestimated during extraordinary periods like stock market crashes.It is important because undervalued risk can lead to possible financial distress.A good risk measure should not only be conservative and accurate but also be strongly correlated to the portfolio's true risk exposure.Efficient and timely computation is very important because the results are used by supervisors and internal risk managers to influence traders' incentives.A more efficient model allows more precise capital allocation and signals to traders.The Basel Committee on Banking Supervision at the Bank for International Settlements formed a VaR-based method for measuring a financial institution's risk using their own internal risk-management models.Internal risk-management models are subject to qualitative and quantitative restrictions.Quantitative criteria include: calculating VaR dailyusing a ten trading day horizon, assuming 99 percent confidence, and accounting for changing delta and impact of time-changing volatility on option prices.Qualitative criteria include: banks are to have independent risk-management divisions reporting directly to senior management, and VaR reports need to be considered when settling trading risk limits.Financial firms need to allocate capital against their estimated risk exposure.Capital requirements are based on VaR values generated by an institution's own risk models, and have drastically increased the demand for more accurate models to prevent over- or underestimate. Backtesting procedures are included in the amendment made to the Basel advisory report in 1996.This gives incentive to financial firms to have measures to improve accuracy to obtain regulators' approval.The testing would formally assess the risk models performance and compare them with actual risk levels in the market.This ensures the VaR values produced by the model meets the minimum econometric reliability requirements.The Basel Committee stated that the percentage of outcomes that VaR model measures should be consistent with given 99 percent confidence.The number of exceptions lying outside the theoretical expectation are counted then the ratio of exceptions to total that lie within 99 percent coverage is determined to evaluate the VaR model.Backtesting is often very difficult because distinguishing between unpredictable errors from poor risk management behavior is necessary.It attempts to remove model risk, or risk from inappropriate use of a theoretical model for the wrong purpose.Model risk happens when an asset-pricing model fails to account for some relevant price variation factor or inaccurately tries to duplicate certain variables by a deterministic process or assumes a normal distribution of price variations.The bank is needed to compensate for forecasting weaknesses by applying a multiplication factor to its reserved capital if the VaR model is inaccurate.

The topic should be further studied because many methodologies are available to calculate VaR with their limitations.Many past methods included a normality assumption which can produce misleading figures as most financial returns are characterized by skewness through asymmetry and leptokurtosis from fat-tails.Third and fourth moments are used to estimate the shape index parameter of the tail to capture these two characteristics.EVT focus on extreme values to calculate the tail ends of a distribution.There is no one particular model that is best for computing VaR although all of the models have proven to capture fat-tailed nature better than a normal distribution by emphasizing the benefits and limitations of the Student-t, ARCH family of models and EVT.One must combine good risk management rationale to determine which model is best applied to certain scenarios.VaR allows a stronger enterprise level for risk management because risk managers now have increasing responsibilities to observe and report daily risk of an entire firm.Financial institutions that are best able to determine possible risks efficiently use modern information technology and apply the most appropriate model will have a competitive edge over other institutions.Answering the question on what is the most appropriate risk measure to use for the appropriate probability distribution during crisis is crucial.

Fig 3.1 is the conceptual development map explaining how the thesis is written as portfolio optimization, portfolio measurement, data representation and risk measurement.

**III.6 STRUCTURAL CHANGE TESTING**

Li (2012)’s research problem is the global sub-prime crisis in 2007-2009 seriously impacted the economy of many countries. United States is not a close trading partner of New Zealand but affects her trading partners and indirectly affects her economy and real estate prices.Quarterly data including Housing Price Index and building permit, Gross Domestic Product, unemployment rate, currency exchange rate, building permit in New Zealand from 1988 to 2010 is collected to locate the date of structural change in housing prices in New Zealand during the global financial crisis. The Chow test demonstrates that a structural break took place in the first quarter of 2008 in New Zealand housing prices.

The key findings of previous research arethe Federal Reserve reduced its target federal funds rate slowly to 1% in 2003 after the stock market reached the peak in 2000.Investors seek high yield investments because of the low interest rates.Collateralized debt obligations (CDOs) backed by subprime mortgages appealed to these investors.Prime or subprime mortgages appeared to be safe investments because a borrower in trouble could refinance or sell the property out to repay the mortgage.Rate of defaults rose because house prices leveled off in 2006 and interest rate escalated.Some companies had difficulty financing their positions.The underwriting standards for subprime mortgages lowered after a decade passed.Mortgage brokers lent to households without sufficient assets or income to service the mortgages.Many investors took the triple - A ratings at face value and invested their portfolios with CDOs.Some companies like Citigroup created structured investment vehicles (SIVs) as off-balance-sheet entities to hold CDOs.The abovementioned CDOs became long-term assets for the SIVs because mortgages return principal slowly over years.Investors demanded to receive cash and the financial market contracted when the financial crisis broke in August 2007.New Zealand was a highly protected domestic economy with strong support from the agricultural sector.The international competitor competes with the country keenly.Reforms carried out included an extensive program of asset sales; abolition of wage and price controls; deregulation of financial markets; corporatization of government departments; and dismantling of import quotas and export subsidies. Agricultural sectors and traditional manufacturing industries were most adversely affected leading to a significant increase in unemployment from 4.2 to 10.1% in 1986 and 1991 respectively as a result of the reforms.New Zealand's economy benefits greatly from the global economic growth in the recent 10 years.China's fast expansion has caused a significant increase in commodity price.Exports increased and lowered those of its manufactured imports.New Zealand had a significant income increment and output over the past 14 years.Its real GDP rises by 3.5 per cent per annum on average even during the 1997 Asian crisis.The long period expansion was underpinned by five important factors:

1) High net immigration resulted in a rapid working-age population growth and a substantial increase in labour income, employment rate and salary growth;

2) The thriving commodity export prices like dairy products and presence of cheap manufactures from China result in a strong terms of trade gains;

3) Fiscal consolidation and structural reforms successfully boosted per capita potential growth;

4) A housing boom because of easy credit and increase immigration and a sufficient supply of global savings which give ready access to credit at a not so high domestic interest rates;

5) Rapid expansion of export markets in China.

However, New Zealand was affected by reversal of this supportive atmosphere severely during the global subprime financial crisis.This was mainly because of a sudden drop in commodity prices and shrunk in export.The supply of foreign credit was tightened and external deficit was very big by international standards.Thus, the New Zealand government has implemented fiscal policies to mitigate the economic problems like personal tax cuts and speeding up the public investments.New Zealand emphasized on subsidy for purchase of private housing and home construction since the 19th century. The First Labour Government started the development of state housing to solve the long term housing problems in the 30s.This type of housing was usually built at a relatively high standard and became an icon.It was the country's early and innovative welfare given to their citizens.New Zealand became the largest mortgage funds owner in the country and a lender of last resort in the 1970s.The Housing Corporation was started in 1974 to manage the state's rental stock of houses and give mortgage finance to low income households.It became the major tool for government integration in the residential market.Its home ownership rate reached 74% even though state housing accounted for only 5% of dwellings in New Zealand by the mid-80s.New Zealand began to start some policies shifting the country away from state owned enterprises, regulation of foreign exchange rate controls, business investment, interest and price controls, public provision of services.The currency was permitted to float and their centralized labor agreements were replaced by voluntary unions and bargaining at the individual and enterprise and levels, monetary policy was focused on maintaing low inflation.The recent growth of inner-city apartment market appear to be strange in New Zealand with very low population densities even in urban areas.The largest New Zeland cities have been experiencing urban spral from 1980s to 1990s a sharp contrast to the continuing relative decay of many US inner cities.Many New Zealand citizens started to blame the high employment rate of 24% because of homeownership.New Zealand's home ownership rate stayed high at 67% with 1.5 million owner occupied dwellings, over 8,000 were transacted and monthly value of residential changing hands were $3.4 million.It implied that one out of every fifteen houses were sold in 2006 assuming trading volumes of 96,000 annually and there was a building stock of 1.5 million dwellings.Housing assets as a percentage of total assets are also high partly because of high ownership rates.New Zealand is strongly aligned with market dominance of single detached housings located on pieces of land to minimize potential of overproduction because of speculative activity in downturn.

The important gapin knowledge and research is structural breaks will be best displayed using graphs.

The trends and themes of the research related to the topic are the financial tsunami in 2007 to 2009 caused a loss of confidence in the financial market.The previous Federal Reserve Chairman Alan Greenspan calls it a once in a half century, probably once in a century kind of event.The United States President Barack Obama held a negative view that the economy entered into a lost decade comparable to the Japanese recession in the 1990s.The sub-prime financial crisis which started in the U.S. subprime mortgage greatly impacted the real estate in the world.One of the most open economies in the world, New Zealand, was impacted by it.Changes in the price of self-occupied housing have multiple impacts on housing owners.Such changes affect many households' wealth because home equity constitutes a significantly large percentage of household wealth.Changes in residential property prices also impact the risk level of a homeowners' portfolio.Thus, knowing the date of housing price turning point in times of poor economy may give some useful insights when the next financial crisis arrives. The causes and consequence of the subprime lending in the US and how it affects the New Zealand real estate market will be discussed.

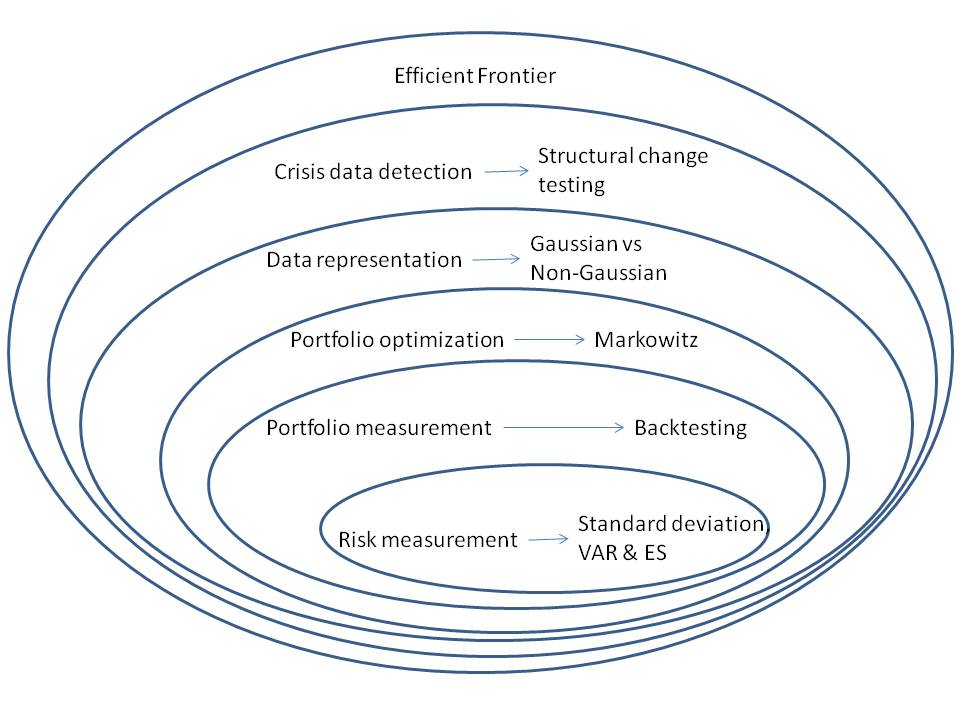
The relationships between key concepts are the graph showing the relationship between time and housing price is a continuous line when there is no special event occuring over time.However, time series of data can often have a structural break because of change in policy, for example, housing supply policy or sudden shock to the economy like 1987 stock market crash.The Chow test is often used to test for structural break.Chow's first test is used because the second test is used in predictions.The model uses an F-test to determine whether a single regression is more efficient than two separate regressions which splits the data into two sub-samples.It is a famous econometric test which split the sample into two sub-periods, estimates the parameters for each of the sub-periods and then tests if the two sub periods are equal with the help of F-statistics.It basically tests whether the single regression line or two separate regression lines fit the data best.

The contradiction is whether chow test can be used on generalized hyperbolic distribution for crisis situation.

The research designs or methods seemedinadequate are factors leading to the rise and fall of housing price changes.Previous research shows that a higher unemployment rate discourages individuals from purchasing home.Itconcurs that sustainable homeownership rates depend mainly on the expansion of housing supply rather than demand.Other research shows that existence of bubbles may be because of exchange rate volatility.The mortgage interest rate significantly affects an individual's decision to buy a residential unit.An increase in mortgage rate or mortgage payment including both the principal and interest discourages people from home purchase and thus decreases its demand.Significantimpactof interest rate on consumer spending are expected when houses serve as collateral.Decrease in interest rate leads to an increase in housing prices.The negative real interest rates caused sharp fluctuations in house prices in Hong Kong in the early 90s.There is a positive relationship between Australian Real Estate and Stock Market prices.It was evidence that first housing price boom in the early 70s was caused by sudden increase in oil price leading to an increase in construction costs in Taipei.An increase in urban housing demand and price was initiated by Japan's highest population growth about 40 years ago in Tokyo in 1947. Movements in per capita real GDP and total housing stock have correlations with real housing prices in the long run.Housing prices react more to an exogenous change in expected productivity or the world interest rate.Residential housing prices show strong downward price stickiness because homeowners usually have high reservation prices or resist selling their housing units below a certain price level during recessions.Home prices tend to decrease through inflation rather than decrease in nominal price. Quarterly data from 1988 to 2010 is collected which includes 1) Housing Price Index and building permit, 2) Gross Domestic Product, 3) unemployment rate, 4) currency exchange rate, and 5) building permit in New Zealand which provides information on housing supply.Gross Domestic Product from 1997 to 2007 increased by nearly 2 times while housing price in New Zealand increased by twice.The House Price Index attained the peak of 1519 and Gross Domestic Product attained the peak of NZD 46004 million in 2007.Sub-prime financial crisis occurred in 2007.The fall in housing price in US caused a loss of confidence to the financial system sucking out credit and leading to a liquidity shortfall in the financial system.The lack of liquidity causes the economy to contract and slows down international trade.US real output level reduced by over 10% while unemployment rate increased to 10%. The results of the chow test show that the structural break point occurred at first quarter of 2008.The first set of equations, that is, equation before the first quarter of 2008 display that RGDP, unemployment rate, exchange rate are positively related to Housing Price Index in the list of independent variables of this regressionmodel before using it to explore the structural pattern of the dependent variable.Building permit number represents the supply of housing shows a negative relationship with the housing price.On the contrary, all the above mentioned relationship between price and other factors are refuted when the second half of the equations.New Zealand exchange rate is negative significant related to the housing price index just opposite to the equation before first quarter 2008.

The opinion regarding the quality and importance is the global subprime crisis happened in the United States 2007 to 2009 had seriously affected many countries' economy.The financial tsunami affects the US trading partners like New Zealand even though they are not close trading partners and their far geographical distance from each other.It indirectly affects her economy and real estate prices.It therefore indirectly affects her economy and real estate prices.Unemployment rate climbed up and housing supply dropped as shown in the decrease in building permit.

The topic should be further studied because the chow test shows that the structural break occurred in the first quarter of 2008 in New Zealand housing price.Although correlation between various macro economic factors like real GDP and building supply are represented by the number of building permits strongly correlates with housing prices, many of them were insignificantly correlated with them.The interest rate showing the cost of borrowing in home purchase, economic factor in Australia in view of close relation between Australia and New Zealand economy and some other related variables will be used in the chow test model.The study is limited to New Zealand only in terms of geographical location.It would be better if more countries' data can be used in the study.



**Fig. 3.1**: Conceptual Development Map

**Literature Review Table 1**

|  |  |
| --- | --- |
| Topic Area | Representation of financial data during crisis |
|  |  |
| Sub-Topic | Gaussian versus non-gaussian distribution |
|  |  |
| Study’s Focus | The Generalized hyperbolic skew Students t-distribution |
|  | Check through references needed again. |
|  |  |
| Study | Aas & Haff, 2006 |
|  |  |
| Research Problem | - Daily or weekly return from financial markets e.g. forex rates, interest rates and |
|  | stock prices are skewed with polynomial and exponential tails. |
|  | - Generalized Hyperbolic (GH) Skew Student’s t-distribution is the best fit. |
|  | - VaR and expected shortfall show how polynomial and exponential tails is important. |
|  | - MLE estimators is computed with expectation-maximization (EM) algorithm and |
|  | GH Skew Student's t-distribution. |
|  |  |
| Key findings of | - Daily or weekly return from financial markets e.g. forex rates, interest rates and |
| previous research | stock prices are skewed with polynomial and exponential tails. |
|  | - Generalized Hyperbolic (GH) Skew Student’s t-distribution is the best fit. |
|  |  |
| Important gaps in | No exact time horizon of the data. |
| knowledge/research/data |  |
|  |  |
| Trends and themes of | - Daily or weekly return from financial markets e.g. forex rates, interest rates and |
| the research related to | stock prices are skewed. |
| the topic |  |
|  |  |
| Relationships | - The GH skew Student’s t-distribution provides the best fit for norwegian stocks, |
| between key concepts | international bonds, EUR/NOK exchange rate and European 5-year interest rate. |
|  | - At 5% significance level for the kupiec test, the null hypothesis is twice rejected |
|  | for Azzalini’s skew Student’s t-distribution, once for the normal inverse Gaussian |
|  | (NIG) distribution but never for the GH skew Student’s t-distribution. |
|  | - GH skew Student’s t-distribution has the lowest Dα value for each distribution and level. |
|  | - This is used to backtest the predicted expected shortfall value for confidence level α. |
|  | - It is the best compared to other distributions in predicting expected shortfall for |
|  | the given test data. |
|  |  |
| Inconsistencies | - GH distribution is seldom used because it is not particularly analytically tractable; |
| or contradictions | for very big sample sizes, it can be hard to differentiate between different |
|  | parametric values determining the subclass. |
|  | - It is because of the flatness of the GH likelihood function in those values. |
|  | - The MLE utilizing the EM algorithm can lead to local maximum. Picking the |
|  | right initial values to avoid that is important. |
|  |  |
| Research designs or | - The GH skew Student’s t-distribution skewness and kurtosis are not defined |
| methods seemed | when ν ≤ 6 and ν ≤ 8. |
| improper, insufficient, | - For risk measurement, VaR only measures a quantile of the distribution ignoring |
| or inadequate | the tails beyond them. Expected shortfall measures the tail risks better. |
|  |  |
| Opinion regarding the | - It is important to apply to data from emerging markets and extend the quality |
| quality and importance | of the data to include stocks, bonds, commodities and currencies during crisis. |
|  |  |
| Why the topic should | - Further studies should be conducted to apply the data during crisis to understand |
| be further studied | the impact on financial market returns. |
|  |  |

**Literature Review Table 2**

|  |  |
| --- | --- |
| Topic Area | Representation of financial data during crisis |
|  |  |
| Sub-Topic | Gaussian versus non-gaussian distribution |
|  |  |
| Study’s Focus | Exponentially decreasing distributions for the logarithm of the particle size |
|  |  |
| Study | Barndorff-Nielsen, 1977 |
|  |  |
| Research Problem | - Continuous type distribution is a hyperbola e.g. logarithm of probability density function. |
|  | - It is a hyperboloid in several dimensions and is investigated. |
|  | - Such distributions is a mixture of normal distributions. |
|  | - Focuses on the mass-size distribution of Aeolian sand deposits referring to the findings of |
|  | R. A. Bagnold |
|  |  |
| Key findings of | - This distribution family is applicable to empirical distributions pertinent to size distributions e.g. |
| previous research | of mined diamonds or of personal incomes. |
|  | - When studying size phenomena, the sizes of the single items cannot be observed directly but the |
|  | data is made up of grouped moment distributions. |
|  | - When working with moment-size distribution, the sizes of the single items sampled is not |
|  | completely independent random variables. |
|  | - A plot of histogram of mass-size distribution with logarithmic scales will strongly suggest a |
|  | theoretical curve which first increases almost linearly and after a smooth transition decreases |
|  | almost linearly. |
|  | - The hyperbola is the simplest mathematical curve representing such histogram. |
|  | - The fitted curve usually lies below the observed points in both tails of the distribution and above |
|  | the highest observed point(s) if the model gives a good description. |
|  | - Theoretical probability distribution commonly used in connection with empirical particle size or |
|  | mass-size distribution is the log-normal. |
|  | - The mean value and variance occurs with a brownian motion with drift. |
|  | - This is the clue to the derivation of the theoretical distribution through the stochastic process model. |
|  | - Pareto’s Law states that how people are distributed based on income, of firms according to size and |
|  | so forth has a strong tendency to decrease in the upper tail like a power of the variate studied. |
|  | - It is not mentioned whether the hyperbola distribution more suitable during crisis situation or not. |
|  |  |
| Important gaps in knowledge/research/data | - We are far from fully understanding and modeling the dynamical processes resulting in the  regularities in the distribution based on the size of the particles of wind blown sands. |
|  | - For a size distribution, if the *n* observations is considered as independent, then the procedure will |
|  | coincide with that of maximum likelihood. However, independence is not always a tenable |
|  | assumption in size distribution investigations. |
|  | - Maximum likelihood estimation assumes that the parameters can be expressed as logarithmic  returns. In the real world, this may not be so. |
|  |  |
| Trends and themes of | - Logarithm of probability density function during crisis is hyperbolic. |
| the research related to | - This is a non-gaussian distribution. |
| the topic |  |
|  |  |
| Relationships | - The mean and variance with a drifted brownian motion allows the derivation of the theoretical |
| between key concepts | distribution using the stochastic process model. |
|  |  |
| Inconsistencies | - Maximum likelihood estimation assumes that the parameters can be expressed as |
| or contradictions | logarithmic returns. In the real world, this may not always be so. |
|  |  |
| Research designs or | - The maximum likelihood estimation should be tested on logarithmic of probability density function. |
| methods seemed |  |
| improper, insufficient, |  |
| or inadequate |  |
|  |  |
| Opinion regarding the | - The quality of the data used can be improved if applied to financial market returns |
| quality and importance | from stocks, bonds, currencies and commodities. |
|  | - It is important to know that financial market returns from stocks, bonds, currencies and  commodities can be hyperbolic. |
|  |  |
|  |  |
| Why the topic should | - Further studies should be done to show how such distributions fit financial |
| be further studied | market returns better during crisis. |
|  |  |

**Literature Review Table 3**

|  |  |
| --- | --- |
| Topic Area | Representation of financial data during crisis |
|  |  |
| Sub-Topic | Gaussian versus non-gaussian distribution |
|  |  |
| Study’s Focus | Normal inverse gaussian distributions and stochastic volatility modeling |
|  |  |
| Study | Barndorff-Nielsen, 1997 |
|  |  |
| Research Problem | - The normal inverse Gaussian distribution is a variance-mean mixture of a normal distribution with |
|  | inverse Gaussian as the mixing distribution. |
|  | - It determines a homogeneous Levy process which is representable through subordination of |
|  | brownian motion by inverse Gaussian process. |
|  | - The Levy type decomposition of the process is determined. |
|  | - The relationship of the normal inverse Gaussian to classes of generalized hyperbolic and inverse |
|  | Gaussian distributions is reviewed briefly. |
|  | - A discussion on the potential of normal inverse Gaussian distribution and Levy process for |
|  | modelling and analysing statistical data with specific reference to extensive observations from |
|  | turbulence and from finance. |
|  | - The need to extend the inverse Gaussian Levy process to account for certain, frequently observed, |
|  | temporal dependence structures. |
|  | - Extensions of the stochastic volatility type are built through an observation-driven method |
|  | to stage space modelling. |
|  | - Generalizations to multivariate settings are shown. |
|  |  |
| Key findings of | - A normal variance-mean mixture distribution (normal inverse Gaussian distribution) is to construct |
| previous research | stochastic processes of interest for statistical modelling especially in turbulence and finance. |
|  | - Univariate normal inverse Gaussian distribution create homogeneous Levy processes. |
|  | - The normal inverse Gaussian Levy process may be represented through random time change of a |
|  | Brownian motion because of the mixture representation of the normal inverse Gaussian distribution. |
|  | - The normal inverse Gaussian Levy process is a subordination of Brownian motion by the inverse |
|  | Gaussian Levy process. |
|  | - Levy decomposition analysis proves that the processes can be viewed as a superposition of |
|  | weighted independent Poisson processes, weights of all numerically small sizes occurring. |
|  | - The small jumps are dominating the behavior of the normal inverse Gaussian Levy process. |
|  | - The norming constant is to ensure the probability density function sums up to 1. |
|  | - The normal inverse Gaussian and hyperbolic distribution are related to λ = -1/2 and λ = 1  respectively. |
|  | - The autocorrelations of an observed series from financial asset returns are 0 but the squared series |
|  | have positive autocorrelations decreasing slowly to 0. |
|  |  |
| Important gaps in | - The model presented is analogous to ARCH models but the conditional law of the volatility at |
| knowledge/research/data | time *t* with observations from previous time points is here non-degenerate. |
|  |  |
| Trends and themes of | - Normal inverse Gaussian distribution approximates most hyperbolic distribution very closely. |
| the research related to | - It can describe considerably heavier tail behavior than log linear rate of decrease characterizing |
| the topic | hyperbolic shape. |
|  | - This is especially so during financial crisis. |
|  | - Normal inverse Gaussian distribution has more tractable probabilistic properties than hyperbolic |
|  | distribution. |
|  |  |
| Relationships | - Financial crisis is represented with a lot of jumps during statistical modelling. |
| between key concepts | - These jumps are best modelled with Levy processes. |
|  | - Levy decomposition analysis proves that the processes can be viewed as a superposition of |
|  | weighted independent Poisson processes, weights of all numerically small sizes occurring. |
|  | - The small jumps are dominating the behavior of the normal inverse Gaussian Levy process. |
|  |  |
| Inconsistencies | - It is a contradiction to build the normal inverse gaussian distribution on top of gaussian distribution |
| or contradictions | because financial data distribution is non-gaussian during crisis. |
|  |  |
| Research designs or | - Only theoretical proof is given for the model described. |
| methods seemed | - Empirical proof to support the theoretical proof is missing. |
| improper, insufficient, |  |
| or inadequate |  |
|  |  |
| Opinion regarding the | - Normal inverse gaussian distribution plays an important role in modelling stochastic volatility. |
| quality and importance | - Statistical significance testing like log-likelihood and akaike information criteria should be given to |
|  | determine how well the normal inverse gaussian dsitribution fit the real financial data obtained |
|  | during crisis. |
|  |  |
| Why the topic should | - Whether normal inverse gaussian distribution can model stochastic volatility precisely during |
| be further studied | crisis should be further studied to understand risk taken commensurate with the return wanted. |
|  |  |

**Literature Review Table 4**

|  |  |
| --- | --- |
| Topic Area | Representation of financial data during crisis |
|  |  |
| Sub-Topic | Gaussian versus non-gaussian distribution |
|  |  |
| Study’s Focus | Normal variance-mean mixtures and z distributions |
|  |  |
| Study | Barndorff-Nielsen, Kent, & Sorensen, 1982 |
|  |  |
| Research Problem | - General properties of normal variance-mean mixtures including various new results are surveyed. |
|  | - The class of self-reciprocal normal variance mixtures is rather wide. |
|  | - Some tauberian results are established where relationships between the tail behavior of a normal |
|  | variance-mean mixture and its mixing distribution may be deduced. |
|  | - The generalized hyperbolic distribution and modulated normal distributions give examples of  normal variance-mean mixtures whose densities can be in terms of well-known functions. |
|  | - It is proven that the z distributions including hyperbolic cosine and logistic distributions are normal |
|  | variance-mean mixtures. |
|  | - Z distributions are the class of distributions from beta distribution through logistic transformation |
|  | after using location and scale parameters. |
|  | - Some properties of the associated mixing distributions are derived. |
|  | - The z distributions are self-decomposable. |
|  |  |
| Key findings of | - The general properties of normal variance-mean mixtures, generalized hyperbolic and modulated |
| previous research | normal distributions are surveyed. |
|  | - Delineate a broad class of self-reciprocal distributions. |
|  | - Elementary results about normal variance-mean mixtures is stated. |
|  | *- r*-dimensional generalized hyperbolic distributions is a quite wide class of normal variance-mean |
|  | mixtures. |
|  | - It can be the reciprocal of a gamma distributed random variable, *r*-dimensional *t*, gamma, McKay's |
|  | Bessel function, skew Laplace, normal Laplace, generalized inverse Gaussian and normal  distributions because of a number of useful mathematical and statistical properties. |
|  | - The family of generalized hyperbolic distributions is closed under margining, conditioning with |
|  | marginals and affine transformations. |
|  | - Variance-mean mixture distributions are self-decomposable. |
|  | - Generalized hyperbolic distributions are not self-decomposable because of the restricted, |
|  | homothetical definition of self-decomposability of multivariate distributions. |
|  | - All generalized hyperbolic distributions are infinitely divisible. |
|  | - An intuitive argument is given on why normal variance mixtures are capable of describing the  changes in many real data sets. |
|  | - It connects the mixing process to a number of active elementary errors randomly. |
|  | - The argument generalizes to an argument for normal variance-mean mixtures use if the elementary |
|  | errors are allowed a nonzero mean. |
|  | - The modulated normal distribuions which are examples of normal variance mixtures is studied as  an example. |
|  | - There are two types of modulated normal distributions: I and II. |
|  | - Type II modulated normal distribution is also self-decomposable. |
|  | - Mixtures of normal distributions are limiting distributions in generalizations of central limit problem |
|  | to nonindependent summands. |
|  | - The (unconditional) limiting distribution of the maximum likelihood estimator in nonergodic  stochastic processes is a normal mixture usually. |
|  |  |
| Important gaps in | - *x* is a random vector of normal variance-mean mixture distribution where Δ is a symmetric, positive- |
| knowledge/research/data | definite *r x r* matrix with determinant one. |
|  | - |Δ| = 1 is imposed to avoid an unidentifiable scale factor. |
|  | - This means it is possible to get an unidentifiable scale factor. |
|  | - The value of λ in Theorem 5.1 is unique but the function *L(u)* is unique up to any asymptotically |
|  | equivalent function. |
|  | - The description in Theorem *5.1* is sufficient for most purposes and *L(u)* is often asymptotically |
|  | equal to a constant. |
|  | - The conditions of Theorem *5.2* appear very mild but can be difficult to check in practice. |
| Trends and themes of | - The focus is on normal variance mixtures with a continuous type mixing disitribution. |
| the research related to | - This is a special type of normal distribution. |
| the topic | - *Z* distributions are surveyed and new properties from them are presented. |
|  | - *Z* distributions are normal variance-mean mixtures. |
|  | - Their mixing distributions are specified. |
|  | - *Z* distribution has log linear tails. |
|  | - When the density function is plotted on a logarithmic scale for the ordinate axis, the lower tail is a |
|  | asymptotical straight line with slope *α/σ but the slope of the asymptote of the upper tail is -β/σ.* |
|  | - The distribution is symmetric when *α = β.* |
|  | - It is negatively skewed when α > β or positively skewed when α < β. |
|  | - The full location-scale class considered that several common distributions or transformations of  such distributions are included in the class with limits tending to infinity to discriminate between  them. |
|  | - The log gamma model, logistic, generalized logistic and hyperbolic cosine distributions are all  classes of z distributions. |
|  | - *Z* distributions are useful as logarithmic income distributions. |
|  | - The Champernowne distributions has only two points of intersection with *z* distributions e.g. logistic |
|  | and hyperbolic cosine distributions. |
|  | - *Z* distributions are proven they can be represented as normal variance-mean mixtures. |
|  | - *Z* distributions belong to the extended Thorin class and therefore are self-decomposable. |
|  | - *Z* distributions are infinitely divisible previously established for the hyperbolic cosine and logistic |
|  | distributions. |
|  | - The extended Thorin class is closed under weak limits, the log gamma model belongs to the  extended Thorin class. |
|  | - It can be directly proven that the log gamma distribution and therefore the *z* distributions belong to |
|  | the the extended Thorin class. |
|  | - This can be easily deduced from product representation of the characteristic function of the log |
|  | gamma distribution. |
|  |  |
| Relationships | - In the theory and practice of statistics, mixtures of normal distributions are important. |
| between key concepts | - They are typical limit distributions in asymptotic theory for dependent random variables. |
|  | - They are used in data analysis for various heavy-tailed and skewed empirical distributions. |
|  | - Infinite divisibility of mixing distribution F implies that distribution P is infinitely divisible. |
|  | - Self-decomposability of mixing distribution *F* is not sufficient to ensure self-decomposability of |
|  | distribution P. |
|  | - If *F* is self-decomposable, then *P* is self-decomposable only if β = 0. |
|  | - It is deduced that *P* belongs to the extended Thorin class. |
|  | - Self-reciprocal distributions are those whose density and characteristic functions are proportional. |
|  | - They are normal, hyperbolic cosine and *r*-dimensional generalized hyperbolic distributions |
|  | with *λ = r/4*, *μ = 0* and *k = δ.* |
|  | - The class of self-reciprocal normal variance mixtures with *Δ = I* and absolutely continuous mixing |
|  | distribution can be generated. |
|  | - The extended function is integrable and can be normalized to integrate to 1 to get the probability |
|  | density function. |
|  | - The normal variance mixture with structure matrix I and the probability density function as mixing |
|  | distribution are self-reciprocal. |
|  | - Self-decomposability is a special statistical interest because only self-decomposable distributions |
|  | can occur as one-dimensional marginal distributions of stationary autoregressive schemes. |
|  |  |
| Inconsistencies | No conclusion and further research are given. |
| or contradictions |  |
|  |  |
| Research designs or | - Study the way how the tail behaviour of a normal variance-mean mixture depends on the tail |
| methods seemed | behaviour of the mixing distribution in the one-dimensional case. |
| improper, insufficient, | - No empirical proof is provided. |
| or inadequate |  |
|  |  |
| Opinion regarding the | Empirical proof should be carried out to support the theoretical proof given. |
| quality and importance |  |
|  |  |
| Why the topic should | To give the empirical proof to support the theoretical model proposed. |
| be further studied |  |
|  |  |

**Literature Review Table 5**

|  |  |
| --- | --- |
| Topic Area | Representation of financial data during crisis |
|  |  |
| Sub-Topic | Gaussian versus non-gaussian distribution |
|  |  |
| Study’s Focus | Hyperbolic distributions in finance |
|  |  |
| Study | Eberlein & Keller, 1995 |
|  |  |
| Research Problem | - Distributional assumptions for the returns on underlying assets is key in theories of valuation for |
|  | derivative securities. |
|  | - Investigate the distributional form of compound returns from daily prices of 30 DAX shares over |
|  | three years. |
|  | - Some of the standard assumptions cannot be justified after conducting some statistical tests. |
|  | - Introduce class of hyperbolic distributions which fitted to empirical returns with high accuracy. |
|  | - Discussed two models built upon hyperbolic levy motion. |
|  | - Derive a valuation formula for derivative securities after studying Esscher transform of the process |
|  | with hyperbolic returns. |
|  | - The results shows correction of standard Black Scholes pricing especially options close to expiration. |
|  |  |
| Key findings of | - The distributional form of returns of underlying assets is key in valuation theories for derivative |
| previous research | securities in finance. |
|  | - Introduce a model which fits the data with high accuracy and conclude about option pricing after |
|  | investigating classical assumptions especially the normality hypothesis. |
|  | - The Black Scholes formula is multiplicative and complete. |
|  | - The complete property allows duplication of cash flow of derivative securities. |
|  | - The valuation of these products by arbitrage is done. |
|  | - The qualitative picture stays the same if the time scale is changed because of the self similarity of |
|  | brownian motion as the source of randomness. |
|  | - Real stock price paths change greatly if examined on different time scales. |
|  | - Thus, discrete models with price changes at equidistant discrete time points is needed. |
|  | - This is a first approximation of reality where price changes at random time points. |
|  | - The correct return distributions for discrete models have to be determined. |
|  | - The assumption of normality fails when tests are applied to the real data. |
|  | - Hyperbolic distributions can fit the empirical distributions with great accuracy. |
|  |  |
| Important gaps in | - Daily KASSA prices of 10 of 30 stocks from german stock index (DAX) during 3 year period |
| knowledge/research/data | (2 October 1989 to 30 September 1992) |
|  | - Time series of 745 data points for each return is obtained. |
|  | - They are corrected for dividend payouts. |
|  | - Dividends are paid once per year for german stocks. |
|  | - Two unusual price changes occurred during the period: crash on 16 October 1989 and drop because |
|  | of Moscow coup on 19 August 1991. |
|  | - The 10 stocks were chosen because of their large trading volume and of specific activity of the |
|  | company to get a good representation of the market. |
|  | - Markets from US and Asia should be considered to give a comprehensive coverage on whether |
|  | hyperbolic distributions work well for them or not. |
|  |  |
| Trends and themes of | - Normal distribution is a poor model for stock returns, in this case, BASF and Deutsche Bank, |
| the research related to | especially during crisis. |
| the topic | - Quantile-quantile plots is used to test the goodness of fit for the stock returns distribution. |
|  | - Deviation from straight line representing normality is obvious. |
|  | - Corresponding normal and empirical density plots are completed too. |
|  | - There is more mass around the origin and in the tails than standard normal distribution can cover. |
|  | - Χ-square test for normality is done. |
|  | - Three different estimation procedures were done to avoid problems from partition sensitivity. |
|  | - Compute certain functions of the moments of the sample data then compare them with expected |
|  | values for a normal population. |
|  | - Kurtosis and skewness are used because of scale and location invariance to test the composite  hypothesis. |
|  | - They are both zero under assumption of normality. |
|  | - The hypothesis is rejected at 1% level for all stocks. |
|  | - The studentized range test is for testing normality of return distribution too. |
|  | - It is rejection at smallest level α = 0.005. |
|  | - Symmetric stable pareto distribution is represented by SP(α) where α is the characteristic component |
|  | is considered too. |
|  | - α = 2 provides the normal distribution and α = 1 provides the cauchy distribution. |
|  | - When α < 2, stable distributions are more peaked around the center than normal ones and have |
|  | arbitrarily heavy tails. |
|  | - When α < 2, variance is infinite. |
|  | - When α ≤ 1, the first moment does not exist. |
|  | - Models of stock returns for blue chips should have finite moments. |
|  | - The observed daily price changes are less than 20% for them, thus their variables are bounded. |
|  | - The stability under addition property is used to test the stable hypotheses because of the analytic |
|  | properties of this class of distributions. |
|  | - The return values are split into groups of increasing size with each group is summed. |
|  | - The characteristic exponent is estimated for each resulting distribution. |
|  | - The stable hypotheses is rejected if the the estimated characteristic component increases with each |
|  | increasing sum size. |
|  | - The serial correlation problem between successive returns is overcome with the data randomized |
|  | before building groups. |
|  | - If there is serial correlation, a higher kurtosis of monthly returns is induced. |
|  | - The value of the estimated alpha reaches 2 or is close to 2 for most of the shares concludes that gaussian |
|  | distributions for monthly returns is apt. |
|  |  |
| Relationships | - The standard continuous time model for stock prices is described by geometric brownian motion. |
| between key concepts | - It is what the Black Scholes formula is built upon. |
|  | - Returns from the geometric brownian motion are brownian motion process increments. |
|  | - They are independent and normally distributed. |
|  | - Hyperbolic distributions have hyperbolic log densities whereas normal distribution has parabolic log |
|  | density. |
|  | - The hyperbolic distributions can be expressed as normal, symmetric and asymmetric laplace, |
|  | generalized inverse gaussian and exponential distributions. |
|  | - α and β shapes the distribution. |
|  | - δ and μ are scale and location parameters. |
|  | - A maximum likelihood estimation is performed assuming independent and identically distributed  variables. |
|  | - Kolmogorov Smirnov test was carried out with values between 0.70 and 1.20 for all. |
|  | - Hyperbolic distributions are infinitely divisible. |
|  |  |
| Inconsistencies | - It is a contradiction to build the hyperbolic distribution on top of gaussian distribution because |
| or contradictions | financial data distribution is non-gaussian during crisis. |
|  |  |
| Research designs or | - Focus on empirical study to determine the correct distributions for the financial returns. |
| methods seemed | - Examined the normal, stable pareto, student and finite discrete mixtures of normals distributions. |
| improper, insufficient, | - The excellent candidate which provides a more realistic model is the class of hyperbolic distributions. |
| or inadequate | - The distributions are not tested during crisis to determine whether they still work as well. |
|  |  |
| Opinion regarding the | The study is important and the quality of the study can be improved by extending the data during crisis. |
| quality and importance |  |
|  |  |
| Why the topic should be further studied | The topic should be further studied by applying it to data during crisis to capture loss distribution accurately. |
|  |  |

**Literature Review Table 6**

|  |  |
| --- | --- |
| Topic Area | Portfolio optimization during crisis |
|  |  |
| Sub-Topic | Markowitz Optimization |
|  |  |
| Study’s Focus | Optimal Portfolio Selection Based on Expected Shortfall Under Generalized Hyperbolic Distribution |
|  |  |
| Study | Surya & Kurniawan, 2013 |
|  |  |
| Research Problem | - It talks about optimal portfolio selection problems using Expected Shortfall as the risk measure. |
|  | - The multivariate Generalized Hyperoblic distribution is the joint distribution for risk factors of  underlying portfolio assets like stocks, currencies and bonds. |
|  | - The optimal portfolio strategy is found using the multivariate Generalized Hyperbolic distribution. |
|  |  |
| Key findings of previous research | - The RORC optimization model is crucial because it is a tool to compare the performance of portfolios  used with a benchmark portfolio. |
|  | - JKSE portfolio is used as the benchmark. |
|  | - The portfolio produced using Generalized Hyperbolic distribution model performs better than JKSE  portfolio with performance measured by RORC with Expected Shortfall as risk measure. |
|  | - This shows that the optimization method used is superior to one used by JKSE. |
|  |  |
| Important gaps in | - Financial data are often not normally distributed. |
| knowledge/research/data | - They show properties that normally distributed data do not have. |
|  | - Empirical return distributions almost always show excess kurtosis and heavy tail. |
|  | - The logarithm of relative price variations on financial and commodity markets show a heavy-tailed  distribution. |
|  | - A Levy process with Variance Gamma distributed increments to model log price processes is proposed. |
|  | - Variance Gamma is a special case of Generalized Hyperbolic (GH) distribution. |
|  | - Other subclasses of GH distribution were proven to give an excellent fit to empirically observed  increments of financial log price processes especially log return distributions like Hyperbolic distribution,  the Normal Inverse Gaussian Distribution and the Generalized Hyperbolic skew Student's t distribution. |
|  | - The student's t and normal distributions are limit distributions of GH. |
|  | - These are reasons for the popularity of Generalized Hyperbolic family distributions because they give a  good fit to financial return data and are also extensions to the well-known student's t and normal  distributions. |
|  | - It has a critical problem. |
|  | - It can lead to a centralized portfolio when applied to nonelliptical distributions though it is coherent for |
|  | elliptical distributions against the diversity principle. |
|  | - It is also a generally nonconvex function of portfolio weights causing portfolio optimization to be an |
|  | expensive computational problem. |
|  | - Expected shortfall (ES) is the more recent and commonly used risk measure. |
|  | - It was made popular because of VaR's downside. |
|  | - It always results in a diversified portfolio as a coherent risk measure compare to VaR. |
|  | - It takes into account the behaviour of return distributions at and beyond a selected point. |
|  | - It displays the behaviour of the distributions' tails with a much wider scope like VaR. |
|  | - These attributes make it more favourable than its classic counterpart and only this measure is focused. |
|  |  |
| Trends and themes of | - Alternative risk measures must be considered because return data are nonnormal with heavy tails and  volatility is not designed to capture extreme large losses. |
| the research related to | - Value-at-Risk (Var) as a risk measure can satisfy this. |
| the topic | - It determines the point of relative loss level that is exceeded at a specified degree not measuring return  deviations from its mean. |
|  | - It can measure the behavior of negative return distributions at a point far away from the expected return |
|  | when appropriately adjusted. |
|  | - It can take into account the extreme movements of assets return. |
|  | - It gives an easy representation of potential losses because it is none other than the quantile of loss  distribution for a continuous distribution. |
|  | - This mainly contributed to its popularity. |
|  |  |
|  | - The Generalized Hyperbolic (GH) distribution is based on the Generalized Inverse Gaussain distribution  (GIG) and Multivariate Normal Mean-Variance Mixture Distribution (MNMVM). |
| Relationships | - The Generalized Hyperbolic distribution possess the linearity property. |
| between key concepts | - The general properties of Expected Shortfall relevant with uncertain loss in a portfolio because of the  volatility of financial market is discussed. |
|  | - Risk meaurement must account for the randomness of loss and be used to determine the capital reserve |
|  | to account for future loss. |
|  | - The subadditivity and positive homogeneity attributes together imply the convexity of the Expected  Shortfall which is very useful in dealing with portfolio optimization problems. |
|  | - The most critical aspect in portfolio optimization is modelling portfolio risk. |
|  | - The portfolio loss function is defined because the risk comes from portfolio loss value over the holding  period. |
|  | - The risk factors are chosen to be logarithmic price of financial assets, yields or logarithmic exchange  rates because the distribution models of their time increments have been empirically known. |
|  | - First-order approximation gives a convenient computation of loss because it shows loss as the linear  combination of risk-factor changes. |
|  | - It is best used when risk-factor variations have small time horizon and when portfolio value is almost |
|  | linear in risk factors. |
|  | - The loss of the stock portfolio is defined. |
|  | - The loss of the zero coupon bond portfolio is defined. |
|  | - The loss of the fixed coupon bond portfolio is defined. |
|  | - The loss of the currency portfolio is defined. |
|  | - The weight for assets valued in foreign currency varies slightly from weight of other domestic assets. |
|  | - This weight can be considered as usual weight multiplied by exchange rate of currency of which it is  denominated. |
|  | - Such interpretation is consistent with conversion process of its foreign value to its base value. |
|  | - The distribution of the portfolio loss using the linearity property of Generalized Hyperbolic distribution  is determined because the risk mapping for each of the portfolio's assets have been done. |
|  | - Only the linearised portfolio loss is considered because of this linearity property to ensure future  calculations are tractable. |
|  | - One of the advantages of modelling risk-factor increments with Generalized Hyperbolic distribution as  linearised portfolio is also Generalized Hyperbolic distributed. |
|  | - Both portfolio loss and profit function are approximated by its linearised counterpart argued by linearity |
|  | property of Generalized Hyperbolic distribution. |
|  | - Portfolio optimization for both symmetric and asymmetric cases are defined. |
|  |  |
| Inconsistencies or contradictions | It is uncertain whether expected shortfall on generalized hyperbolic distribution will work well during crisis. |
|  |  |
|  |  |
| Research designs or | - Numerical results for 4 stocks, 3 foreign currencies, 2 Indonesian government-issued zero coupon bonds |
| methods seemed | and 1 Indonesian government-issued international fixed coupon bond are discussed. |
| improper, insufficient, | - The base currency is IDR and the stocks are chosen from international blue-chip companies. |
| or inadequate | - First-order approximation of portfolio loss is accurate and robust, and is justified. |
|  | - Only the asymmetric Generalized Hyperbolic model is focused because portfolio |
|  | optimization in symmetric framework can be solved analytically. |
|  | - An advantage of modelling with asymmetric model is that it gives an additional degree of freedom |
|  | without fixing the parameter gamma to 0. |
|  | - From observing the calibration result whether value of gamma and its skewness are close to 0 or not will |
|  | determine whether the better model is asymmetric or symmetric. |
|  | - Calibration results are percentage risk-factor increments of assets presented for both univariate |
|  | and multivariate asymmetric Generalized Hyperbolic distributions. |
|  | - The calibration is done using the EM algorithm. |
|  | - Goodness of fit of calibrated parameters of Generalized Hyperbolic on data are analysed. |
|  | - Some unvariate data examples will be first analysed to gain some confidence that Generalized |
|  | Hyperbolic gives a good fit. |
|  | - Comparison between histogram of empirical distribution and pdf of theoretical distributions are done to look at how the kurtosis and skewness of theoretical distributions match those from the actual distributions. |
|  | - Numerical results of markowitz optimization problem are obtained. |
|  | - Numerical results of RORC optimization problem are obtained. |
|  | - Backtesting is done by comparison with JKSE index. |
|  | - This is to check whether the portfolio's performance in a frictionless world can surpass the stock |
|  | market's performance. |
|  | - The in-sample period is from 6 February 2008 until end of 2010 and the out-of-sample period is from start of 2011 until 4 march 2011. |
|  | - The JKSE composite index data is first fit and multivariate risk increment data from the portfolio |
|  | within the in-sample period with the Generalized Hyperbolic distribution. |
|  | - Next, the expected return of the JKSE index is extracted from the calibrated distribution and value input |
|  | to optimization engine for portfolio to find the minimum expected shortfall that can be obtained and |
|  | the optimal portfolio weights too. |
|  | - The Expected Shortfall of daily losses between that of obtained optimal portfolio is compared with |
|  | the one from JKSE composite index in the out-of-sample period. |
|  |  |
| Opinion regarding the | - A way to solve portfolio optimization problems when expected shortfall is used as risk measure and |
| quality and importance | when asset return distribution is modelled by Generalized Hyperbolic distribution is developed. |
|  | - Analytical solutions to Markowitz and RORC portfolio optimization problems are obtained in the |
|  | framework of symmetric Generalized Hyperbolic. |
|  | - Such solutions can be obtained wih the linearity property of Generalized Hyperbolic. |
|  | - The optimal weights from Markowitz optimization model with Expected Shortfall are equal to |
|  | those from using volatility as risk measure by linearity of Generalized Hyperbolic distribution. |
|  | - The problem is reduced to classical Markowitz optimization problem. |
|  | - Optimization problems using Expected Shortfall are solved numerically in asymmetric framework. |
|  | - Initially, evaluating Expected Shortfall as function of portfolio weights is evaluating high dimensional integral. |
|  | - This almost intractable problem can be greatly simplified into a one dimensional integral problem |
|  | with the linearity property of the Generalized Hyperbolic distribution. |
|  | - The Markowitz-optimal composition from using Expected Shortfall have also been found to be |
|  | not necessarily equal to those from using volatility. |
|  | - The compositions are very similar for high expected returns when no shortings are allowed. |
|  | - The nonconvexity of the RORC problem can be avoided by changing it into a number of convex |
|  | Markowitz optimization problems. |
|  | - That depends on the desired accuracy. |
|  | - Optimal weights cannot be achieved for RORC optimzation version when asset shortings are allowed |
|  | because the existence condition for maximimum RORC is not satisfied although it tends to asymptotic value. |
|  | - There is no existence condition of the maximum RORC when Expected Shortfall is used in asymmetric framework. |
|  | - The trend of the plot is forever increasing to an asymptotic value. |
|  | - The plot behaves similarly with plot when volatility is used as risk measure is produced where |
|  | optimal weights cannot be achieved. |
|  | - Such findings give the confidence that maximum RORC initially is unachievable and RORC tends |
|  | to some asymptotic value. |
|  |  |
| Why the topic should | - It is crucial to apply expected shortfall with generalized hyperbolic distribution during crisis to check |
| be further studied | whether it still works or not. |
|  |  |

# CHAPTER IV.DISSERTATION DESIGN AND METHODS

## IV. 1 INTRODUCTION

The standard continuous time model for financial data like stock, bond and currency prices is described by geometric Brownian motion which is highly uncertain and random. Returns from the geometric Brownian motion are Brownian motion process increments. They are independent and normally distributed. However, during crisis, financial data do not follow the Gaussian or normal distribution because of the large and frequent jumps. They display properties that Gaussian financial data do not have. Logarithmic relative price changes on financial and commodity markets display heavy-tailed distributions (Mandelbrot 1963). The original generalized hyperbolic (GH) distribution is introduced where variance gamma is a special case (Barndorff-Nielsen1977). A lévy process with variance gamma distribution of increments is provided to model logarithmic price processes (Madan and Seneta1990). Subclasses of GH distribution proved to be excellent fits to empirically observed financial log price processes increments especially logarithmic return distributions. Hyperbolic distribution is discussed (Eberlein and Keller1995).

Normal inverse Gaussian distribution is a variance-mean mixture of a normal distribution with inverse Gaussian as mixing distribution. It determines a homogeneous Levy process represented by the subordination of Brownian motion by inverse Gaussian process. The canonical Levy kind decomposition of the process is determined (Barndorff-Nielsen1997).A set of stylized empirical facts appear during the statistical analysis of price variations in various kinds of financial markets. Various statistical properties of asset returns are: distributional properties, tail properties and extreme fluctuations, pathwise regularity, linear and nonlinear dependence of returns in time and across stocks. Empirical return distributions often display excess kurtosis and heavy tail (Cont2001).Generalized hyperbolic skew student’s t distribution has the important property of two tails exhibiting different behavior. One is polynomial, while the other is exponential. It is the only subclass of generalized hyperbolic distribution with this property. This serves as a perfect fit to skew financial data exhibiting such tail behaviors (Aas and Haff2006).Student’s t and Gaussian distributions are limit distributions of GH. GH family distributions are popularly used because they provide a good fit to financial return data and are extensions to common student’s t and normal distributions.Multivariate generalized hyperbolic distributions possess diminishing tails growing very fast for multivariate modelling. Robust and fast estimation procedures are rare in a limited data environment. Its alternative class is proposed with random vectors that are stochastically independent and generalized hyperbolic marginals affine-linearly changed. They have good approximation attributes and have appealing reliance framework. Dependence of extreme events (tail dependence) can be modelled with them. The essential approximation and arbitrary number creation methods are given (Schmidt et al. 2006). GH distribution is used to figure out the appropriate distribution for the portfolio loss distribution (Surya and Kurniawan 2013). There are two mains reasons for doing that. First, this is because of the linearity property of GH distribution. Only linearized portfolio loss functions will be considered to ensure future calculations are tractable because of this special property. Second, it encompasses the Generalized Inverse Gaussian distribution and Multivariate Normal Mean Variance Mixture distribution. This allows the flexibility to model a wide range of portfolio loss distribution. Normality of observations and regression disturbances tests through Lagrange multiplier procedure or score test on the Pearson family of distributions are obtained (Jarque and Bera 1987). They have optimum asymptotic power properties and good finite sample performance. They prove to be useful tools in statistical analysis of multivariate GH distribution because of their simplicity. The commutativity of robust estimators of multivariate location by applying estimators after a preliminary transformation to principal components coordinates or to sphericized coordinates is addressed (Gnanadesikan and Kettenring 1972). The transformation is robustified with a data-dependent transformation namely when the sample covariance or correlation matrix is used for obtaining the transformation. The objectives of techniques for detecting multivariate outliers are intertwined with those related to methods of assessing the joint normality of multiresponse data. The probability distribution of stock price changes is studied by scrutinizing the trades and quotes to record every trade for all stocks in 3 US stock markets from Jan 1994 to Dec 1995. A sample of 40 million data points is extracted substantially, substantially larger than studied hitherto. They found that an asymptotic power-law behavior for the cumulative distribution with an exponent α ≈ 3 outside the Levy regime (0 < α < 2) (Gopikrishnan et al. 1998).An accurate scientific representation of the portfolio returns distribution composed of random joint multivariate distribution asset is carried out. A non-linear fractional covariance matrix is generalized by a non-linear transformation with returns as Gaussian variables using covariance matrix to measure dependence between the non-Gaussian returns. It becomes the definite fat tail framework of the fundamental marginal distributions for firmness and good control. The portfolio distribution is a mapping to particle physics field theory with a substantial treatment using Feynman diagrammatic approach and large divergence theory for multivariate Weinbull distributions. Reducing the portfolio variance i.e. small risks may enlarge risks measured by higher normalized cumulants. Substantial empirical tests on the foreign exchange market prove the theory. An ample prediction of risks of a portfolio hinges much more on the appropriate description of the tail structure not on their interdependence for fat tail distributions (Sornette et al. 2000).

Alternative risk measures have to be considered because financial return data are non-Gaussian with heavy tails and volatility cannot capture extreme large losses. Volatility only measures financial return deviations from its mean. Value-at-Risk (VaR) is a good example. It determines the point of relative loss level exceeded at a specified degree. It can measure the behavior of negative return distributions at a point far from the expected return when adjusted suitably. It accounts for extreme movements of financial assets return. It is the quantile of loss distribution in a continuous distribution. However, it has a serious disadvantage. It can lead to a centralized portfolio when applied to non-elliptical distributions violating the diversity principle (Artzner et al. 1999). It leads to a generally nonconvex function of portfolio weights. Portfolio optimization becomes an expensive computational problem.

Expected shortfall (ES) as a risk measure responds to VaR’s disadvantage (Artzner et al. 1999). It is a coherent risk measure which always leads to a diversified portfolio. It takes into account the behavior of financial return distributions at and beyond a selected point. It shows how the distributions’ tails behave like VaR with a much bigger scope. These attributes make it more favorable than VaR. We will focus on ES in this paper.

## IV.2 DISSERTATION METHOD

The proposed dissertation is carried out in four phases from Fig. 4.1. The first phase is posing　the right questions about the appropriate risks to be quantified and managed efficiently and optimally to produce the propitious expected return wanted during crises. Phasetwo attempts to capture the real world involved using the mathematical model formulated subjected to the appropriate assumptions and constraints. Phasethree is about the computation of the formulated mathematical model. Phasefour is about verification of the formulated mathematical model against the real world.

**Fig. 4.1**: Dissertation Method

In the world of finance, take care of the risks and the propitious expected returns will take care of itself not vice versa during crisis. This is especially true in portfolio management consisting of stocks, bonds and currencies. Each asset class has its own unique risk factor. For stocks and currencies, logarithmic return is sufficient to explain that. For defaultable corporate and government bonds, in addition, the probability of default through the hazard rate is necessary to explain that based on the well-known Vasicek Model. The recovery rate determines how much of the portfolio will be recovered during default. The generalized hyperbolic distribution is more adept at fitting asymmetric data distribution common during financial crisis. In addition, it also allows linearization of risk factors which beautifully simplifies the computation process later. The optimal efficient frontier using generalized hyperbolic distribution is constructed by minimizing the expected shortfall subject to the defined risk factors. My mathematical model as follows attempts to capture that.

**IV.3 GENERALIZED HYPERBOLIC DISTRIBUTION**

McNeil, Frey, & Embrechts (2005) mentioned the Generalized Hyperbolic Distribution is built upon the Generalized Inverse Gaussian and Multivariate Normal Mean Variance Mixture Distributions.

**Definition IV.3.1 Generalized Inverse Gaussian Distribution (GIG).**

The random variable, Z, is a GIG represented by Z∼ N−(λ, χ, ψ) if its probability density function is:

, *z, ,> 0,* (4.1)

represents a modified Bessel function of the third kind with index λ fulfilling the parameters:

λ< 0

λ = 0

λ> 0

**Definition IV.3.2 Multivariate Normal Mean Variance Mixture Distribution(MNMVM).**

A random variable X ϵ is MNMVM if it is represented by the following:

γ + (4.2)

with γϵand A ϵ, a matrix, as distribution parameters, Z ∼ Nk (0, Ik) is a standard multivariate normal random variable. *W* is non-negative, scalar mixing random variable independent of *Z*. Σ *:= AA’* must be positive definite. In a univariate model, Σ is replaced by σ2.

This was a new class of distribution first proposed by Barndorff-Nielsen (1977) for multivariate GH distribution. It was further developed by Barndorff-Nielsen, Kent, and Sorensen(1982). Its parameters are: μ, location parameter; γ, skewness parameter; Σ, scale parameter; , shock factor for skewness and scale.

The GH distribution is defined from (4.2). It is linked to the lévy process where the levy-ito decomposition allows it to do linear transformation of risk factors.

**Definition IV.3.3 Generalized Hyperbolic Distribution (GH).**

A random variable *X*ϵ is GH-distributed if it is represented by

X∼GH(λ, χ, ψ, ) (4.3)

iff it has (4.2) with ∼ N−(λ, χ, ψ) is a scalar GIG distributed random variable. X is symmetric iff

(4.4)

The above pdf is consistent with the definition of GH first proposed by Barndorff-Nielsen(1977). It has the following normalizing constant:

(4.5)

(4.2) contributes significantly to the linearity property of GH distribution. The following theorem is central to solving optimal portfolio selection problems discussed with greater details in Section 6.

**Theorem 4.1**

If ***X****∼ GHd (λ, χ, ψ, )* and ***Y*** *=* ***BX*** *+* ***b*** given ***B***ϵ and **b**ϵ, then

***Y****∼ GHk (λ, χ, ψ, )* (4.6)

The theoretical proof for Theorem 4.1 is found from Proposition 3.13 (McNeil et al. 2005).

**IV.4 EXPECTED SHORTFALL**

This section will talk about the general properties of expected shortfall as risk measure relevant to uncertain loss in a portfolio because of extreme volatility in the financial market during crisis. It must consider the randomness of the loss to determine the certainty of return amidst the uncertainty. It will be discussed as follows.

**Definition 4.1 Expected Shortfall (ES) for Continuous Distributions.**

*ES* is the expectation of portfolio loss *L* conditional on itself being at least its with *α ϵ (0, 1)*.

ESα(L):(4.1)

Acerbi and Tasche(2002) proved that ES is a coherent risk measure.

**Theorem 4.2**

*ESα(L), expected shortfall of a portfolio loss L, is a coherent risk measure for all α ϵ (0, 1).*

*ESα(L + l) = ESα(L) + ESα(l) if l is constant (translation-invariance criterion)*

*ESα(λL) = λESα(L) if λ is a positive constant (positive homogeneity criterion)*

*If is another portfolio loss,*

*ESα(L + ) ≤ ESα(L) + ESα() (subadditivity criterion)*

*ESα(L) ≤ ESα()if L ≤ (monotonicity criterion)*

The theoretical proof for Theorem 4.2 is found from 6.1.1 The Axioms of Coherence(McNeil et al. 2005).

The convexity of the ES is implied by the subadditivity and positive homogeneity criteria considered together. This is very useful during portfolio optimization problems in later sections.

**IV.5 PORTFOLIO STRUCTURE**

Modelling of portfolio risk plays a critical role in portfolio optimization problems. Defining the loss function of a portfolio is important because risk comes from portfolio loss value over a holding period.

*V(s)* is the portfolio value at calendar time *s*. Given time horizon ∆, the loss of the portfolio over period [s, s + ∆] is defined as

(5.1)

∆ is assumed to be a fixed constant in constructing the portfolio theory. It is more convenient to use the following definition

(5.2)

Time series notation is . From here on, any random variables with subscript *t* are assumed to be defined in similar way.

If calendar time is measured in years and daily losses are being considered, is set to 1/250. *t* is measured in days, while is a time unit conversion from days to years. and is the portfolio value on days *t* and *t + 1* respectively. is the loss from day *t* to day *t + 1*.Equation (4.2) will be used to define portfolio loss with *t* measured in time horizon *∆*.

is modelled as a function of time *t* and a *d*-dimensional random vector of risk factors assumed to follow discrete stochastic process.is represented as

(5.3)

for measurable function f :. The selection of *f* depends on the assets in the relevant portfolio. The risk factors commonly used are logarithmic price of financial assets, yields or logarithmic exchange rates. For this paper, the risk factors chosen take one of these forms because the distribution models of their time increments are empirically known in the introduction.

The increment process, , is defined as With the mapping from (5.3), the portfolio loss can be expressed as

A first-order approximation of (5.4) can be considered if f is differentiable.

(5.5)

where partial derivatives is denoted by subscripts to *f*. The first-order approximation provides a loss computation because it represents loss when the linear combination of risk-factor changes. It is used best when the risk-factor changes are probably small i.e. the risk measured is in small time horizon and when the portfolio value is almost linear in the risk factors i.e. the function *f* has small second derivatives.

**IV.5.1 Stock Portfolio**

We will look at a fixed portfolio of stocks and is the number of stock *i* in the portfolio at time *t*. The stock *i* price process is .  *:= ln*  is the risk factor because one of the GH subclass, the Variance Gamma distribution, have been used to model the logarithmic price process of financial assets in the past by Madan and Seneta (1990) to fit in the GH framework.

The increment of the risk factor is assumed to be the stock’s logarithmic return,  *:= ln*.

Therefore,

= (6.1)

and

= . (6.2)

From first order approximation in equations (6.1) and (6.2), the loss function is linearized as

= = . (6.3)

is the stock portfolio weight of stock *i* at time *t*. Equation (6.3) is the linearized risk mapping of the stock portfolio. If the stock’s logarithmic return is generally small, the error from such linearization is small.

**IV.5.2 Zero Coupon Bond Portfolio**

**Definition 5.1 (Zero Coupon Bond Portfolio)**

A zero coupon bond or zero with maturity date *T* and face value *K* is a contract promising its holder payment of amount *K* paid at date *T*with price at time *t* where *0 ≤ t ≤ T* expressed as .

**Definition 5.2 (Continuously Compounded Yield)**

A continuously compounded yield at time *t* with *0 ≤ t ≤ T*, , for a zero coupon bond with maturity date *T* and face value *K* is defined as factor *y* satisfying

(7.1)

or equivalently

. (7.2)

is assumed to be the risk factor in zero coupon bond portfolio because it is the logarithmic price of a zero coupon bond.

We will look at a fixed portfolio with zero coupon bonds. Each is with maturity and face value for i = 1,2,…, . The current time *t* is . is the number of bonds with maturity in the portfolio. := is the risk factor. The increment of risk factors is

The value of the portfolio at time *t* is

= (7.3)

The linearized loss can be obtained using first order approximation similar to that in the stock portfolio as

=

= (7.4)

is the zero coupon bond portfolio weight of bond *i* at time *t*.

**IV.5.3 Currency Portfolio**

We will look at a currency portfolio with number of foreign currencies and is the value of currency *i* in corresponding currency denomination at time *t*. The currency *i*exchange rate process in foreign or domestic value is . The risk factor is assumed to be  *:= ln* . The portfolio value at time *t* is

= (8.1)

After notation replacements, equations (6.1) – (6.3) can be used to derive the risk mapping for linearized currency portfolio loss,

= (8.2)

is the currency portfolio weight of currency *i* at time *t*.

Let us look at a fixed portfolio with *d* non-currency assets valued in foreign currency. Asset *i* is valued in currency *i*. is the price of asset *i* depending on risk factor . is the amount of asset *i*. is the exchange rate of currency *i* with respect to the base currency. The portfolio value at time *t* in base currency is

= (8.3)

Each asset of the portfolio contains two risk factors. They are the intrinsic risk factor and logarithmic exchange rate. The linearized portfolio loss is obtained from e quation (5.5) to be

=

= (8.4)

is the asset’s weight. and are the risk-factor increments for asset and foreign currency respectively. The weight for assets valued in foreign currency is different from the weight of other local assets. It is the usual weight multiplied by the exchange rate of the currency it is denominated in. This aligns with the conversion process of its foreign value to base value.

**IV.6 LOSS DISTRIBUTION**

The determination of the distribution of the portfolio loss is using the linearity property of GH distribution in theorem 4.1 after the risk mapping for each of the portfolio’s assets. Only the linearized portfolio loss function is considered to ensure future calculations are tractable because of this linearity property. We will consider a fixed portfolio with stocks, currencies and zero coupon bonds. We assume that all assets are in the base currency. Employing the formula of linearized loss from preceding sections, the linearized loss of the portfolio is

(9.1)

*:= ln, := ln* and are the risk factors for stocks, currencies and zero coupon bonds respectively. Let as assumed.

*X*ϵ with + + . With equation (9.1), it is clear that

(9.2)

is the weight vector to each portfolio assets arranged in similar fashion as *X.*

***B***ϵ and **b**ϵare diagonal matrix and constant vector respectively.

(9.3)

and

Using the same case except some of the non-currency assets are in foreign currency. Using the arguments in section IV.5.3, equation (9.2) is used to represent the linearized loss of this portfolio except each component of the weight vector corresponding with assets valued in foreign currency must be multiplied by the foreign exchange rate at time *t* and the diagonal matrix *B* must be modified into a sparse matrix with same diagonal entries previously, the non-diagonal entries

(9.5)

with and *j*

Using theorem 4.1, the previous representation is simplified into

(9.6)

with *∼ GH (λ, χ, ψ,)*. By theorem 4.1, *∼ GH (λ, χ, ψ,)*.

This is one of the pros of modelling risk-factor increments with GH distribution. The linearized portfolio is also GH distributed. Equation (9.6) represents portfolio losses for optimization purposes. A portfolio loss function is represented by its weight vector and the vector of risk-factors increments through equation (9.6).

GH covers more general processes than jump-diffusion process. λ,χ, ψ are from the mixing distribution. are the location, scale and skewness parameters respectively. Such parameters already take into account skewness, kurtosis and jumps of the distribution. Only static not dynamic portfolio optimization is done. Thus, it is not necessary to consider the jump-diffusion process.

**IV.7 PORTFOLIO OPTIMIZATION**

Both portfolio loss and profit functions are approximated by their linearized counterparts from the linearity property of GH distribution. The notation L will be used as linearized portfolio function from now on to replace . The portfolio optimization problem is set up as follows:

***X****∼ GH (λ, χ, ψ, ) (10.1)*

is a *d*-dimensional GH random variable. ***X*** is thevariable in equation 9.6 after adjusting the parameters of ***X***because equation (9.6) can be used to represent portfolio losses. *ϵ ϵ R}* is the set of portfolio losses and domain for Expected Shortfall *E*. The distribution L is a function of portfolio weights as discusses in the previous section. This includes expectation and risk measure.

**Definition 7.1 (Markowitz Optimization Problem)**

min *ς*

subject to *R =*  and ***w’1*** *= 1*, (P1)

is the targeted expected return. ***w*** is the portfolio weights.

*ςE (10.2)*

*RE. (10.3)*

The portfolio loss *L* has a one-to-one correspondence with , the portfolio weights from equation (9.6). It follows that *ς* is convex on because *E* is convex. This approach is appropriate during crisis where the risk can be present throughout the whole loss distribution especially expected shortfall capturing tail risk. The main goal during crisis is to minimize the portfolio loss *L* given the targeted expected return .

In contrast, in a normal situation, the main goal is to maximize the targeted expected return given the standard deviation σwhere the risk may not be prevalent throughout the whole loss distribution.

**IV.7.1 Optimization in Asymmetric Case**

*E* cannot be expressed in basic functions in the asymmetric GH framework because it is not a linear transformation of volatility. The factor *q* in the equation contains the term .

*E∼ GH (λ, χ, ψ, 0). (10.4)*

It is expressed in integral form as follows for numerical computation

*E*

*=*

=∼ GH (λ, χ, ψ,μ, **Σ**, γ). and are the cumulative distribution function and probability density function of Y respectively. The computation of is by numerical root funding methods. The integral of equation (10.5) is by numerical integration methods.

Transforming the normalizing constant c from vector to scalar:

From

To

(10.6)

Transforming vector to scalar from

to  
 (10.7)

*χ,ψ > 0, λ ϵ R*

Comparison of portfolio optimization under GH and other distributions like Gaussian can be found in V.3Numerical Results of Markowitz Optimization Problem.

The purpose of the objective function, , allows risk-adjustment to be carried out based on the risk profile of the investors or speculators: risk averse, risk neutral, or risk taker. Minimizing the objective function will allow the speculator or investor to minimize the riskand increase the return for an efficient investment portfolio. A major dissertation contribution will be determining the relationships between these constraints.

Risk is always present in financial instruments. It cannot be removed. It can only be hedged by transferring to counterparties with premiums paid.The best long-term valuationis to valuate the portfolio as if it consists only of stocks. This is pertinent to investors because they are more concerned about the future earnings of the corporations issuing those stocks. The best short-term valuation is to valuate the portfolio as if it consists of bonds. This is pertinent to speculators because they are more concerned about the current cash liquidity of the corporation issuing the bonds to realize immediate profits. The key question that can be answered by my mathematical model is:

1. How to create an optimal efficient frontier using generalized hyperbolic distribution during crises?

## IV.8 DISSERTATION DESIGN

## IV.8.1 SELECTION OF SAMPLE

For simulation, 10 stocks, 1 foreign currency, 1 US government-issued zero coupon bond and 1 S&P 500 market return tracked by SPDR S&P 500 (SPY) ETF are used. USD is the base currency. The data obtained for the abovementioned assets are their trading prices from 1 January to 31 December 2008. Stocks and currencies’ logarithmic returns are calculated directly. Prices are percentage of par values for bonds. Equation (7.2) calculates the yields of zero coupon bonds. Their trading prices is used as the value of . The US stocks from the 10 different companies are chosen based on the stock selection for defensive investor from Graham (2003) and being part of the S&P 500 and Dow Jones Industrial Average (DJIA) companies:

1. Adequate Size of the Enterprise: All the minimum figures must be arbitrary especially in the matter of size needed. The idea is to exclude small companies which may be subject to more than average vicissitudes especiallyin the industrial field. (There are often good possibilities in such enterprisesbut they are not considered suitable to the needs of the defensive investor.) Round amounts are used: not less than USD 100 million of annual sales for an industrialcompany, and not less than USD 50 million of total assets for a public utility.

2. A Sufficiently Strong Financial Condition: Current assetsshould be at least twice current liabilities – a two-to-one currentratio for industrial companies. Long-term debt should not be more than the net current assets (or “workingcapital"). The debt should not be more than twice the stock equity (at book value) for public utilities.

3. Earnings Stability: Some earnings for the common stock in each of the past ten years.

4. Dividend Record (D): Uninterrupted payments for at minimum the past 20 years.

5. Earnings Growth: A minimum increase of at least one-third in per-share earnings in the past ten years using three-year averages at the start and end.

6. Moderate Price/Earnings Ratio: Current price should not be more than 15 times average earnings of the past three years.

7. Moderate Ratio of Price to Assets: Current price should not be more than 1.5 times the book value last reported. On the contrary, a multiplier of earnings below 15 could justify a correspondingly higher multiplier of assets. It is suggested that the product of the multiplier times the ratio of price to book value should not exceed 22.5 as a rule of thumb. This figure corresponds to 15 times earnings and 1.5 times book value.It would permit an issue selling at only 9 times earnings and 2.5 times asset value, etc.

Table 4.1 exhibits the various asset classes like stocks, bond, currency and S&P market return.

The foreign currencies chosen are major reserve currencies in the world:

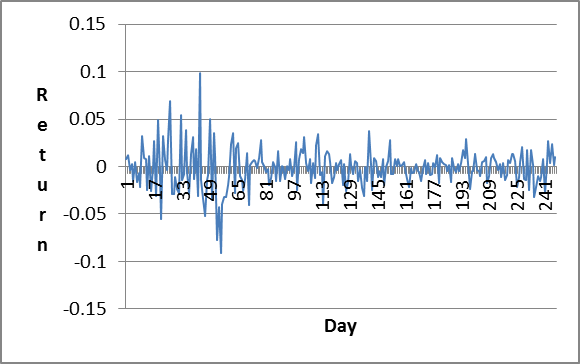
1. EUR (Euro)
2. USD (US dollar)

The zero coupon bond is government issued of series

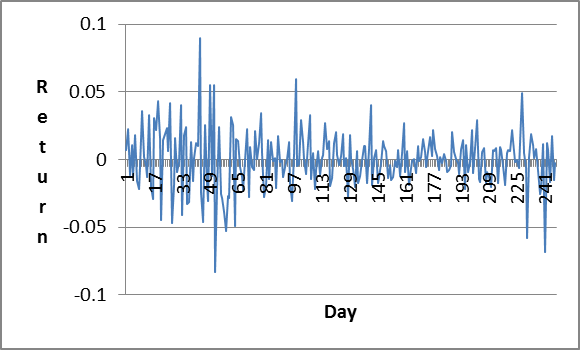
1. 52 weeks USD t-bills

**Table 4.1T**he various asset classes invested in the portfolio

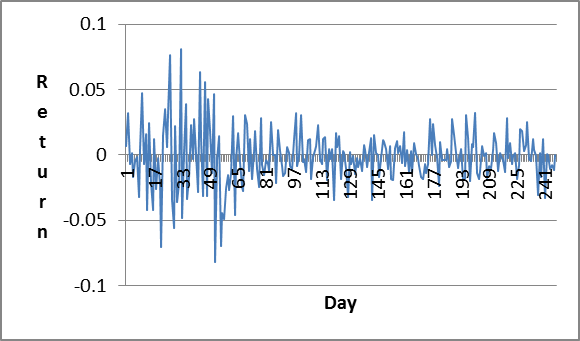
|  |
| --- |
| **Stocks** |
| Bank of America Corporation (D),( DJIA), (S&P 500) ,Ticker symbol: BAC |
| Berkshire Hathaway, Ticker symbol: BRK-B |
| General Electric Company (D), (DJIA), (S&P 500) ,Ticker symbol: GE |
| The Coca-Cola Company (D), (DJIA), (S&P 500) ,Ticker symbol: KO |
| Lowe’s Companies Inc. (D), (S&P 500) ,Ticker symbol: LOW |
| McDonald’s Corp (D), (DJIA), (S&P 500) ,Ticker symbol: MCD |
| 3M Company (D), (DJIA), (S&P 500) ,Ticker symbol: MMM |
| Regions Financial Corporation (D), (S&P 500) ,Ticker symbol: RF |
| Target Corp. (D), (S&P 500) ,Ticker symbol:TGT |
| Wal-Mart Stores Inc. (D), (DJIA), (S&P 500) ,Ticker symbol:WMT |
| **Bond** |
| T-bill |
| **Currency** |
| EUR/USD |
| **S&P 500** |
| SPY |



**Fig. 4.2** Daily logarithmic return of The Coca-Cola Company



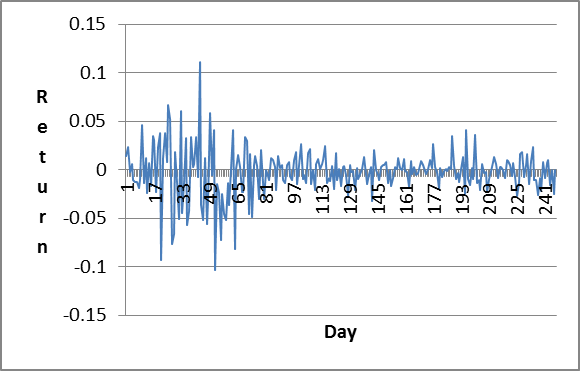
**Fig.4.3** Daily logarithmic return of McDonald’s Corp



**Fig. 4.4** Daily logarithmic return of 3M Company

**Fig. 4.5** Daily yield increment of T-Bill

**Fig. 4.6** Daily logarithmic return of EUR/USD



**Fig. 4.7** Daily logarithmic return of S&P 500

Fig. 4.2 to Fig. 4.7 show the dynamics is different for different asset classes during crisis especially big jumps and volatility as stated in the introduction of the dissertation.

No one really knows what the outcome will be with certainty based on any investment or speculation decision taken to produce the propitious expected return wanted in a portfolio. This leads to risks. The risk factors in the portfolio represent the risks. The increments of those risk factors are assumed to be GH distributed. The increments are their logarithmic returns for stocks and currencies respectively. It is the continuously compounded yields for zero coupon bonds. The calculations of time to coupon maturities are under the actual/365 rule.

Asset weights can be unconstrained or constrained. It is unconstrained if asset short-sellings are allowed. It is constrained if asset short-sellings are not allowed.

## IV.8.2 SOURCES OF DATA

The data set for the mathematical model will come from US stocks in USD.The sources of data are obtained from Yahoo Finance.

## IV.8.3 COLLECTION OF DATA

The data are downloaded from Yahoo Finance.

## IV.8.4 DATA MANAGEMENT

The data is categorized into stocks, bonds, and currencies.

## IV.8.5 DATA ANALYSIS STRATEGIES

Market price alone is inappropriate and insufficient to measure risk during crises. Market risks must be identified and measured.The data is analyzed with Chi-Square Test, Scatter Plot, Anderson-Darling Test, Kolgomorov-Smirnov goodness-of-fit hypothesis Test and Jarque-Berra hypothesis Test of composite normality on Multivariate distributions. My data was also fitted to the multivariate Generalized Hyperbolic distribution based on log-likelihood and akaike information criteria (AIC) tests.

## IV.8.6 EXPECTED RESULTS

The expected results will be obtained from the eightstages mentioned in my dissertationcontribution.Generalized Hyperbolic distributions are proven to be very useful in finance to model return variance or volatility of major asset classes like equity, fixed income, and foreign exchange.Understanding the behavior of the variance of the return process is important for forecasting and pricing option-type derivative instruments because variance is a proxy for risk.To model univariate time series like short-term prediction of asset prices or returns or to test the market-efficiency hypothesis, Generalized Hyperbolic distribution models are a first starting point and serve as a benchmark against more complex methods.

**Table 4.2**Summary of Expected Results

| **No** | **Stages** | **Expected Results** |
| --- | --- | --- |
| 1. | Measure the portfolio returns appropriately and precisely to create a more efficient portfolio. | Values of lognormal returns. |
| 2. | Creates the mathematical model i.e. risk factors equations to construct anoptimal efficient frontier using generalized hyperbolic distribution during crisis. | Values of the equations in the mathematical model calculated. |
| 3. | Portfolio optimization | Optimal weights for portfolio obtained. |

# CHAPTER V. DISSERTATION DATA ANALYSIS

## 

## V.1 INTRODUCTION

## Fitting the financial data to the right probability distribution is crucial during portfolio optimization. In this case, this is done using the EM algorithm. The fitted parameters are then used to perform the portfolio optimization.

## 

## V. 2 CALIBRATION RESULTS

The paper is on portfolio optimization. Symmetric optimization can be solved analytically. The focus is on asymmetric GH model. The advantage of modelling with the asymmetric model gives an additional degree of freedom because it is unnecessary to fix the parameter to 0. From the calibration result, it can be observed whether the value of and its skewness are close to 0 or not to decide whether the model is better asymmetric or symmetric. The calibration results are in percentage risk-factor increments of assets used i.e. risk-factor increments multiplied by 100%. Both univariate and multivariate asymmetric GH distributions are presented as such. The order of elements in the GH random vector for multivariate GH calibration of data is displayed in Table 5.1.

|  |  |
| --- | --- |
| Stocks | Bond |
| 1. BAC logarithmic return | 11. T-bill yield increment |
| 2. BRK-B logarithmic return |  |
| 3. GE logarithmic return | Currency |
| 4. KO logarithmic return | 12. EUR/USD logarithmic return |
| 5. LOW logarithmic return |  |
| 6. MCD logarithmic return | S&P 500 |
| 7. MMM logarithmic return | 13. SPY logarithmic return |
| 8. RF logarithmic return |  |
| 9. TGT logarithmic return |  |
| 10. WMT logarithmic return |  |

**Table 5.1**Elements of asset classes in numeric sequential order

The calibration is using the EM algorithm. As a word of caution, calibration can only be done with fixed λ. χ-algorithm is used for positive λ. ψ-algorithm is used for negative λ. The calibration is terminated when the likelihood increments between the current and previous iterations is less than a specified value for each λ. is the observed Generalized Hyperbolic random vector. is the Generalized Hyperbolic estimated parameters at iteration *k*. The calibration process is terminated at iteration *k* + 1 given positive є if

is the likelihood function. The EM algorithm is combined with a one dimensional optimization method to find the value of λ yielding the highest likelihood value.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Asset | λ |  | μ | Σ | Γ |
| EUR/USD (ANIG) | N.A | 1.0000000000 | 0.0004739236 | 0.0085387701 | -0.0006753919 |
| t-bill (ANIG) | N.A | 1.0000000 | 1.5409076 | 2.6200804 | -0.7617573 |
| SPY (AGH) | 0.2239090449 | 0.2870898254 | 0.0004625918 | 0.0243432147 | -0.0029855295 |
| BAC (AGH) | -0.477185331 | 0.353540165 | -0.008634634 | 0.063934749 | 0.003967910 |
| BRK-B (AGH) | -0.245982148 | 0.436622469 | -0.003462836 | 0.024071016 | 0.001931927 |
| GE (AGH) | 0.039715025 | 0.246245279 | -0.001035980 | 0.036014197 | -0.002046741 |
| KO (ANIG) | N.A | 1.0000000000 | -0.0014827553 | 0.0200286982 | -0.0001966238 |
| LOW (ANIG) | N.A | 1.00000000 | -0.01130192 | 0.03497425 | 0.01059927 |
| MCD (AGH) | -1.6779805875 | 0.8850530489 | 0.0008357937 | 0.0210506265 | -0.0010050472 |
| MMM (ANIG) | N.A | 1.000000000 | -0.002433906 | 0.021224737 | 0.000641435 |
| RF (ANIG) | N.A | 1.000000000 | -0.008118692 | 0.070563886 | 0.003682139 |
| TGT (ANIG) | N.A | 1.000000000 | -0.003337907 | 0.037108178 | 0.001365540 |
| WMT (AGH) | -1.8483568568 | 0.2010683939 | 0.0008411315 | 0.0208971861 | -0.0003862582 |

**Table 5.2**Calibrated parameters

|  |  |  |
| --- | --- | --- |
| Asset | Log-Likelihood | Akaike Information Criteria |
| EUR/USD (ANIG) | 861.6396 | -1717.2790 |
| t-bill (ANIG) | -571.0845 | 1148.169 |
| SPY (AGH) | 598.1572 | -1178.3880 |
| BAC (AGH) | 365.8162 | -721.6325 |
| BRK-B (AGH) | 598.8894 | -1187.7790 |
| GE (AGH) | 512.0288 | -1014.0580 |
| KO (ANIG) | 626.1371 | -1246.2740 |
| LOW(ANIG) | 487.7584 | -969.5168 |
| MCD (AGH) | 613.8968 | -1217.7940 |
| MMM (ANIG) | 611.0669 | -1216.1340 |
| RF (ANIG) | 310.8207 | -615.6415 |
| TGT (ANIG) | 475.2530 | -944.5060 |
| WMT (AGH) | 626.2028 | -1242.4060 |

**Table5.3**Statistical test results

Table 5.2 displays the calibrated parameters of the univariate Generalized Hyperbolic distribution for all of the assets used. ANIG refers to Asymmetric Normal Inverse Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution.

Table 5.3 shows that univariate Generalized Hyperbolic distribution give very high log-likelihood values greater than 6.63, the probability of the result happening by chance is less than 1%.. Very low akaike information criteria gives good values. ANIG refers to Asymmetric Normal Inverse Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution.

Fitting risk-factors for stocks, bond, currency and S&P 500 benchmark to multivariate generalized hyperbolic distribution:

Asymmetric Generalized Hyperbolic Distribution (AGH) :

The calibrated parameters are:

λ:

|  |
| --- |
| 2.000000e+00 |

:

|  |
| --- |
| 2.097363e-08 |

μ:

|  |
| --- |
| -0.0100798777 |
| -0.0003040603 |
| -0.0056802810 |
| -0.0007707272 |
| -0.0071344429 |
| -0.0016861231 |
| -0.0031424134 |
| -0.0143091944 |
| -0.0028836965 |
| -0.0010959269 |
| -0.3613469721 |
| 0.0014453081 |
| -0.0018320762 |

Σ:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | BACreturn | BRKBreturn | GEreturn | KOreturn | LOWreturn | MCDreturn | MMMreturn | RFreturn | TGTreturn | WMTreturn | TBill return | EURUSDreturn | SPY return |
| BAC  return | 0.003182 | 0.00057 | 0.001205 | 0.000429 | 0.001321 | 0.000611 | 0.000723 | 0.003133 | 0.001275 | 0.000679 | -0.0089451254 | -0.00005 | 0.000941 |
| BRKB  return | 0.00057 | 0.000531 | 0.000252 | 0.000091 | 0.000242 | 0.000145 | 0.000177 | 0.000562 | 0.000277 | 0.000129 | -2.404253e-03 | -0.000017 | 0.000218 |
| GE  return | 0.001205 | 0.000252 | 0.000961 | 0.000295 | 0.000675 | 0.000375 | 0.000481 | 0.001207 | 0.000702 | 0.00038 | -0.005707 | -0.000022 | 0.000543 |
| KO  return | 0.000429 | 0.000091 | 0.000295 | 0.000388 | 0.000305 | 0.000213 | 0.000211 | 0.000476 | 0.000325 | 0.000206 | -2.160700e-04 | -0.000004 | 0.000249 |
| LOW  return | 0.001321 | 0.000242 | 0.000675 | 0.000305 | 0.001218 | 0.000402 | 0.000442 | 0.00136 | 0.000963 | 0.000486 | -0.0034125402 | -0.000035 | 0.000558 |
| MCD  return | 0.000611 | 0.000145 | 0.000375 | 0.000213 | 0.000402 | 0.000437 | 0.00028 | 0.000631 | 0.000458 | 0.000274 | 2.677799e-04 | -0.000015 | 0.000321 |
| MMM  return | 0.000723 | 0.000177 | 0.000481 | 0.000211 | 0.000442 | 0.00028 | 0.000441 | 0.000709 | 0.000466 | 0.000271 | -1.093396e-03 | -0.000015 | 0.000367 |
| RF  return | 0.003133 | 0.000562 | 0.001207 | 0.000476 | 0.00136 | 0.000631 | 0.000709 | 0.005247 | 0.001316 | 0.00074 | -0.003974 | -0.000074 | 0.000946 |
| TGT  return | 0.001275 | 0.000277 | 0.000702 | 0.000325 | 0.000963 | 0.000458 | 0.000466 | 0.001316 | 0.001314 | 0.000549 | -2.945187e-03 | -0.00003 | 0.000571 |
| WMT  return | 0.000679 | 0.000129 | 0.00038 | 0.000206 | 0.000486 | 0.000274 | 0.000271 | 0.00074 | 0.000549 | 0.000443 | -3.329112e-04 | -0.000016 | 0.000319 |
| TBillYield | -8.945125e-03 | -2.404253e-03 | -5.672770e-03 | -2.160700e-04 | -3.412540e-03 | 2.677799e-04 | -1.093396e-03 | -3.944774e-03 | -2.945187e-03 | -3.329112e-04 | 6.177207e+00 |  |  |
| EURUSD  return | -0.00005 | -0.000017 | -0.000022 | -0.000004 | -0.000035 | -0.000015 | -0.000015 | -0.000074 | -0.00003 | -0.000016 | -2.068806e-05 | 0.000081 | 0.000002 |
| SPY  return | 0.000941 | 0.000218 | 0.000543 | 0.000249 | 0.000558 | 0.000321 | 0.000367 | 0.000946 | 0.000571 | 0.000319 | -1.067450e-03 | 0.000002 | 0.000478 |

γ:

|  |
| --- |
| 0.0054881237 |
| -0.0012454045 |
| 0.0026331864 |
| -0.0009197011 |
| 0.0064054945 |
| 0.0015374528 |
| 0.0013748392 |
| 0.0101126316 |
| 0.0009224791 |
| 0.0015702659 |
| 1.1805805513 |
| -0.0015017929 |
| -0.0007046468 |

Log-Likelihood:

|  |
| --- |
| 7339.334 |

Akaike Information Criteria:

|  |
| --- |
| -14442.67 |

The stocks and bonds have negative returns during crisis. The currency pair has positive return during crisis.

Financial data using logarithmic returns or losses from risks taken tend to display a hyperbola i.e. the return will increase to a certain level, taper off, then decrease to a certain level because of herd instinct. Herd instinct is about investors or speculators thinking taking higher risk will lead to higher return. If every investor or speculator think in this manner, lower return will be achieved from higher risk instead This is a profound and paradoxical observation. The more return an investor or speculator makes from the risk taken, the more likely new competition will be faced because the high returns shows clearly that the risk can be easily taken. The new competition will lead to higher risk and lower returns. This is usually overlooked by overenthusiastic investors or speculators who believed that early investors or speculators would sustain their advantage indefinitely. Analagously, Graham (2003) observed this when applied to continued success of outstanding companies in highly competitive areas.

1. Prove that = 1 using 1 asset

Using daily EUR/USD 2008: lower = -0.04, upper = 0.04, 0.9952274 with absolute error < 3.6e-06.

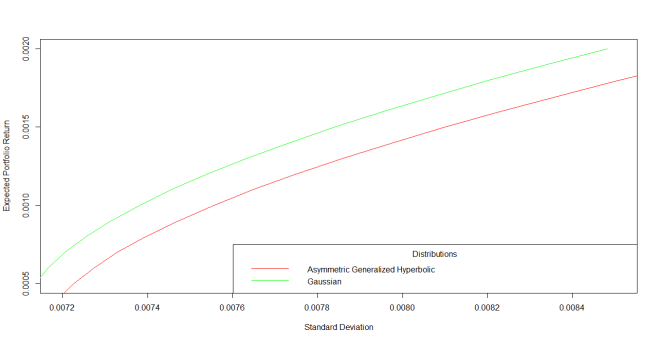
Using daily t-bill return 2008: lower = -6, upper = 4, 0.897949 with absolute error < 2.4e-08.

1. Prove that = 1 using 2 assets with weight of 0.5 on each asset.

Using daily t-bill and EUR/USD returns 2008: lower = -3, upper = 2,0.8980362 with absolute error < 2.4e-08.

**V.3 NUMERICAL RESULTS OF MARKOWITZ OPTIMIZATION PROBLEM**

In this subsection, the focus is on the results of the numerical portfolio optimizations under the asymmetric GH model. The calibrated parameters of the multivariate GH distribution presented in the previous subsection are used for the optimizations. Results of optimization using standard deviation is given as a tool to understand the similarity or difference between the results of optimization using Expected Shortfall. Expected returns, standard deviation and Expected Shortfalls are given in percentage of the portfolio value.

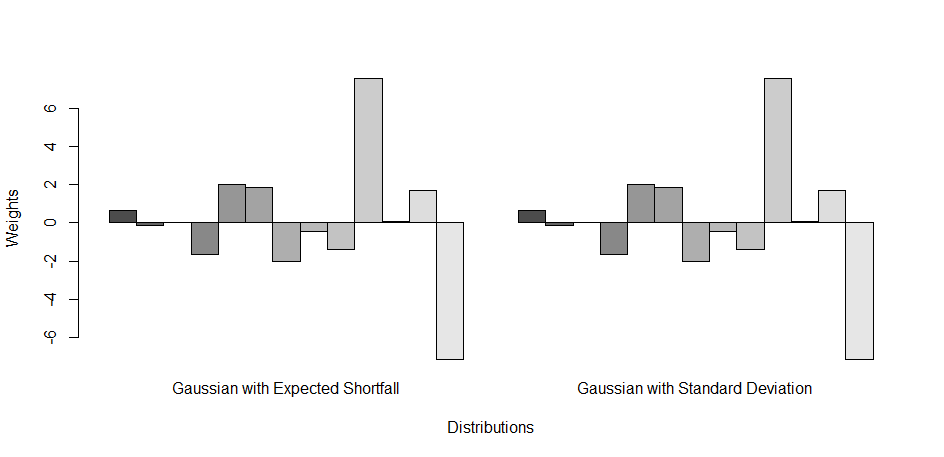


**Fig.5.1** Portfolio efficient frontiers with unconstrained weights using standard deviation as risk measure for Gaussian and Asymmetric Generalized Hyperbolic distribution

Fig. 5.1 shows that for the same standard deviation, the expected portfolio return is higher for Gaussian compared to Asymmetric Generalized Hyperbolic distribution. For the same expected portfolio return, the standard deviation is higher for Asymmetric Generalized Hyperbolic compared to Gaussian distribution. This shows that during crisis, Gaussian distribution is over-optimistic in its display of the risk-return relationship in investment.

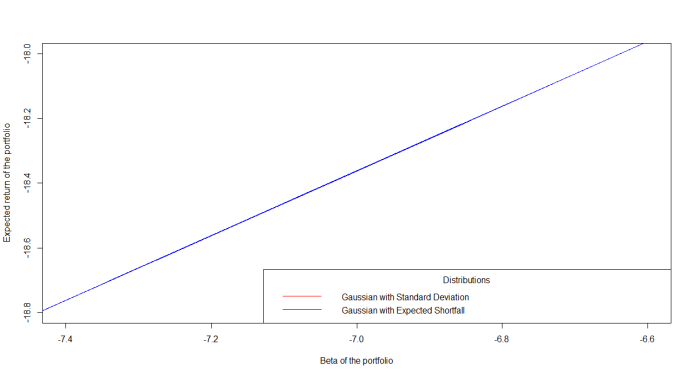
During portfolio optimization, asymmetric generalized hyperbolic distribution using expected shortfall does not converged. Asymmetric generalized hyperbolic distribution using standard deviation and Gaussian using expected shortfall converged. During crisis, one needs to look at the whole probability distribution for risk. Expected shortfall only looks at the tail risk. Standard deviation is a deviation from the mean and thus looks at the whole probability distribution. Gaussian distribution using expected shortfall is an over-optimistic perception of risk, thus by itself is already sufficient.

Target return = 0.10:



**Fig.5.2** Weights of assets using Expected Shortfall and standard deviation as risk measures on Gaussian distribution

When the target return is 0.10, the portfolio alpha for Gaussian distribution with standard deviation and expected shortfall are 18.40684 and 22.5973 respectively. This means the portfolio has outperformed the S&P 500 benchmark by 18.40684% using Gaussian distribution with standard deviation but has outperformed S&P 500 benchmark by 22.5973% using Gaussian distribution with expected shortfall.



**Fig.5.3** Security Market Line for Gaussian distribution with combined risk measures of standard deviation and expected shortfall

Fig. 5.2 display the same outcome for Gaussian distribution using both standard deviation and expected shortfall as risk measures because optimal weights from the assets are the same for target return of 0.10 from Fig. 5.3 and as shown:

|  |
| --- |
| 0.623349350 |
| -0.162460809 |
| -0.004882113 |
| -1.645713259 |
| 2.000161640 |
| 1.858336890 |
| -2.017350216 |
| -0.436629790 |
| -1.420685157 |
| 7.553442366 |
| 0.089988053 |
| 1.701185367 |
| -7.138742322 |

The expected shortfall0.6411494 is larger than the standard deviation0.2623484 using efficient frontiers after portfolio optimization to generate the security market lines for Gaussian distribution using expected shortfall and standard deviation. This shows that tail risk measured by expected shortfall is a more rigorous method than standard deviation from the mean on Gaussian distribution during crisis.

The theoretical proof for the phenomenon is found in Surya and Kurniawan (2013). From 6.1 Optimization in Symmetric Case, proposition 6.3 (Equality of Markowitz-Optimal Weights) is proven where in a symmetric Generalized Hyperbolic, the optimal portfolio composition from Markowitz optimization using Expected Shortfall at confidence level β ≥ 0.5 is equal to that using volatility as risk measure.

When the target return is 0.0020, the portfolio alpha for Asymmetric Generalized Hyperbolic distribution with standard deviation is23.23316. This means the portfolio has outperformed the S&P 500 benchmark by 23.23316%.The portfolio alpha for Gaussian distribution with standard deviation using target return 0.0020 is22.57892. This means the portfolio has outperformed the S&P 500 benchmark by 22.57892%.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Asset | λ |  | μ | Σ | γ |
| Refer to Table 5.2 (AGH) | 2.000000e+00 (SD) | 2.097363e-08 (SD) | 7.174630e-05 (SD) | 8.713960e-03 (SD) | 1.928254e-03 (SD) |
| Refer to Table 5. 2 (ANIG) | N.A | 0.7442727620 (SD) | 0.0002977526 (SD) | 0.0092197537 (SD) | 0.0017022474 (SD) |
| Refer to Table 5.2 (AVG) | 1.3746387325 (SD) | N.A. | 0.0000272378 (SD) | 0.0089418357 (SD) | 0.0019727622 (SD) |
| Refer to Table 5. 2 (G) | N.A | N.A | 0.002000000 (SD & ES) | 0.008481577 (SD & ES) | N.A |

**Table 5.4** Calibrated parameters for multivariate Generalized Hyperbolic distribution for all of the assets using standard deviation and expected shortfall with target return = 0.0020. ANIG refers to Asymmetric Normal Inverse Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution.G refers to Gaussian distribution. AVG refers to Asymmetric Variance Gamma distribution. SD refers to Standard Deviation. ES refers to Expected Shortfall.

|  |  |  |
| --- | --- | --- |
| Asset | Log-Likelihood | Akaike Information Criteria |
| Refer to Table 5.2 (AGH) | 7339.334 | -14442.67 |
| Refer to Table 5.2 (ANIG) | 7363.595 | -14491.19 |
| Refer to Table 5. 2 (AVG) | 7347.435 | -14458.87 |
| Refer to Table 5.2 (AST) | 7359.184 | -14482.37 |
| Refer to Table 5.2 (G) | 6896.577 | -13585.15 |

**Table 5.5**Statistical test results for multivariate Generalized Hyperbolic distribution for all of the assets. ANIG refers to Asymmetric Normal Inverse Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution. AVG refers to Asymmetric Variance Gamma distribution. AST refers to Asymmetric Student-t distribution. G refers to Gaussian distribution.

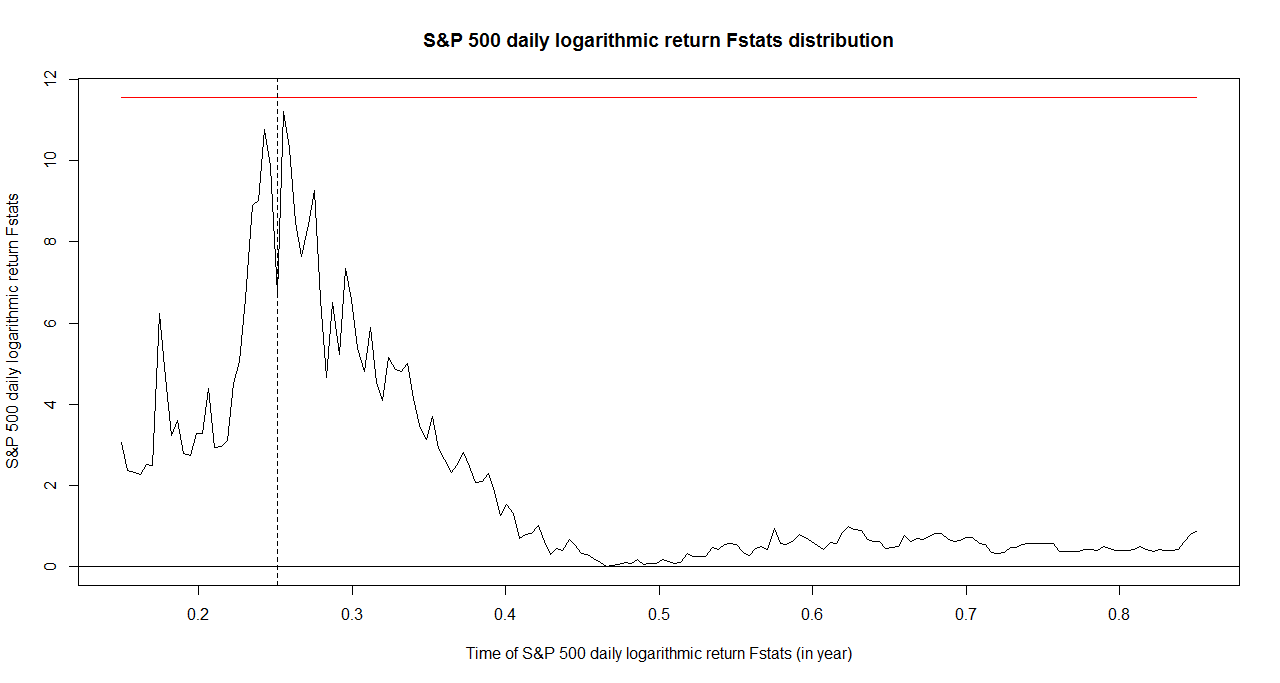
Table 5.4 displays the calibrated parameters of multivariate Generalized Hyperbolic distributions for all of the assets with standard deviation and expected shortfall with target return = 0.0020.

Table 5.5 shows that multivariate Generalized Hyperbolic distribution give very high log-likelihood values greater than 6.63, the probability of the result happening by chance is less than 1%. Very low akaike information criteria gives good values.

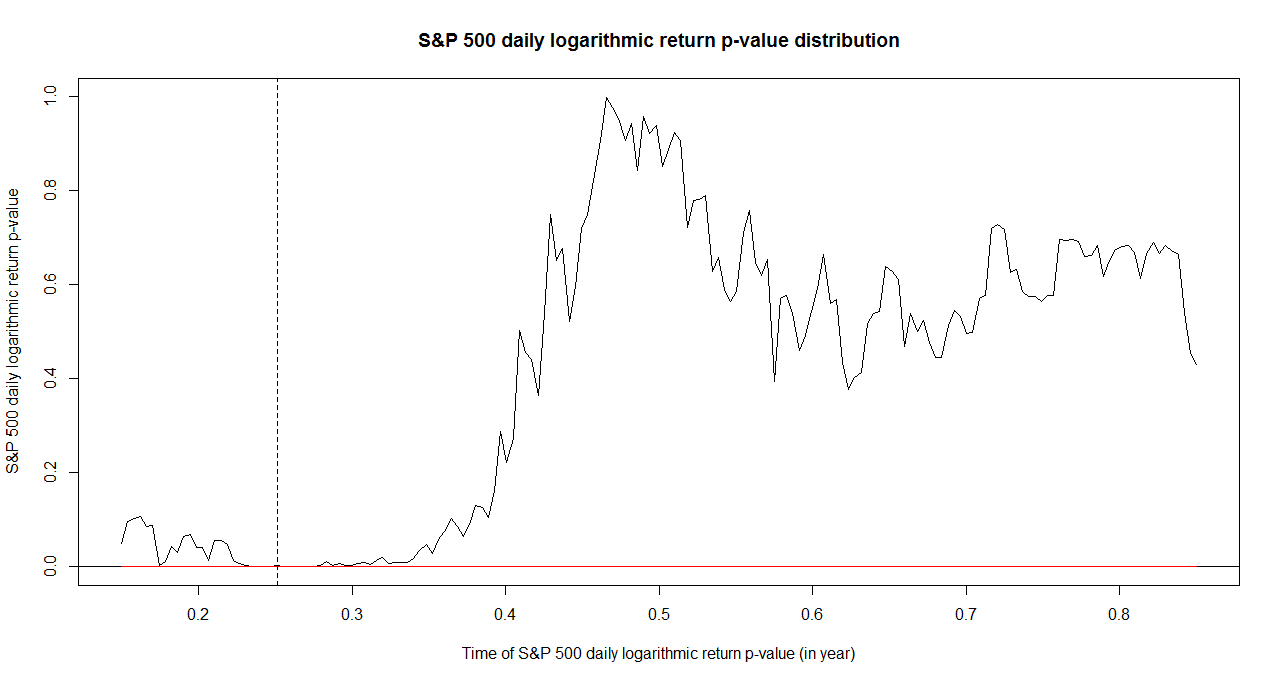
**V.4 STRUCTURAL CHANGE TESTING FOR CRISIS PERIOD 2008 TO 2009**

1. The crisis period is from1 January to 31 December 2008. This is to check for crisis data used during V.5.1 In-sample Analysis.

1. S&P 500



**Fig. 5.4** S&P 500 daily logarithmic returnFstats distribution.



**Fig. 5.5** S&P 500 daily logarithmic return p-value distribution.

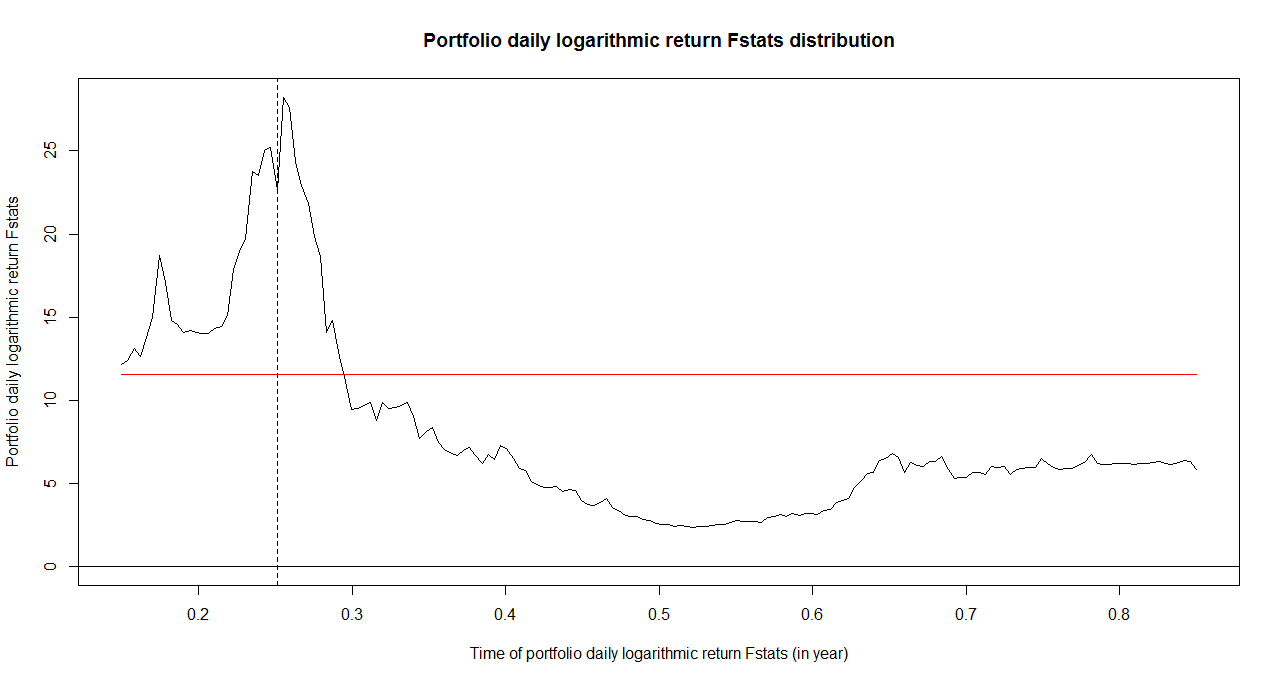
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| SPY (GLM) | -4.052e-04 | -3.914e-05 | 8.500e-06 | 4.216e-05 | 4.624e-04 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | 5.532e-04 | 8.536e-04 | 0.648 | 0.518 |  |  |
| date\_dailylogreturn\_sp500 | -4.007e-08 | 6.070e-08 | -0.660 | 0.510 |  |  |
| Dispersion parameter for gaussian family | 9.892303e-09 |  |  |  |  |  |
| Null deviance | 2.4279e-06 on 246 degrees of freedom |  |  |  |  |  |
| Residual deviance | 2.4236e-06 on 245 degrees of freedom |  |  |  |  |  |
| AIC | -3847.6 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -1.752e-05 | -1.382e-05 | -1.023e-05 | -1.023e-05 | -6.645e-06 | -2.978e-06 |
| Structural change test: | supF test | data | sup.F | p-value |  |  |
|  |  | dlrsp500 | 11.2209 | 0.05762 |  |  |

**Table 5.6** Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for S&P 500 daily logarithmic return.

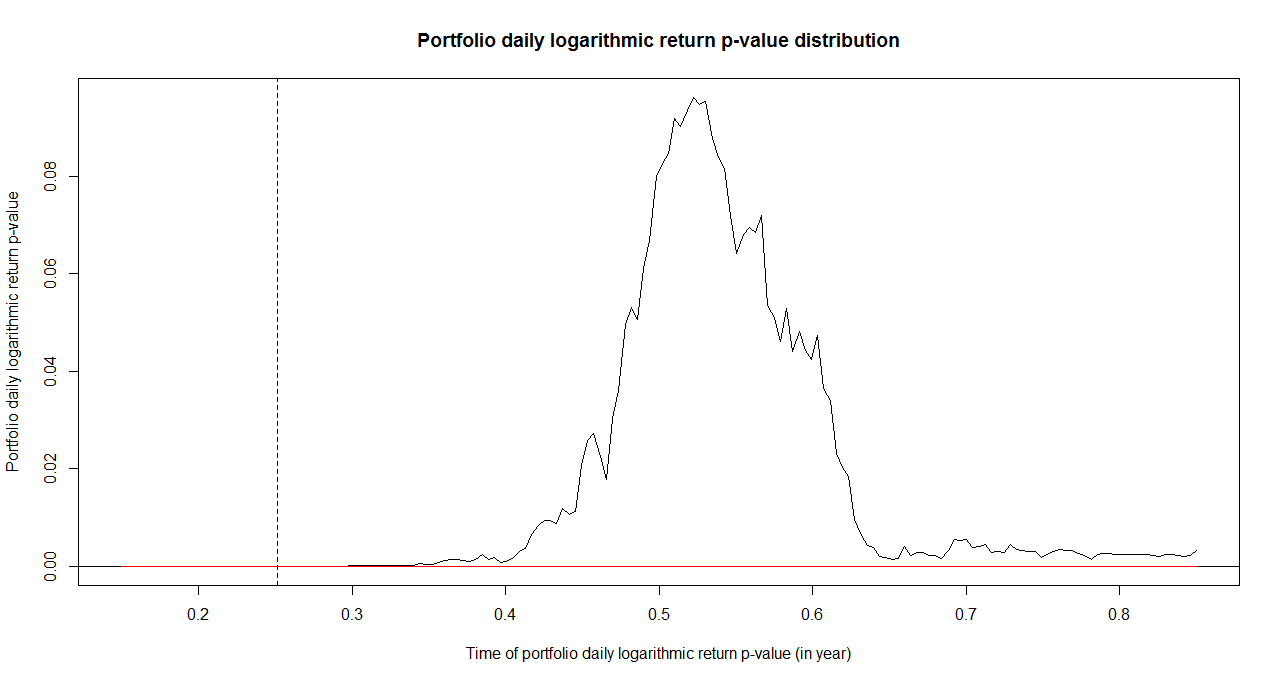
Formula = daily\_logreturn\_sp500 ~ date\_dailylogreturn\_sp500is based on the generalized linear model of how the daily logarithmic return of S&P500 changes with date. Table 5.6 shows a very low akaike information criteria giving good value.

Fig. 5.4 and 5.5 shows the sup.F and p-value where the structural change occurs.

1. Asymmetric Generalized Hyperbolic Distribution with standard deviation



**Fig. 5.6** Portfolio daily logarithmic return Fstats distribution.



**Fig. 5.7** Portfolio daily logarithmic return p-value distribution.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| Portfolio (GLM) | -0.033355 | -0.003712 | 0.000296 | 0.004472 | 0.036055 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | 6.948e-01 | 7.730e-02 | 8.989 | <2e-16 \*\*\* |  |  |
| date\_dailylogreturn\_agh\_sd | -4.958e-05 | 5.497e-06 | -9.021 | <2e-16 \*\*\* |  |  |
| Dispersion parameter for gaussian family | 8.111108e-05 |  |  |  |  |  |
| Null deviance | 0.026473 on 246 degrees of freedom |  |  |  |  |  |
| Residual deviance | 0.019872 on 245 degrees of freedom |  |  |  |  |  |
| AIC | -1621.7 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -0.011490 | -0.006908 | -0.002470 | -0.002466 | 0.001968 | 0.006505 |
| Structural change test: | supF test | data | sup.F | p-value |  |  |
|  |  | dlragh | 28.218 | 2.39e-05 |  |  |

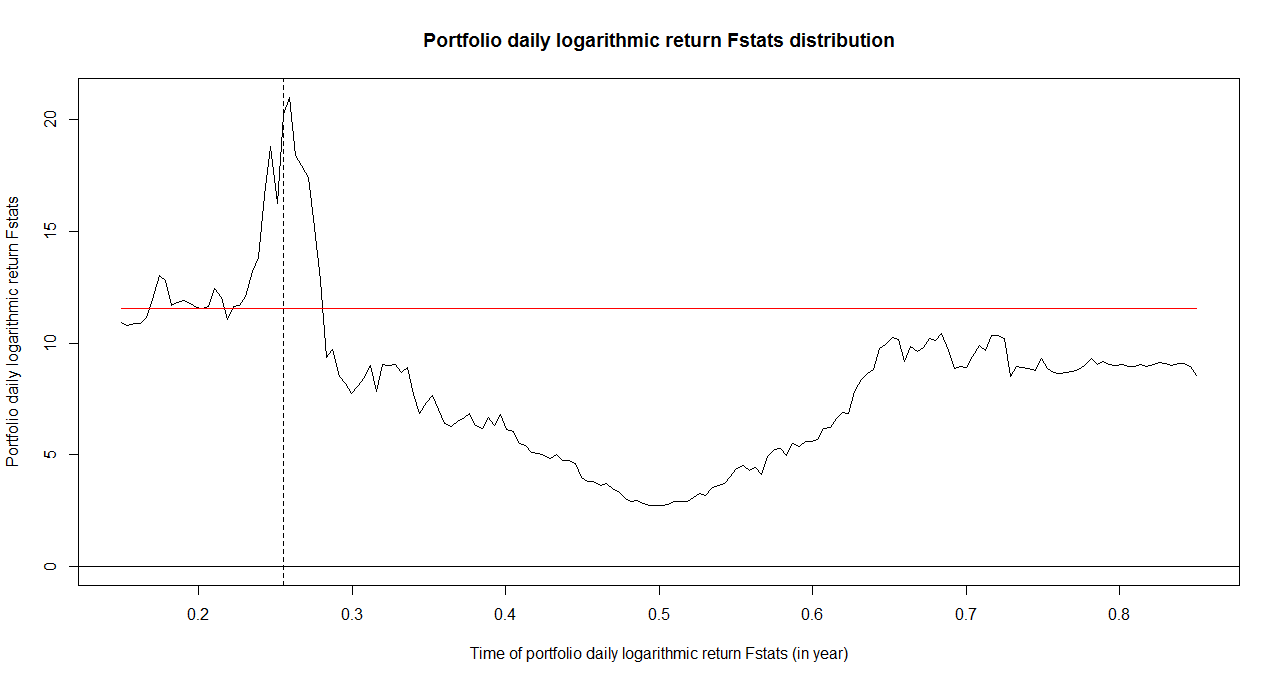
**Table 5.7** Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for

Portfolio daily logarithmic return where signif.codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1.

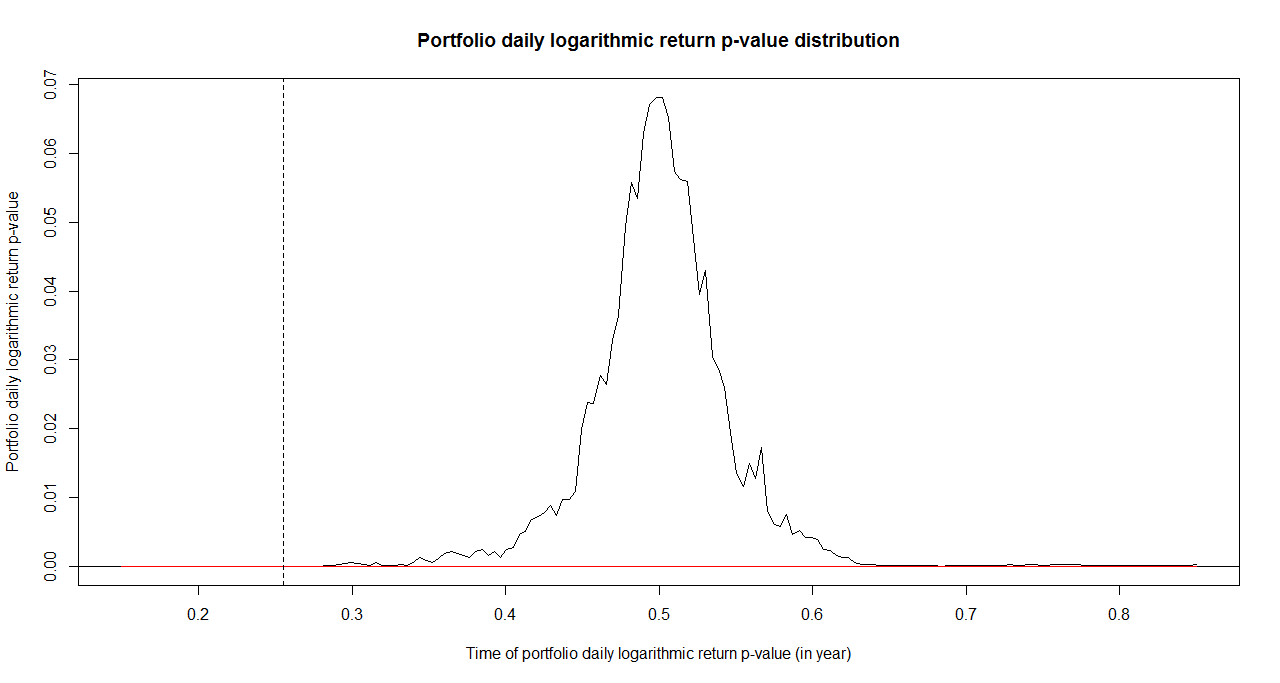
formula = daily\_logreturn\_agh\_sd ~ date\_dailylogreturn\_agh\_sdis based on the generalized linear model of how the daily logarithmic return of the portfolio changes with date. Table 5.7 shows a very low akaike information criteria giving good value.

Fig. 5.6 and 5.7 shows the sup.F and p-value where the structural change occurs.

1. Gaussian Distribution with expected shortfall



**Fig. 5.8** Portfolio daily logarithmic return Fstats distribution.



**Fig. 5.9** Portfolio daily logarithmic return p-value distribution.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| Portfolio (GLM) | -0.028565 | -0.003929 | 0.000754 | 0.004560 | 0.033787 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | 7.791e-01 | 7.808e-02 | 9.978 | <2e-16 \*\*\* |  |  |
| date\_dailylogreturn\_g\_es | -5.559e-05 | 5.552e-06 | -10.012 | <2e-16 \*\*\* |  |  |
| Dispersion parameter for gaussian family | 8.276147e-05 |  |  |  |  |  |
| Null deviance | 0.028572 on 246 degrees of freedom |  |  |  |  |  |
| Residual deviance | 0.020277 on 245 degrees of freedom |  |  |  |  |  |
| AIC | -1616.7 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -0.012730 | -0.007585 | -0.002610 | -0.002606 | 0.002365 | 0.007451 |
| Structural change test: | supF test | data | sup.F | p-value |  |  |
|  |  | dlrg | 21.0041 | 0.0007371 |  |  |

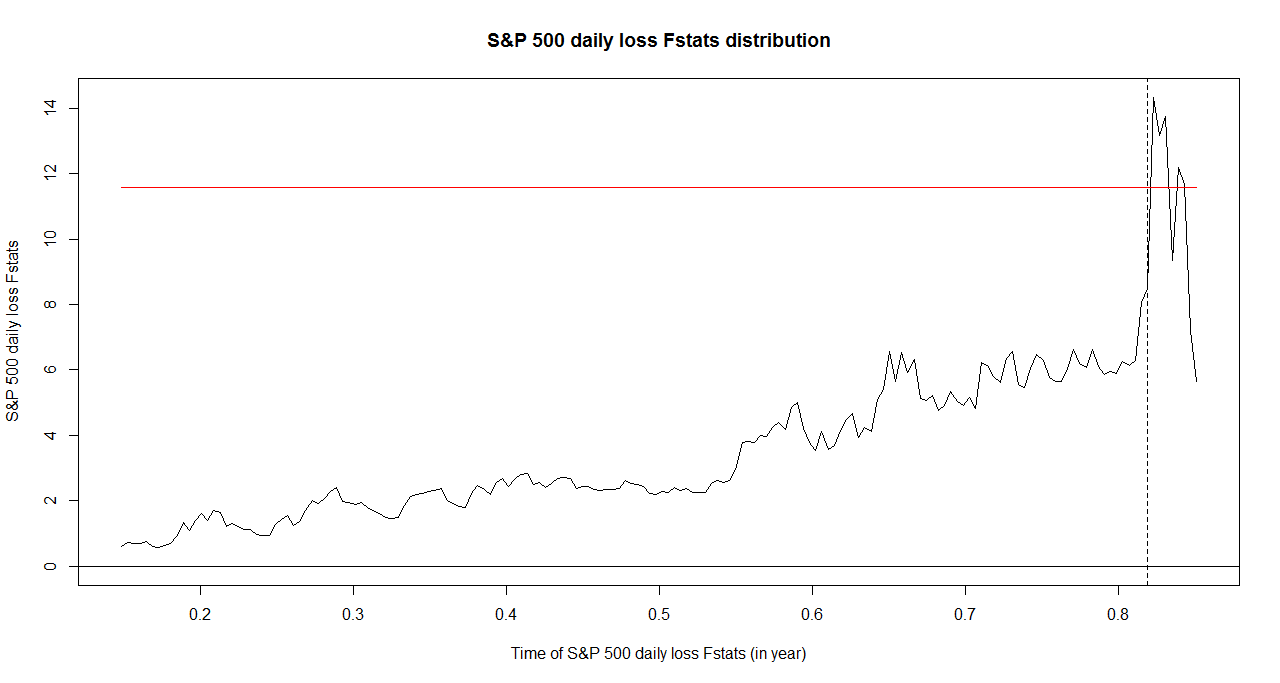
**Table 5.8**Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for

Portfolio daily logarithmic return where signif.codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1.

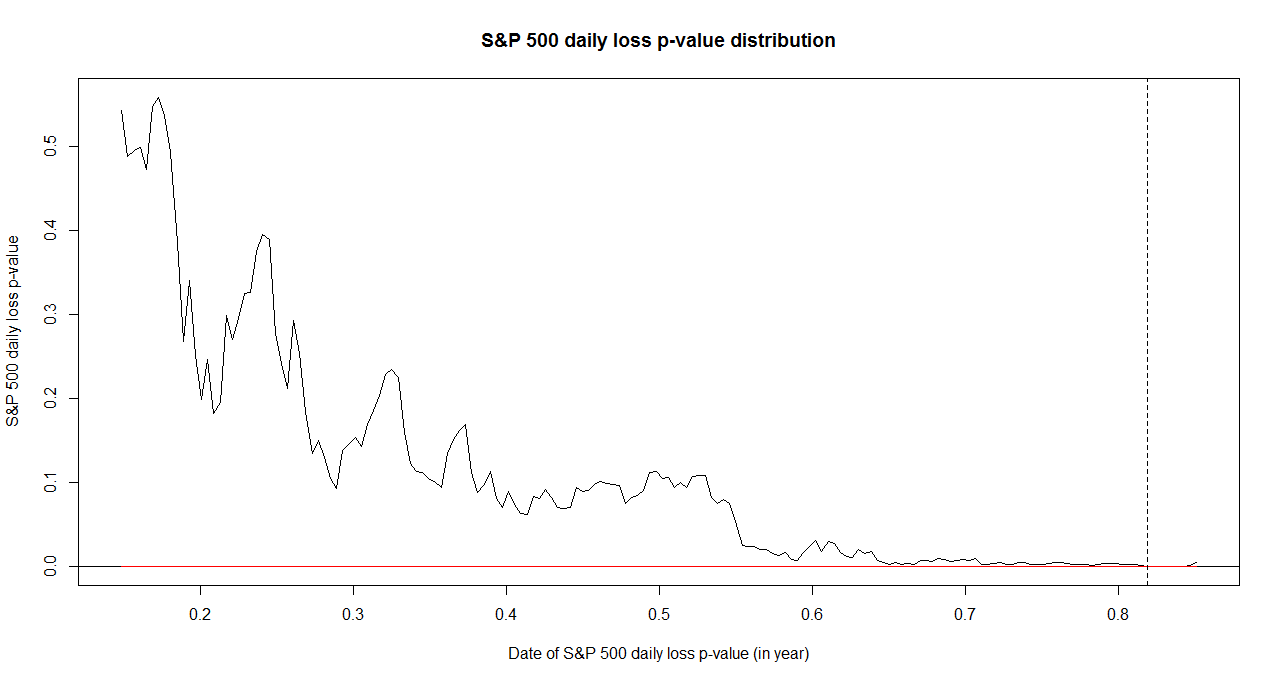
Formula = daily\_logreturn\_g\_es ~ date\_dailylogreturn\_g\_es is based on the generalized linear model of how the daily logarithmic return of the portfolio changes with date. Table 5.8 shows a very low akaike information criteria giving good value.

Fig. 5.8 and 5.9 shows the sup.F and p-value where the structural change occurs.

1. The crisis period is from 1 January to 31 December 2009.This is to check for crisis data used during V.5.2Out-Of-Sample Analysis.
2. S&P 500



**Fig. 5.10** Portfolio daily loss Fstats distribution.



**Fig. 5.11** Portfolio daily loss p-value distribution.

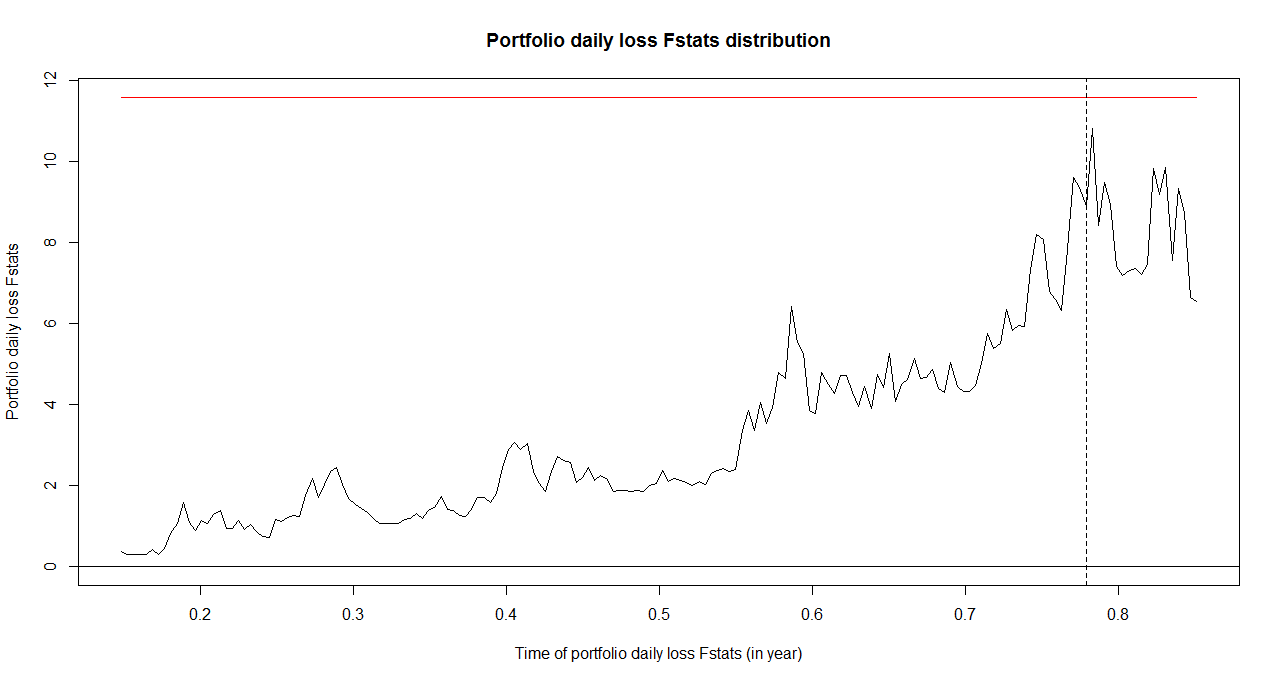
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| Portfolio (GLM) | -2.140e-04 | -3.082e-05 | 1.411e-06 | 3.482e-05 | 2.799e-04 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | -5.765e-04 | 5.927e-04 | -0.973 | 0.332 |  |  |
| date\_dailyloss\_sp500 | 4.015e-08 | 4.108e-08 | 0.977 | 0.329 |  |  |
| Dispersion parameter for gaussian family | 4.554658e-09 |  |  |  |  |  |
| Null deviance | 1.1294e-06 on 248 degrees of freedom |  |  |  |  |  |
| Residual deviance | 1.1250e-06 on 247 degrees of freedom |  |  |  |  |  |
| AIC | -4071.9 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -4.410e-06 | -8.768e-07 | 2.737e-06 | 2.779e-06 | 6.350e-06 | 1.004e-05 |
| Structural change test: | supF test | Data | sup.F | p-value |  |  |
|  |  | sp500 | 14.3282 | 0.01539 |  |  |

**Table 5.9** Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for

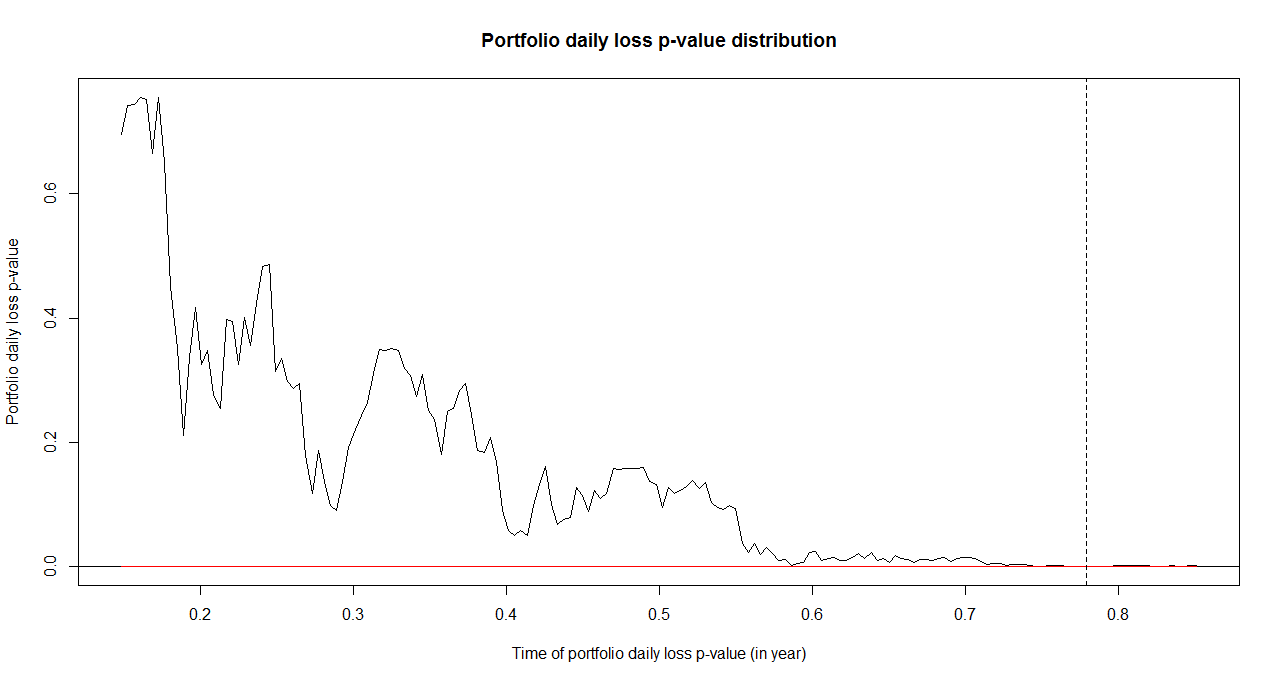
Portfolio daily loss distribution.

Formula = daily\_loss\_sp500 ~ date\_dailyloss\_sp500 is based on the generalized linear model of how the daily loss of S&P 500 changes with date. Table 5.9 shows a very low akaike information criteria giving good value. Fig. 5.10 and 5.11 shows the sup.F and p-value where the structural change occurs.

1. Asymmetric Generalized Hyperbolic Distribution with standard deviation



**Fig. 5.12** Portfolio daily loss Fstats distribution.



**Fig. 5.13** Portfolio daily lossp-value distribution.

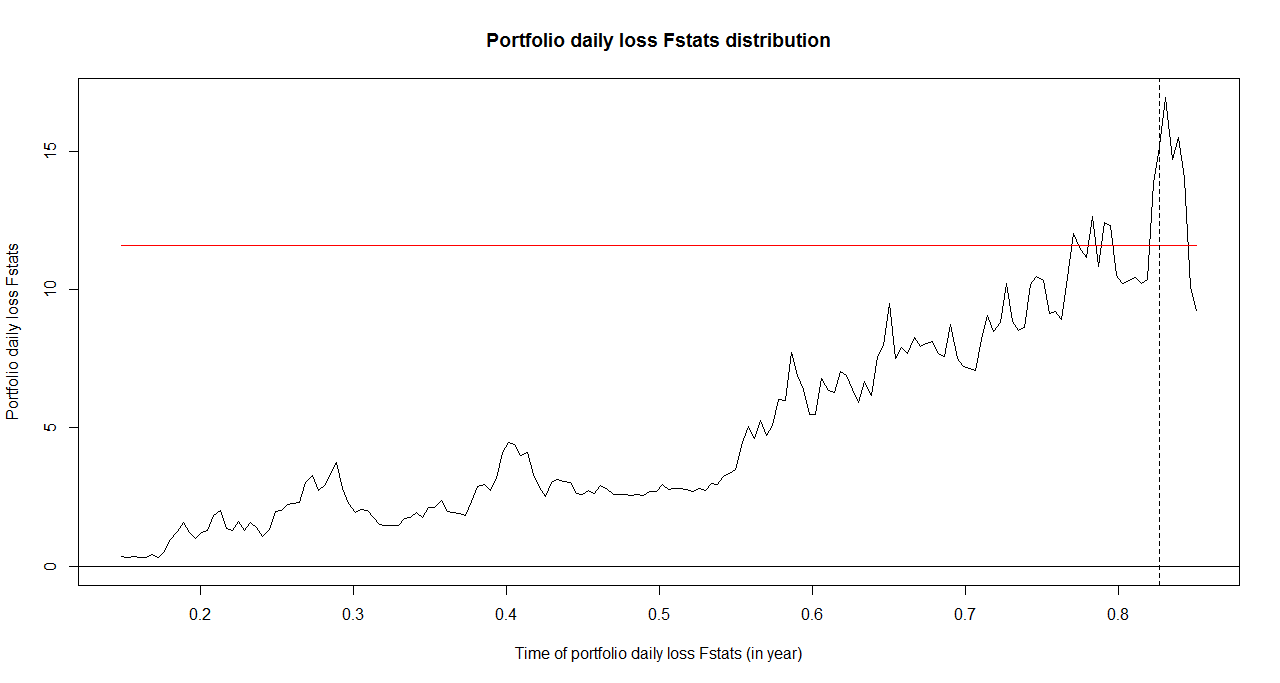
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| Portfolio (GLM) | -0.0232857 | -0.0058188 | 0.0000028 | 0.0057987 | 0.0289475 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | 8.074e-02 | 7.737e-02 | 1.043 | 0.298 |  |  |
| date\_dailyloss\_agh\_sd | 6.085e-06 | 5.362e-06 | -1.135 | 0.258 |  |  |
| Dispersion parameter for gaussian family | 7.762121e-05 |  |  |  |  |  |
| Null deviance | 0.019272 on 248 degrees of freedom |  |  |  |  |  |
| Residual deviance | 0.019172 on 247 degrees of freedom |  |  |  |  |  |
| AIC | -1645.8 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -0.008158 | -0.007598 | -0.007050 | -0.007057 | -0.006503 | -0.005967 |
| Structural change test: | supF test | Data | sup.F | p-value |  |  |
|  |  | dlagh | 10.8128 | 0.06855 |  |  |

**Table 5.10** Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for

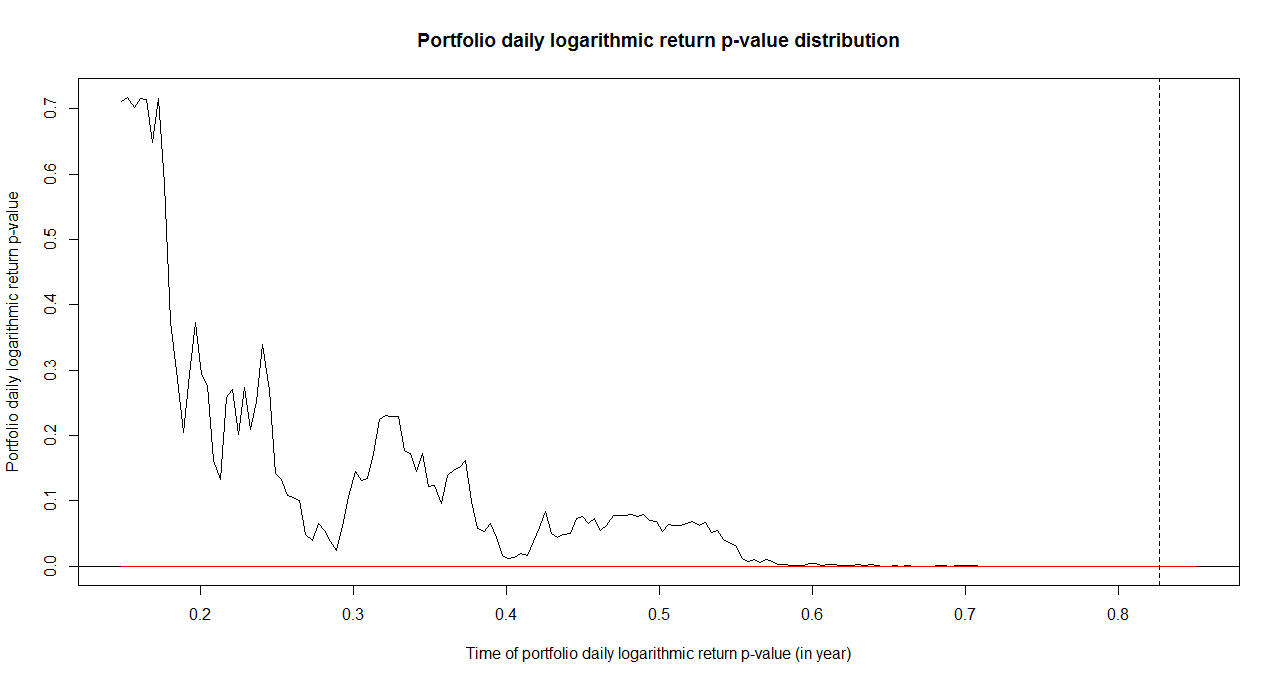
portfolio daily loss distribution.

Formula = daily\_loss\_agh\_sd ~ date\_dailyloss\_agh\_sd is based on the generalized linear model of how the daily portfolio loss changes with date. Table 5.10 shows a very low akaike information criteria giving good value. Fig. 5.12 and 5.13 shows the sup.F and p-value where the structural change occurs.

1. Gaussian Distribution with expected shortfall



**Fig. 5.14** Portfolio daily loss Fstats distribution.



**Fig. 5.15** Portfolio daily loss p-value distribution.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset/Deviance residuals | Min | 1Q | Median | 3Q | Max |  |
| Portfolio (GLM) | -0.029975 | -0.007056 | 0.000547 | 0.007119 | 0.048399 |  |
| Coefficients | Estimate | Std. Error | T value | Pr (> |t|) |  |  |
| (Intercept) | 6.510e-02 | 9.586e-02 | 0.679 | 0.498 |  |  |
| date\_dailyloss\_g\_es | -5.097e-06 | 6.644e-06 | -0.767 | 0.444 |  |  |
| Dispersion parameter for gaussian family | 0.000119154 |  |  |  |  |  |
| Null deviance | 0.029501 on 248 degrees of freedom |  |  |  |  |  |
| Residual deviance | 0.029431 on 247 degrees of freedom |  |  |  |  |  |
| AIC | -1539.1 |  |  |  |  |  |
| Number of Fisher Scoring iterations | 2 |  |  |  |  |  |
| Fitted parameters | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|  | -0.009372 | -0.008903 | -0.008444 | -0.008449 | -0.007985 | -0.007537 |
| Structural change test: | supF test | Data | sup.F | p-value |  |  |
|  |  | dlg | 16.9431 | 0.004807 |  |  |

**Table 5.11**Generalized Linear Model (GLM) calibrated parameters, fitted parameters and supF test for

portfolio daily loss distribution.

Formula = daily\_loss\_g\_es ~ date\_dailyloss\_g\_es is based on the generalized linear model of how the daily portfolio loss changes with date. Table 5.11 shows a very low akaike information criteria giving good value. Fig. 5.14 and 5.15 shows the sup.F and p-value where the structural change occurs.

**V.5 BACKTESTING WITH S&P 500 INDEX**

The markowitz optimization performance of the portfolio will be compared with the S&P 500 index tracked by SPDR S&P 500 (SPY) ETF. The S&P 500 index is a capitalization-weighted stock index representing all major industries in the USA which measures performance of the domestic economy through changes in aggregate market value. We want to know whether the portfolio’s performance can beat the S&P 500 index.The in-sample period is from 1 January to 31 December 2008 whereas the out-of-sample period is from 1 January to 31 December 2009. First, the S&P 500 indexdata and multivariate risk increment data from the portfolio in the in-sample period is fitted with GH distribution. Second, the expected return of the S&P 500 index from the calibrated distribution is extracted and becomes the input as the optimization engine for the portfolio to determine the minimum Expected Shortfall and optimal portfolio weights. The expected shortfall of daily losses between optimal portfolio and S&P 500 index is compared.

Expected return, expected shortfall and volatility levels for each portfolio and S&P 500 index will be given. Portfolio weights are provided too.

**V.5.1 In-sample Analysis**

S&P index can be the standardized value for a stock portfolio and behaves like stock prices. Its logarithmic index is assumed to be the risk factor and fitted the logarithmic return with GH distribution. The results of calibration to S&P 500 data is presented to justify the use of GH distribution to fit the index daily logarithmic return.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Asset | λ |  | μ | Σ | Γ | Expected Value |
| SPY (AGH) | 0.2239090449 | 0.2870898254 | 0.0004625918 | 0.0243432147 | -0.0029855295 | -0.002522938 |

**Table5.12**Parameters and expected value of calibrated AGH distribution for S&P 500 daily logarithmic return. AGH refers to Asymmetric Generalized Hyperbolic distribution.

|  |  |  |
| --- | --- | --- |
| Asset | Log-Likelihood | Akaike Information Criteria |
| SPY (AGH) | 598.1572 | -1186.314 |
| SPY (G) | 565.7782 | -1127.556 |

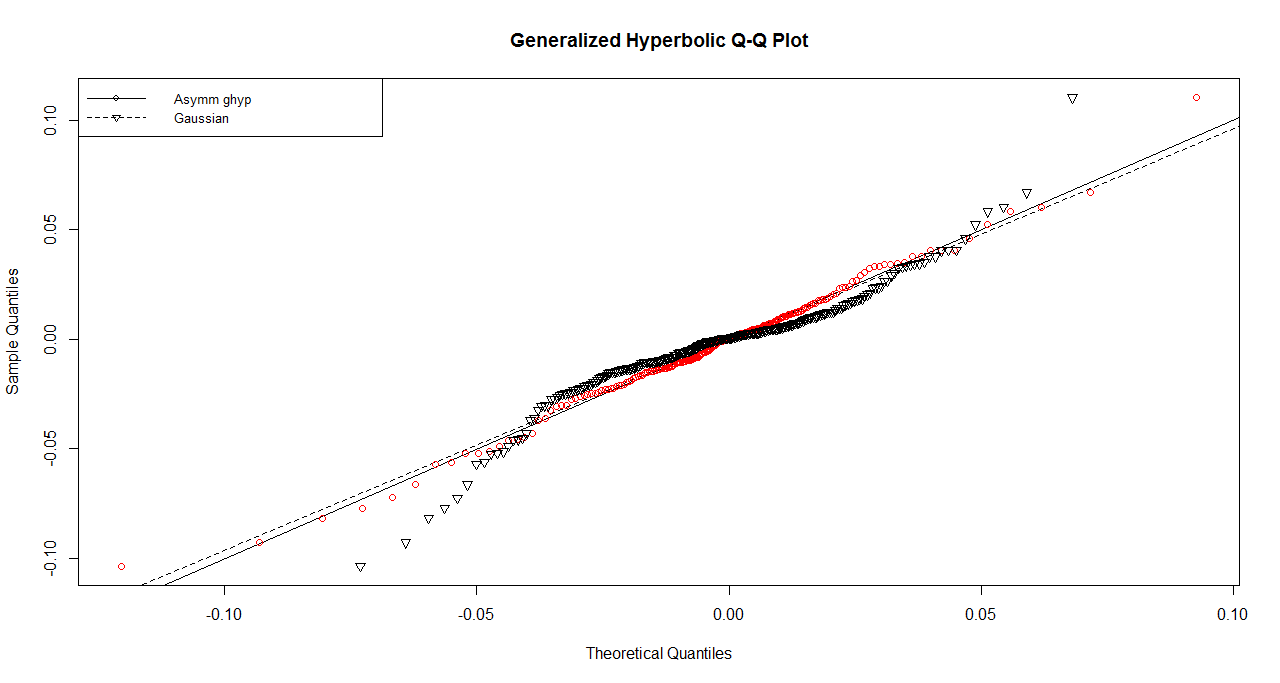
**Table5.13**Statistical test results. AGH refers to Asymmetric Generalized Hyperbolic distribution. G refers to Gaussian distribution.

|  |  |  |
| --- | --- | --- |
| Asset | Standard deviation | ES0.95 |
| Unconstrained (G) | N.A. | 0.0184171 |
| Unconstrained (AGH) | 0.01002933 | N.A. |
| SPY (AGH) | 0.0243432147 | 0.05143167 |

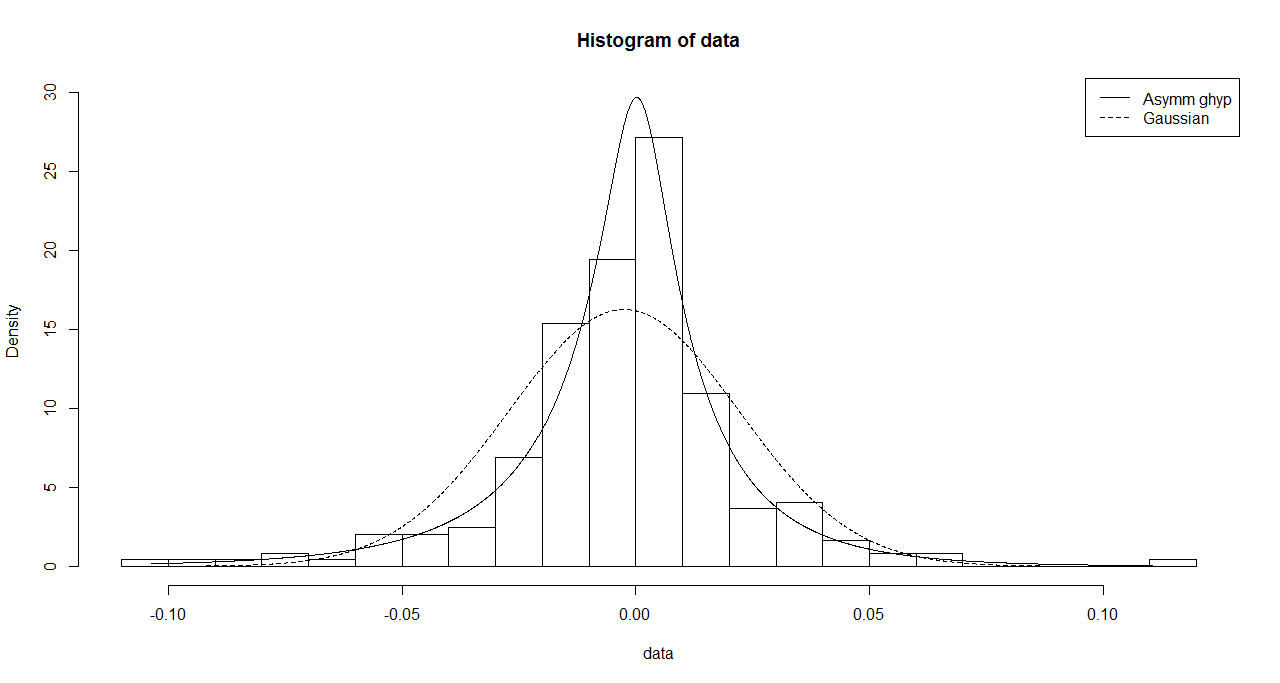
**Table5.14**Comparison between in-sample portfolios at expected return level of -0.002522938 with Expected Shortfall and Standard Deviation as risk measures and S&P 500. G refers to Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Assets | BACreturn | BRKBreturn | GEreturn | KOreturn | LOWreturn | MCDreturn | MMMreturn | RFreturn | TGTreturn | WMTreturn | TBillYield | EURUSDreturn | SPYreturn |
| Unconstrained (G) | -0.022941959 | 0.098709454 | 0.047842160 | 0.146543770 | 0.005662085 | 0.023894159 | 0.192402382 | 0.007517566 | -0.015699166 | -0.089467940 | -0.002439556 | 0.722859093 | -0.114882050 |
| Unconstrained (AGH) | -0.031840964 | 0.105806151 | 0.008156395 | 0.138749176 | -0.017034464 | -0.017089589 | 0.127025190 | 0.012299512 | -0.003733325 | -0.048770753 | -0.002108974 | 0.647939677 | 0.080601970 |

**Table5.15**Percentage optimal weights of portfolio assets at-0.002522938 expected returnlevel of return-expected shortfall and return-standard deviation optimizations. G refers to Gaussian distribution. AGH refers to Asymmetric Generalized Hyperbolic distribution.



**Fig.5.16** QQplot comparison between Gaussian and AGH distribution of S&P 500 daily logarithmic return. AGH refers to Asymmetric Generalized Hyperbolic distribution.



**Fig.5.17** Models for S&P 500 daily logarithmic return.

Table 5.12 provides the parameters of calibrated GH distribution to S&P 500 logarithmic return and its expected value forming the basis of the comparison between the hand-picked portfolio and S&P 500 stock portfolio. Table 5.13 provides the Log-Likelihood and Akaike Information Criteria for S&P 500 with AGHand Gaussian distributions. AGH has better Log-Likelihood and Akaike Information Criteria values than Gaussian distributions.

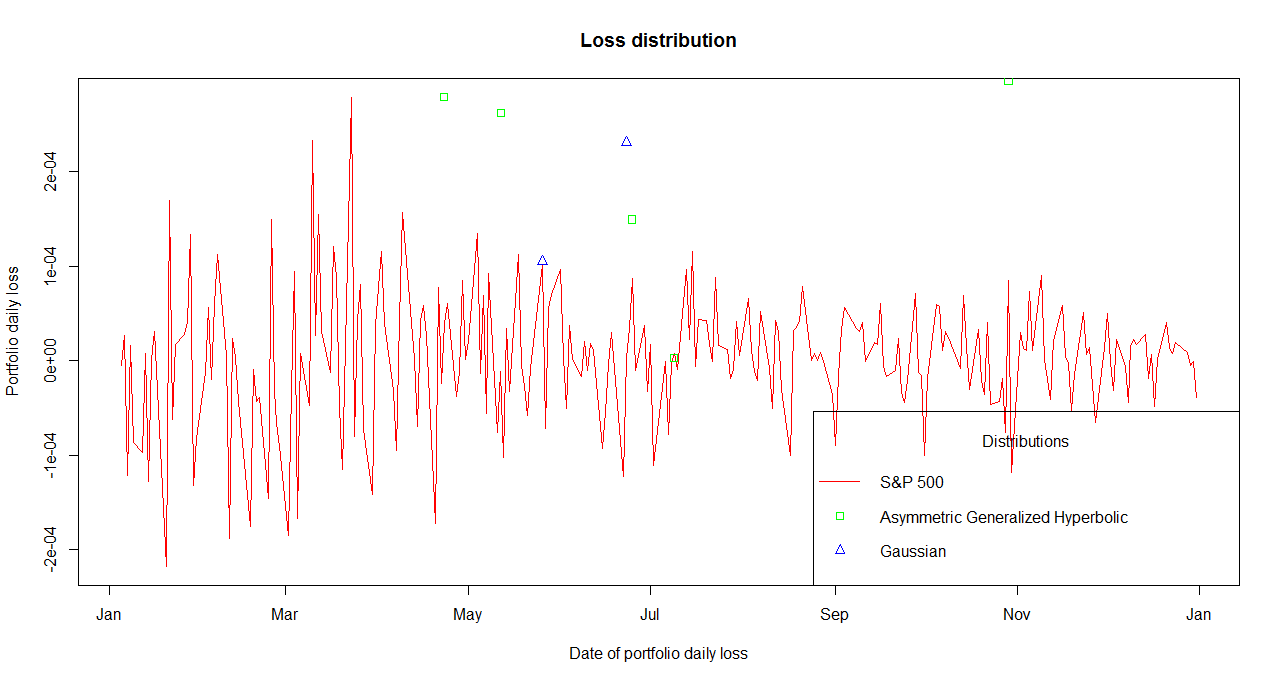
Table 5.14shows the comparison between in-sample portfolios at expected return level of -0.002522938with Expected Shortfall and Standard Deviation as risk measures and S&P 500. Using standard deviation as risk measure, the unconstrained portfolio with AGH has a lower value compared to S&P 500 on AGH. Using expected shortfall as risk measure, the unconstrained portfolio with G has a lower value compared to S&P 500 on AGH. Overall, using standard deviation as risk measure is the lowest.

Table 5.15shows the percentage optimal weights of portfolio assets at -0.002522938expected return level of return-expected shortfall and return-standard deviation optimizations. The unconstrained portfolios on both G and AGH have the highest weightage for EURUSDreturn.

Fig. 5.16 and5.17 give the graphical overviews of fitting results. Normal distribution gives a poor fit because the logarithmic return gives heavier tails. GH distribution gives a much better and good fit to data because it can accommodate heavy tail nature of the data.

**V.5.2 Out-of-sample Analysis**

Using the optimal weights from Table 5.15, the portfolio is fixed from 1 January to 31 December 2009. The daily losses during this period is observed. The daily loss is a fraction of the portfolio value, , at the start of the day to compare with results from Table 5.15.



**Fig.5.18** Daily losses comparison in the out-of-sample period.

Fig.5.18 compares the daily losses between Asymmetric Generalized Hyperbolic and Gaussian portfolios and the S&P 500 index. Both the portfolios exhibit extreme volatility with losses from -0.0002 to 0.0002 whereas the S&P index is mostly between -0.0002 to -0.0001 during the out-of-sample crisis period.

|  |  |  |
| --- | --- | --- |
| Asset | Standard deviation | ES0.95 |
| Unconstrained (G) | N.A. | 0.01106827 |
| SPY (H) | N.A. | -0.0001551778 |
| SPY (GM) | N.A. | -0.0001361373 |
| SPY (M) | N.A. | -0.0001527657 |

**Table5.16**Comparison between out-of-sample portfolios at expected return level of -0.002522938 with Expected Shortfall and Standard Deviation as risk measures and S&P 500. G refers to Gaussian distribution. H refers to historical method for normal Expected Shortfall calculation. M refers to the modified method of Expected Shortfall calculation using Cornish-Fisher estimates if the return series is skewed and/or has excess kurtosis. GM refers to the Gaussian method of Expected Shortfall calculation for Gaussian distribution.

Table 5.16 provides the comparison between out-of-sample portfolios at expected return level of -0.002522938 with Expected Shortfall and Standard Deviation as risk measures and S&P 500. Portfolio optimization with Asymmetric Generalized Hyperbolic distribution using Standard Deviation does not converge. The lowest value of ES0.95 is -0.0001551778 from historical method for normal Expected Shortfall calculation on the SPY which tracks the SPDR S&P 500ETF. Hedging of risks can be done through call and put options to reduce the risk. The S&P 500 is also made up of market capitalizations of 500 large companies with common stock listed on New York Stock Exchange (NYSE) or National Association of Securities Dealers Automated Quotations (NASDAQ) based on a free-float capitalization- weighted index that is fully diversified. The highest value of ES0.95 is 0.01106827 from unconstrained Gaussian distribution. During crisis, the risk is present throughout the entire probability distribution. Expected shortfall only looks at tail risk of the probability distribution. The portfolio chosen is limited in diversification from the asset classes chosen.

**V.6CONCLUDING REMARKS**

This dissertation proves that it is possible to obtain an optimal efficient frontier using generalized hyperbolic distribution during crisis with expected shortfall as risk measure. Multivariate generalized hyperbolic distribution on joint distribution of risk factors from stocks, bonds and currencies as portfolio assets simplifies the calculation of risk factors by allowing them to be linearized. Two approaches to risk measurements during crisis are taken: expected shortfall and standard deviation. Both the appropriate probability distribution and risk measure will determine whether the efficient frontier is optimal or not. Standard deviation considers risk across the whole probability distribution versus expected shortfall which only looks at tail risk of the probability distribution. Thus, during crisis, generalized hyperbolic distribution with standard deviation will produce an efficient frontier. Structural change testing was done to determine the exact crisis time for data used. Backtesting of the Gaussian distribution portfolio using expected shortfall against SPY show that the SPY outperform it because hedging through call and put options help to reduce the risk. The S&P 500 is also made up of market capitalizations of 500 large companies with common stock listed on New York Stock Exchange (NYSE) or National Association of Securities Dealers Automated Quotations (NASDAQ) based on a free-float capitalization-weighted index that is fully diversified whereas the portfolio chosen is limited in diversification from the asset classes chosen.

**V.7LIMITATIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH**

The limitation of the study is the limited or absent data available in the given time period and the constraint of applying it only during crisis periods. Data outside crisis periods should be considered for comparing those during crisis periods. It is highly recommended to use behavioral finance in extension to this research to understand how investors or speculators make their investment and speculative decisions respectively in real-time not after their decisions are carried out. Only static optimization is considered in my dissertation. Dynamic optimization of portfolio should be attempted in future.

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